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THE

ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

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of the Yerkes Observatory

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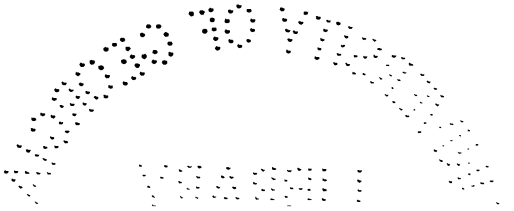

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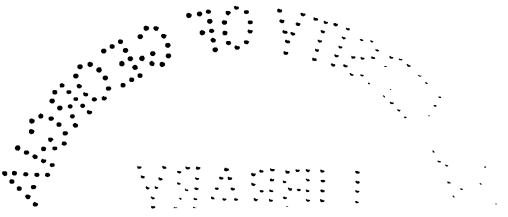

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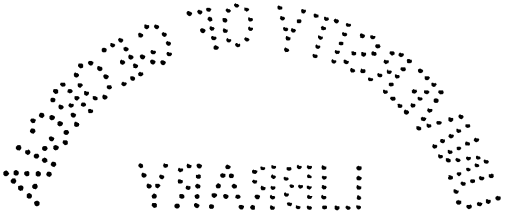

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ON THE OPTICAL CONDITIONS REQUIRED TO SECURE MAXIMUM ACCURACY OF MEASUREMENT IN THE USE OF THE TELESCOPE AND SPECTROSCOPE.

By F. L. O. WADSWORTH.

(*Continued from p. 299, Vol. 16, December 1902.*)

B. Causes of asymmetry in the image of Class B may be subdivided as follows:

B (1). Lack of resolving power due to (*a*) small aperture, (*b*) imperfect definition, or (*c*) imperfect achromatism.

B (2). Errors of adjustment of focus.

B (3). Errors due to an asymmetrical arrangement or use of the instrument itself.

B (4). Errors due to erroneous or imperfect design.

B (1). A frequent cause of error in the measurement of the position of a point or line source is an unsymmetrical broadening of the image by the superposition upon it of another image of a second fainter source too close to the former to be clearly "resolved."

If the two sources vibrate independently and have intensities

i_1, i_2, the distribution in intensity in the superposed images is represented by

$$i_1 \frac{\sin^2 \xi}{\xi^2} + i_2 \frac{\sin^2 (\xi - \kappa)}{(\xi - \kappa)^2} , \qquad (54)$$

κ being the angular interval between the centers of the two sources, and ξ, the angular distance of any point in the image

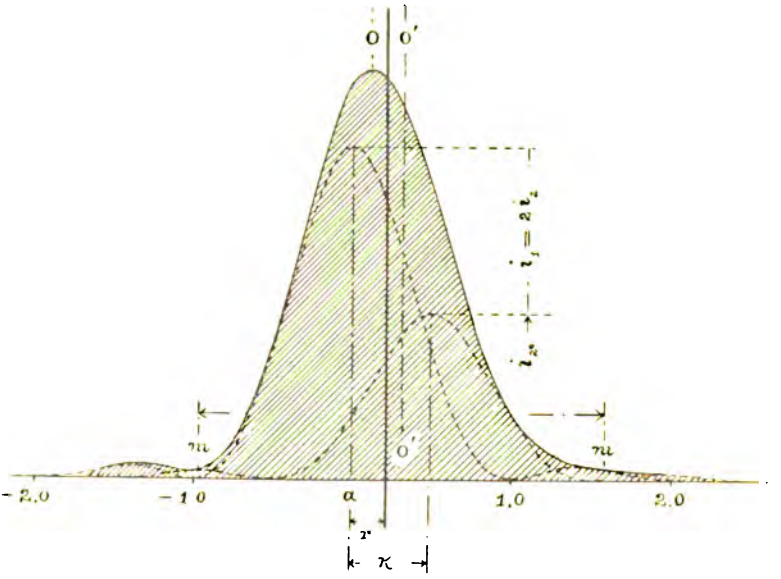


FIG. 9.

from the center of the geometrical image of the principal source S_1. An example of the image of such a double source is shown in Fig. 9. The dotted lines represent the separate diffraction images of the two sources, the full line the resulting focal image of the two together.

In measuring the position of such an image the reference cross-wires are always set too far to the right in the figure, *i. e.*, toward the smaller component, by an amount depending on its intensity in comparison with that of the principal source. Examples of errors of measurement of this kind are found in the determinations of position, proper motion, and parallax of very close double stars.[1] Similar errors in the determination of wave-

[1] Certain interesting cases of this kind which seem to have escaped attention will be considered fully in a subsequent paper. (Note added January 1903.) One of

lengths of spectral lines have been frequently noted.[1] The only way in which such errors can be detected and corrected is by the use of higher resolving power, by which the two components may be separated and measured separately. It appears probable from the work of Michelson[2] and the more recent work of Perot and Fabry,[3] that by the use of sufficiently high resolving powers a large number of the lines which are now regarded as single will be resolved into two or more components, in many cases of very unequal intensity.

In this case the error of setting the cross-wires is a real one, for the reason that we desire to determine in our measurements the position of one of the components of the double source (generally the brighter one) and not the mean position of the two. If we know the separation of the two sources and their relative magnitude, we may indeed correct the results of the measurements of the double source so as to give with a fair degree of approximation the true position of either component. In general the setting of the cross-wire on such an asymmetrical image as is here being considered will be a compromise between the tendency to set on the point of maximum intensity O, Fig. 9, of the compound diffraction image and the point O', midway between those points of equal intensity mm, which are just perceptible to the eye of the observer. In the case represented in Fig. 9, in which $\kappa = \frac{1}{2}a_0$ and $i_1 = 2i_2$, the setting of the cross-wire would be about $r = a + 0.4\kappa$.

these cases was recently investigated by Comstock, whose paper, "The Motion of 85 *Pegasi*," was read on December 30, 1902, at the recent meeting of the Astronomical and Astrophysical Society in Washington.

[1] For example, see Rowland's "Table of Standard Wave-Lengths;" KAYSER and RUNGE, "Ueber die Spectren der Elemente," *Abh. d. K. Akad. d. W.*, Berlin, 1888–1892; HASSELBERG, "Untersuchungen u. d. Spectra der Metalle," *Kongl. Svenska Vet. Akad. Handlingar*, 1894–1898; KAYSER, "Influence of Slit-Width on Comet Spectra," *A. N.*, **3217**; *A. and A.-P.*, **13**, 367; HARTMANN, "Wave-Length of the Nebular Lines," ASTROPHYSICAL JOURNAL, **15**, 292; WRIGHT, *ibid.*, **16**, 57, etc.

[2] "Application of Interference Methods to Spectroscopic Measurements," *Phil. Mag.*, (5) **34**, 280.

[3] *Bull. Astron.*, **16**, 5; ASTROPHYSICAL JOURNAL, **9**, 87; *Jour. Phys.*, (3) 9, 369; ASTROPHYSICAL JOURNAL, **15**, 73, etc.

The corrected reading for S_1 is, therefore,

$$a = r - 0.4\kappa \ , \tag{55}$$

and for S_2 similarly

$$a + \kappa = r + 0.6\kappa \ . \tag{56}$$

B (2). Displacements due to imperfect adjustments for focus.

In considering errors of measurement due to this cause alone we shall at first assume optical conditions to be such that the wave-front which forms the image is symmetrical and concentric about the optical axis on which the image is formed; *i. e.*, that there is no unsymmetrical aberration, either spherical or chromatic, and no eccentricity.

When these conditions are fulfilled a change of focus should result simply in a symmetrical expansion of the diffraction disk or band, which represents the physical image of a point or line.[1] The center of each image will remain fixed on the secondary optical axis joining the center of the geometrical image with the optical center of the lens system. Under such conditions the separation of any two such images at the focal distance z from the optical center of the objective is

$$\xi = 2z \sin \frac{\kappa}{2} \ , \tag{57}$$

where κ, as before, is the angular separation of the two images. Whence for any change in z

$$d\xi = 2 \sin \frac{\kappa}{2} dz = \kappa dz = dz \frac{\xi}{z} \ . \tag{58}$$

This expression is entirely independent of f, the true focal length of the instrument, and hence if z could always be maintained *constant*, dz and, therefore, $d\xi$, would be zero, no matter what the *absolute* value of z might be. This is only possible under absolutely constant temperature conditions. When the temperature changes, z will also change, first, on account of the thermal expansion or contraction of the telescope tube, and, second, on account of the mechanical refocusing which may be necessary in order to compensate for the corresponding tempera-

[1] This is a common way of testing the excellence either of an objective or of atmospheric conditions.

ture change in f. We can best consider these two effects separately.

1. The change in z, due to the thermal change in the length of the telescope tube, for a difference of temperature Δt will be

$$\frac{\delta}{dt} z = \alpha z \Delta t$$
$$= \frac{d\xi}{2 \sin \frac{\kappa}{2}} ,$$

where α is the coefficient of expansion of the material of which the telescope tube is made. But we have also a similar change in the scale of the measuring or recording device at the focal plane, which amounts to

$$- d\xi' = - \alpha' \left(2z \sin \frac{\kappa}{2} \right) \Delta t .$$

The apparent change in the separation of the two images as indicated by the reading of the instrument is therefore

$$d\xi + (- d\xi') = 2z \sin \frac{\kappa}{2} (\alpha - \alpha') \Delta t = \Delta_m \xi . \qquad (59)$$

If the material of the micrometer screw or reference scale is the same as that of which the telescope tube is constructed, then $\alpha' = \alpha$ and $\Delta_m \xi = 0$; *i. e.*, in this case the measured separation of the images at the focal distance, z, remains the same at all temperatures.

If the position of the images is first recorded photographically on a sensitive plate, and afterward measured by a comparator or similar device, we have to consider not only the relative expansions of the glass plate and the telescope tube, but also those of the plate and the scale or screw of the comparator. In this case the total change in the measured distance as indicated by the reading of the comparator at a temperature $T = t_0 + \Delta' t$ is

$$\sum_t^T d\xi = 2z \sin \frac{\kappa}{2} \{ (\alpha - \alpha') \Delta t + (\alpha' - \alpha'') \Delta' t \} . \qquad (60)$$

In order to make (60) vanish we must have

$$(\alpha - \alpha') \Delta t = (\alpha'' - \alpha') \Delta' t .$$

This condition may be satisfied in one of three ways: (a) by making $a'' = a$ and $\Delta' t = \Delta t$; *i. e.*, by making the screw or scale of the comparator of the same material as the telescope tube and measuring the plate at the same temperature at which it was photographed. This plan is generally neither convenient nor practicable; (b) by controlling the temperature, T, of measurement, so that for given values of a, a' and a'' we always fulfill the relation

$$T - t_0 = \frac{a - a'}{a' - a''} = (t - t_0) \ .$$

This is also inconvenient and troublesome, although less so than the preceding; (c) by making $a = a' = a''$; *i. e.*, by constructing both the telescope tube and the comparator scale of glass or platinum. This, while impracticable for large telescopes, is not at all so for small ones, such as are used for spectroscopes.

Instead of attempting to make the correction (60) vanish we may simply determine its amount and apply it to the measurements. In this case we need to determine the degree of accuracy with which the temperatures t_0, t, and T must be known. This is easily done by equating the differential of (60) with the limiting value of ϵ expressed in linear measure, *i. e.*,

$$\epsilon_0 = \epsilon f \cong \epsilon z \ .$$

This gives at once from (60) and (14)

$$\frac{\delta}{dt}\{(a - a')\Delta T + (a' - a'')\Delta' t\}\, dt = \frac{\lambda}{15\, b\kappa} \ . \tag{61}$$

The values of a, a', and a'' for brass, glass, and steel (the materials usually employed) are respectively

$$a \cong 0.000018 \qquad a' = 0.000008 \qquad a'' = 0.000010.$$

In case of the Mills spectrograph the aperture b is about 3.7cm. Hence for an angle $\kappa = 1^{\circ}.5 = 0.027$ [1] we have for (61),

$$dt = \frac{40}{10\frac{\delta}{dt}\Delta t - 2\frac{\delta}{dt}\Delta' t} \ , \tag{62}$$

and if we assume that it is equally easy to measure all three temperatures, t_0, t, and T, (62) indicates that the error (which

[1] See Table II of this paper.

must be taken without regard to sign), in determining any one of them must not exceed 1°.6. It is generally possible to be certain of all these temperatures to this or even a higher degree of accuracy, and it is therefore usually better to observe them and correct the observed readings of the comparator than it is to attempt to eliminate the corrections by any of the methods indicated in the preceding paragraph.

2. *Effect of refocusing:* In large telescopes the alteration of the true focal length, f, for a given change in temperature is so much greater than the corresponding thermal alteration in z alone, that it is necessary to refocus the eyepiece or plate in order to obtain good definition. Such refocusing is also essential, as will be shown later, even in small instruments, if there is any unsymmetrical aberration or eccentricity in the incident wave-front. In such cases we have

$$\begin{aligned} dz &= \frac{\delta}{dt}(z + d\Omega) + d\Omega \\ &= (z + \delta\Omega)\,\alpha\Delta t + d\Omega \\ &= \frac{d\xi}{2\sin\frac{\kappa}{2}} \end{aligned} \tag{63}$$

where $d\Omega$ represents the actual mechanical movement of the eyepiece or plate in refocusing. The change in the separation of the two images at the new focal plane, as indicated by the micrometer screw or comparison scale, now is

$$\Delta\xi = 2\,(z + d\Omega)\sin\frac{\kappa}{2}\left\{(\alpha - \alpha')\,\Delta t + \frac{d\Omega}{z + d\Omega}\right\}, \tag{64}$$

and in the case of comparator readings on photographs

$$\Sigma\xi = 2\,(z + d\Omega)\sin\frac{\kappa}{2}\left\{(\alpha - \alpha')\,\Delta t + \frac{d\Omega}{z + d\Omega} + (\alpha' - \alpha'')\,\Delta' t\right\}. \tag{65}$$

If we assume that the terms of (64) and (65) which represent temperature corrections are computed as before and applied to the measurements, we have left to consider only the effect of the term

$$2d\Omega \cdot \sin\frac{\kappa}{2} = \kappa d\Omega\ .$$

In order that this may be computed also with the same degree of accuracy as already assumed we must have

$$d(d\Omega) \leqq \frac{\epsilon f}{\kappa} = \frac{\lambda}{30\beta\kappa} , \tag{66}$$

β being the semi angular aperture.

The degree of accuracy which it is necessary to observe in refocusing (*i.e.*, in determining the amount of mechanical shift of the eyepiece or plate with reference to the telescope tube) is therefore independent of the size of the instrument and depends only on its form (*i.e.*, its angular aperture), and the angular separation of the two objects measured. We will determine what this is in a few cases of particular interest.

For large visual telescopes and heliometers, $\beta \cong 0.027$. Hence for $\kappa = 0^\circ.35 = 0.0061$,[1] and $\lambda = 0.000056$ cm.,

$$d(d\Omega) \leqq 0.01 \text{ cm} \cong \frac{1}{250} \text{ in.} \tag{67}$$

For photographic telescopes and concave gratings of sufficiently long focus to give full photographic resolution $\beta \cong 1/80 \cong 0.013$. The field κ is, however, correspondingly larger. For Rowland's concave gratings, for example, $\kappa \cong 0.074$ (more than 4°). For such cases therefore

$$d(d\Omega) \leqq 0.002 \text{ cm} , \tag{68}$$

or less than one one-thousandth of an inch.

From the above results, (66), (67), and (68), it would appear necessary that the focusing scales of large telescopes designed for micrometric or heliometric work should be provided with verniers capable of reading to 0.1 mm, while those for the view telescopes of large spectrographs and spectrometers should be capable of reading to at least $1/50$ mm.

The accuracy of focusing demanded in all these cases is considerably higher than is usually considered necessary, or is attained. It is, of course, far in excess of either visual or photographic requirements on the standard of either resolution or general optical definition. But as we have already stated, this

[1] This is the limit imposed by considerations of aberrational distortion in the case of the Lick and Yerkes telescopes (see Table II). This limit has, however, been exceeded in a number of cases of individual measures with these instruments.

standard is an inadequate and unsafe one to apply as a test of the *accuracy* of an optical instrument.

In the case of a telescope used as a collimator the accuracy of focusing necessary depends on the nature and method of use of the instrument.

In general, any lack of exact collimation may be compensated by altering the focus of the observing telescope, and if the change in the latter is not objectionable and everything remains symmetrical with respect to the axis of collimation, no error can be thus introduced. In the case of prism spectroscopes, however, the condition of symmetry with respect to the x axis cannot be fulfilled, and any divergency or convergency in the beam of light coming from the collimator objective will result in the introduction of an unsymmetrical aberration in the wave-front traversing the prism train, the amount of which will depend on the curvature of the incident wave front, the optical perfection of the prisms themselves, and the angles of incidence on the successive faces of the prisms.

The simplest case to consider is that in which the axis of the incident wave-front traverses the train at minimum deviation, the faces of the prisms are optically plane, and the material of which they are composed is of uniform optical density. The amount of longitudinal aberration produced by the passage of a cone of light of semi-angular aperture θ through a single prism under the conditions assumed above has been calculated by Rayleigh, who finds [1]

$$\delta v = 3\theta u \frac{\tan i}{\cos^2 i'} \cdot \frac{(n^2 - 1)}{n^2} , \qquad (69)$$

where i and i' are the angles of incidence on the first and second faces of the prism and u is the radius of the spherical wave-front.

We have also the relation [2]

$$\delta v = \frac{3E}{\beta^2} , \qquad (70)$$

[1] RAYLEIGH, *Phil. Mag.*, (3) 9, 40–49. See also "Theory of the Ocular Spectroscope," ASTROPHYSICAL JOURNAL, 16, 2, July 1902.

[2] RAYLEIGH, *Phil. Mag.*, (3) 8, 411. The result there given is for a symmetrical aberration depending on x^4. For an unsymmetrical aberration depending on x^3 the numerical factor in the numerator is 3, not 4.

where β is the semi-angular aperture of the cone of rays emerging at the second face of the prism. Hence if we denote the aperture of the collimator by b and the length of path of the central ray through the prism by L, we have

$$\theta_1 = \frac{b}{2v} \cong \frac{b}{2(u+nL)} \cong \theta\left(\frac{u}{u+nL}\right) . \tag{71}$$

Also

$$\frac{1}{f} - \frac{1}{f+df} = -\frac{1}{u} , \tag{72}$$

whence

$$u = -\frac{f(f+df)}{df} , \tag{73}$$

and

$$\theta_1 = -\frac{bdf}{2\{f^2 + (f-nL)\,df\}} . \tag{74}$$

In most cases the quantity $(f-nL)$, which represents the difference between the principal focal length of the collimator and the optical path of the central ray through the prism, is so small that the second term of the denominator in (74) is vanishingly small compared to f^2 and may be neglected.

We then obtain at once from (69), (70), (71), and (74)

$$E = \beta^3 \frac{(df)^2}{f} \cdot \frac{\tan i}{\cos^2 i'} \cdot \frac{(n^2-1)}{n^2} , \tag{75}$$

where β as before denotes the semi-angular aperture of the collimator objective.

The limiting value of df will be found by equating (75) and (14), which gives

$$df = \pm \frac{n \cos i'}{4\beta} \sqrt{\frac{f\lambda}{\beta \tan i\,(n^2-1)}} . \tag{76}$$

If we neglect the successive changes of the second order in the value of θ, which are indicated in (71), we have similarly for N prisms

$$E_N = NE ,$$

$$\therefore\ df_N = \frac{df}{\sqrt{N}} . \tag{77}$$

In the case of a spectrometer which has a collimator of focal length $f = 100$cm and aperture $b = 5$ cm, and a prism train consisting of a single 60° prism of light flint of index 1.6 for $\lambda = 0.00005600$,

we have $\beta = 0.025$, $i = 53^\circ\ 7'\ 48''$, and $i' = 30^\circ$. For this case then

$$df = \pm 4.55 \text{ cm} , \tag{78}$$

which shows that under the conditions assumed above a very considerable range is permissible in focusing the collimators of even very large spectrometers.

This conclusion is so at variance with the commonly accepted statement that an accurate focusing of the collimator is very important[1] that it is liable to be misinterpreted unless carefully considered. It is only true when the minimum deviation condition is strictly fulfilled. It is necessary therefore to consider, first, the degree of accuracy with which the various parts of the spectrometer train may be adjusted and maintained in their correct relative positions, and second, the amount by which a given error in adjustment will affect the results given in (75) and (76).

With any given instrument it is always possible by simple hand adjustment alone to bring the prism to a position of minimum deviation with an error not exceeding that which would produce a displacement of the image equal to ϵ, the metrological power of the view telescope. The relation between the angular displacement θ of the prism from the position of minimum deviation and the resulting change in deviation of the central ray has been determined by the writer, who finds for small values of θ,[2]

$$\left.\begin{aligned} \Delta = \Delta_0 + \sin^{-1}\theta^2\left(1 + \frac{\theta^2}{2}\right)\frac{\sin\frac{\phi}{2}(n^2-1)}{n\cos^2\frac{\phi}{2}\sqrt{1 - n^2\sin^2\frac{\phi}{2}}} \\ \text{or}\quad \delta = \Delta - \Delta_0 \cong \theta^2\left[\frac{\sin i}{\cos i\cos^2 i'}\cdot\frac{(n^2-1)}{n^2}\right] \cong C\theta^2 \end{aligned}\right\}, \tag{79}$$

where Δ_0 is the angle of minimum deviation of the ray, and Δ

[1] See for example SCHUSTER, *Enc. Brit.*, article "Spectroscopy;" HARTMANN, ASTROPHYSICAL JOURNAL, 12, 36; FROST, ASTROPHYSICAL JOURNAL, 15, 15, etc.

[2] ASTROPHYSICAL JOURNAL, 2, 280. The expression there given is in error. The term $\sqrt{1 - n^2\sin^2\frac{\phi}{2}}$ should appear only in the denominator and not in the numerator.

the deviation when the prism has been displaced from the correct position for Δ_0 by an angle θ. If, therefore, we put $\delta = \epsilon \cong \frac{1}{16}\frac{\lambda}{b}$ we have for θ

$$\theta = \frac{1}{4} n \cos i \sqrt{\frac{\lambda}{b \tan i (n^2 - 1)}}, \qquad (80)$$

an expression very similar to (76). For the case of the spectrometer above considered

$$\theta = 0.0008 \cong 3' .$$

This is about as accurately as the prism can be maintained at minimum deviation with the ordinary sliding link movement, unless great care is taken in its construction. With such a device therefore it is useless to set the prism with any greater exactness than is indicated above.

If, however, we use a pivoted link minimum deviation device or, better still, a fixed arm prism train,[1] we can control the movement of the prism much more accurately and can then use a more accurate method of initial setting, for example a theodolite, with advantage. We may thus with ease increase both the initial and continuous accuracy of adjustment at least three times, *i. e.*, we can reduce θ to not more than $1'$.

It is next necessary to determine the effect of a given value of θ on the aberration as given by (69) or (75).

The expression for the longitudinal aberration produced by the passage of a cone of light through a prism placed in any position with reference to the axis of the ray can be deduced from the general fundamental equations given by Rayleigh in the paper to which reference has already been made. The expressions thus obtained are, however, complicated and cumbersome to reduce, and for small angles of inclination to the position of minimum deviation the total aberration may be obtained in a more simple way in the following manner:

Let *caa'*, *coo"*, Fig. 10, be the central and edge rays passing through the prism. Owing to the difference θ in the angles of incidence, the total deviation of the two rays in passing through the prism will differ from each other by a small angle $o'a'o'' = f(\theta)$

[1] *Phil. Mag.*, (5) 38. 337, October 1894.

which we will call D. Since the wave-fronts are by supposition very nearly plane, the extreme edge aberration E between the incident wave-front ao and the refracted wave front $a'o''$ will be

$$E = o'o'' \cong ao \cdot D = u\theta D = u\theta f(\theta) \ . \qquad (81)$$

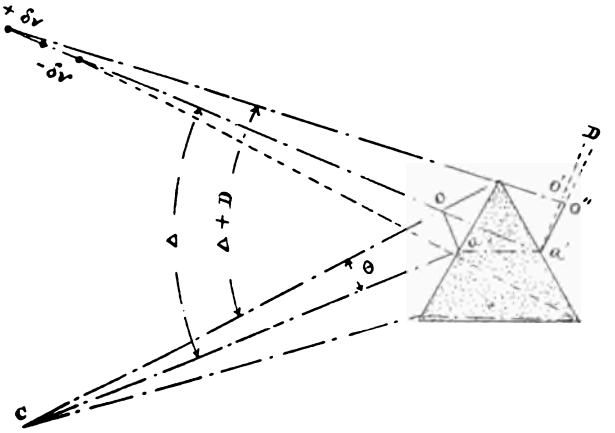


FIG. 10.

When the central ray falls upon the first face of the angle of incidence i for minimum deviation, we have from (79)

$$f(\theta) = \delta = C\theta^2 \ ,$$

and

$$E_o = Cu\theta^3 = \theta^3 u \frac{\tan i}{\cos^2 i'} \cdot \frac{(n^2 - 1)}{n^2} \ , \qquad (82)$$

and from (70)

$$\delta v = 3\theta u \frac{\tan i}{\cos^2 i'} \cdot \frac{(n^2 - 1)}{n^2} \ , \qquad (83)$$

the same expression as obtained by Lord Rayleigh.[1]

The method above indicated of determining the aberration of a prism applies not only to the position of minimum deviation but also to any other position within which equation (79) holds. When the central ray is incident on the prism at an angle

[1] It is perhaps worth while to note that the expression for the longitudinal aberration of a prism deduced by Abbot and Fowle, *Amer. Jour. Sci.*, (4) **2**, 253, and *Annals of the Smithsonian Astrophysical Observatory*, **1**, 78–79, is based upon an erroneous assumption. If equation (12) of their paper were correct there would be no aberration, *i. e.*, the angular deviation of the central and extreme rays would be the same.

other than that for minimum deviation, say at an angle $i \pm \kappa$

$$\Delta_\epsilon = \Delta_0 + f(\kappa)$$
$$= \Delta_0 + C\kappa^2 .$$

And for the extreme ray incident at an angle $i \pm (\kappa + \theta)$ we have similarly

$$\Delta_\epsilon = \Delta_0 + C(\kappa + \theta)^2 .$$

Hence

$$D = C(2\kappa\theta + \theta^2) , \tag{84}$$

and the wave-front aberration E is

$$E_\kappa = Cu\theta^3\left(1 + \frac{2\kappa}{\theta}\right) , \tag{85}$$

$$= \beta^3\frac{(df)^2}{f^2}\cdot\frac{\tan i}{\cos^2 i'}\left(\frac{n^2-1}{n^2}\right)\left(1 - \frac{4\kappa f^2}{bdf}\right) , \tag{86}$$

As before we obtain the limiting values of df for a single prism by equating (86) and (14). This gives us after reduction

$$df = -\frac{\kappa f}{\beta} \mp \frac{n\cos i}{4\beta}\sqrt{\frac{\lambda f}{\beta\tan i(n^2-1)} + \frac{16\kappa^2 f^2}{n^2\cos^2 i'}} , \tag{87}$$

which for large values of κ may be reduced to the form

$$df = \mp\frac{1}{32}\frac{\lambda}{\kappa}\cdot\frac{n^2\cos^2 i'}{\beta^2\tan i(n^2-1)} . \tag{88}$$

When the prism train contains N prisms, each of which is out of the position of minimum deviation by the same amount κ, the total aberration E_N (to the same degree of approximation as before) is N times that for a single prism. If we call the first term under the radical in (87) A and the second term B, we have for N prisms

$$df_N = -\frac{\kappa f}{\beta} \mp \frac{n\cos i}{4\beta}\sqrt{\frac{A}{N} + B} . \tag{89}$$

For $\kappa = 0$ we have as before

$$df_N = -\frac{df}{N} , \tag{90}$$

and for large values of κ

$$df_N = -\frac{df}{N} , \tag{91}$$

i. e., the effect of adding to the number of prisms in the train is

greater as the departure from the position of minimum deviation increases.

The permissible values of df both for a single prism and for a train of three prisms have been computed for the spectrometer already considered (in which $f = 100$ cm, $\beta = 0.025$ and $\mu = 1.6$ for $\lambda = 5600$ tenth-meters) for a series of successively increasing values of κ from $\kappa = 1'$ to $\kappa = 10°$. They are given in columns 2 and 3 of Table III.

TABLE III.

κ	df_κ, $N=1$	df_κ, $N=3$	K	df_K, $N=1$	df_K, $N=3$
	cm	cm		cm	cm
0° 0′ 0″	−4.55	2.62	.....	....	.....
0 1′ 0″	−3.54	1.77	.....	....	.....
0 3′ 0″	−2.25	0.90	.....	....	.....
0° 15′ 0″	−0.57	0.21	.....	....	.....
0 30′ 0″	−0.26	0.12	0° 30′ 0″	0.57	0.21
1 0 0″	−0.15	0.06	1° 0′ 0″	0.26	0.12
1° 30′ 0″	−0.10	0.035	1° 30′ 0″	....	.....
2° 0′ 0″	−0.07	0.023	2° 0′ 0″	0.15	0.06
2 15′ 0″	−0.066	0.022	.....	....	.....
3° 0′ 0″	−0.049	0.016	3° 0′ 0″	0.10	0.035
.....	.....	.....	4° 0′ 0″	0.07	0.023
5° 0′ 0″	−0.029	0.010	.....	....	.....
10° 0′ 0″	−0.016	0.005	10° 0′ 0″	0.03	0.010

As κ increases, the permissible values of df decrease, at first very rapidly and then much more slowly. For large values of κ df must be very small; *i. e.*, the focusing must be very exact, especially with several prisms. This is the reason why Schuster's method of focusing (in which the prism is turned far out of the position of minimum deviation, first on one side and then on the other) gives such accurate results. Thus Table III shows that when the displacement κ is 10°, errors of focusing of only 0.016 cm, or of 1 part in 6,000, become manifest with a single prism. It is probably for this reason also that the statement already alluded to on p. 11 has been so often made.

Further consideration of the values of df for such values of κ as may be likely to occur because of errors of adjustment, shows, however, that so far as this cause is concerned we still have a very considerable latitude in the adjustment of the collimator. Thus if the error of adjustment κ does not exceed 3′, as assumed

on p. 12, the focusing of the 100 cm collimator need not be closer than 2 cm for one prism or 0.9 cm for three prisms. For $\kappa = 1'$, errors of focusing as large as 3.5 cm and 1.75 cm for one and three prisms respectively may be permitted.

In the case of the spectrograph, in which a considerable angular field is under simultaneous observation, it is of course impossible to adjust the prisms to minimum deviation for all parts of the field at once. If we suppose that the prisms are accurately adjusted to minimum deviation for the central image in the field (wave-length λ_0) and that the maximum semi-angular field, corresponding to spectral images of wave-lengths λ and λ', is K, the amount κ by which a single prism is out of minimum deviation for these wave-lengths is

$$\frac{1}{2}(K \pm \delta) = \frac{1}{2}\{K - c(\kappa^2)\}$$
$$\text{and} \quad \kappa \cong \frac{1}{2}K . \tag{92}$$

The required accuracy of focusing is obtained as before from (87). Values of df for $K = 30'$, 1°, 2°, 3°, 4° and 10° ($\kappa = 15'$ to 5°) are given in Table III, column 5.

When the prism train consists of N prisms and the maximum semi-angular field is K_N as before, the amount by which each prism of the train is out of minimum deviation is:

for the first prism, $\kappa_1 \cong \frac{1}{2N} K_N$;

for the second prism, $\kappa_2 = \frac{3}{2N} K_N$;

for the third prism, $\kappa_3 = \frac{5}{2N} K_N$;

and for the Nth prism, $\kappa_N = \frac{2N-1}{2N} K_N$.

The total aberration, E_N, in the train is, under such circumstances [see (85)],

$$E_N = C\alpha\theta^3 \sum_{\kappa_1}^{\kappa_N} \left(1 + \frac{2\kappa}{\theta}\right),$$
$$\cong C\beta^3 \frac{df^2}{f} \left(1 - \frac{4f^2}{bdf} \cdot \frac{K_N}{2}\right) N , \tag{93}$$

and the permissible errors df for the train are therefore the same as for a train of N prisms each of which has an error of adjustment κ equal to

$$\frac{K_N}{2}.$$

The value of df for this case for three prisms and for the same semi-angular fields as before are given in the last column of Table III.

It will be seen that even for semi-angular fields as large as 2° (total 4°) the permissible range in focus of a 1 meter collimator is nearly ¼ mm, a comparatively large quantity compared to that which expresses the permissible range in focus of the view telescope. The statement already made that the accuracy required in focusing the collimator is much less than that required in focusing the view telescope is therefore generally true, not only for single prism spectrometers, but for large multiple prism spectrographs with fields of 3° or 4°.

In this latter case, however, the range of focus is not sufficiently great to permit us to collimate over more than a limited range of wave-lengths or to disregard altogether changes in temperature in the apparatus. In a well-designed objective of the usual two-lens type, of the size and angular aperture assumed above, the change in focus, df_λ, is between $0.0004f$ and $0.0005f$ for a change of wave-length of 600 tenth-meters on each side of the minimum focus. For the three-prism spectrograph having a field of 4° (total) the permissible range of focus, as stated above, is about ¼ mm or $0.00023f$ on each side of the point of exact collimation. By setting the slit at a point about this distance outside the minimum focus we can therefore collimate the rays with the required degree of accuracy over a maximum total range of nearly 1,200 tenth-meters.[1] In this case, in order to *maintain* the accuracy of collimation over the entire range we must refocus the collimator for every change in temperature. On the other hand, if we set the slit at the exact focus of the collimator for any given wave-length and given

[1] This is a little less than the results claimed by Hartmann with the Potsdam three prism spectrograph. ASTROPHYSICAL JOURNAL, 12, 39.

temperature, t, the required accuracy of collimation will be maintained for that particular spectral region over a range of temperature Δt such that

$$\Delta t = \left[\frac{\delta}{dt}(z) - \frac{\delta}{dt}(f)\right] = 0.00023f \ ;$$

or, since $\frac{\delta}{dt}(z) = \alpha z$ and similarly $\frac{\delta}{dt}(f) = \alpha''' f$,

$$\Delta t \cong \frac{0.00023}{\alpha - \alpha'''} \ . \tag{94}$$

For objectives of the usual crown and flint glass the second term of the denominator α''' is about 0.000025 for 1° C. For brass and steel the values of α are about 0.000018 and 0.000010, as already given. Hence we have for Δt_{max}:

$$\Delta t_{max} \cong 15^\circ \text{ C. for steel a tube}$$
$$\Delta t_{max} \cong 30^\circ \text{ C. for a brass tube.}$$

In general the most satisfactory procedure is to limit the range of wave-lengths for which we attempt to secure simultaneous collimation so as to leave a certain range of focus for changes in temperature. If we content ourselves with collimation over a range of 600 tenth-meters (which in the case of most three-prism spectroscopes represents an angular field of about four degrees) we have left to take care of temperature changes a range of focus of about $0.00012f$, which permits a maximum temperature variation

$$\Delta t_{max} \cong 7.5^\circ \quad \text{for a steel tube}$$
$$\Delta T_{max} \cong 15^\circ \quad \text{for a brass tube}$$

without refocusing.

It is very evident from (94) as well as from our previous considerations with reference to the focusing of the view telescope that so far as avoiding the effect of temperature changes is concerned, brass is a much more satisfactory material than steel to use for telescope tubes. Indeed it is possible by using this metal in conjunction with suitable varieties of glass to obtain almost perfect equality between the temperature coefficient of

expansion α' and the corresponding temperature coefficient of focal length α''' so that the range Δt of (94) may be increased and the quantity $d\Omega$ of (63) reduced to zero. Two troublesome adjustments and errors are thus avoided. The more detailed results of this investigation will be given in a subsequent paper.

[*To be concluded.*]

RESEARCHES ON THE ARC-SPECTRA OF THE METALS.

VI. SPECTRUM OF MOLYBDENUM.

By B. HASSELBERG.

(*Concluded from p. 319, Vol. 16, December 1902.*)

RESULTS OF OBSERVATIONS.

AFTER thoroughly taking into account the above comparisons I have collected in the following catalogue the wave-lengths of those lines of the arc-spectrum molybdenum which must be ascribed to this metal as far as may be judged at present. It will not be suprising if there should be among them doubtless a few and perhaps many which further investigation will show to be foreign lines, in view of the richness of the spectrum; nor if the use of stronger currents should make it possible to increase considerably the number of lines characteristic of the metal. The corrections and additions resulting under such circumstances can, however, refer only to the faintest lines, and it may therefore be hoped that this catalogue represents all of importance in respect to the spectrum of the metal.

In regard to the arrangement of the catalogue nothing needs to be added to what I have said in my previous papers. It may only be recalled that as before the wave-lengths given are the means of the results of two wholly independent series of measures, made on different plates, the internal agreement of which may be designated as satisfactory in every respect. The deviations of the values of the two series from each other are as follows:

	Tenth-Meters		Tenth-Meters
In 330 cases	0.00	In 10 cases	0.06
In 462 cases	0.01	In 2 cases	0.07
In 236 cases	0.02	In 3 cases	0.08
In 93 cases	0.03	In 3 cases	0.09
In 45 cases	0.04	In 1 case	0.10
In 16 cases	0.05		

Hence the cases in which this deviation exceed the value of 0.02 constitute only about 14 per cent. of the whole. The probable error of a wave-length therefore would in general not reach the amount of 0.02, an estimate which is shown to be practically correct by other circumstances as will be shown below.

Mo Å	R.	i Mo	i ⊙	Remarks	Mo Å	R.	i Mo	i ⊙	Remarks
3463.78					.....	3532.72			
65.81		1.2			3534.83		1.2		Very diffuse.
66.98		2			37.41		3		*Mo?*
67.13		1.2			39.07		1.2		
68.02		2—			39.62		1		
68.70		1+			42.32		2.3		
69.39		2			42.92		1		
69.80		1			43.27		1+	?	
71.09		1.2			47.57		1+		
.....	3473.43				48.05		1+		
75.19		2			48.88		1		
76.15		2—		76.07 *Cu*	51.12		1.2		
79.60		2			52.57		1	2	*Mo?*
80.26		1.2			.....	3553.62			
81.95		1.2			54.35		2—		
82.55		2			55.58		1.2	2	Coinc. *Mo?*
84.05		2			57.63		1		
90.42		1			58.25		3		At edge of ⊙ line 58.21
.....	3491.46								
91.92		1			59.42		1		
92.05		1+			60.28		2		At edge of ⊙ line 60.28
92.98		1							
93.49		2—			62.26		2—	1	
3498.21		1+			63.30		2.3	1	
3504.55		2.3	2	Also *Fe* 04.56; *V* 3504.57	63.91		2—		
05.45		2+			64.45		1.2		
07.16		1			66.20		2+	?	
07.45		1			66.57		1		
08.26		2.3			66.91		1		
10.93		1.2			70.63		1.2		
.....	3512.78				70.82		2.3		
13.86		1.2			71.42		1.2		
14.93		1+			.....	3573.54			
17.70		2			74.05		2.3	2.3	⊙ has 74.05? 73.97 *Fe*
18.35		2							
21.17		1.2			74.63		1.2		
			1		75.78		1.2		
21.32		1.2			75.88		1.2		
21.56		2.3			76.35		1.2		
22.52		1.2			80.70		2	?	
24.76		2+			81.15		1		
25.11		2+			82.03		3.4		
26.08		1+	?		83.30		4+		
3531.44		1+			3584.42		1.2		

Mo λ	R.	Mo i	⊙	Remarks	Mo λ	R.	Mo i	⊙	Remarks
3585.74		1+			3635.57		2.3		At edge of ⊙ line 35.61 *Ti, Fe*
87.02		2							
89.10		2		At edge of ⊙ line 89.05	35.77		1.2		
90.47		1			.....	3637.40			
90.90		2+			37.08		2	?	⊙ has 37.69. Coinc?
91.55		1.2							
94.73		1			38.35		2.3		
.....	3595.45				38.57		1.2		Also in *V*. Foreign line?
95.71		2—							
95.87		2—			38.72		1.2		
96.54		1—			40.76		2+		
3599.05		2			41.08				
3600.04		1.2			41.16		2		
03.10		2.3			42.37		2—		
03.86		1.2		Between the ⊙ lines 03.92 *Ti* .83 *Cr*	47.03		1+		
					48.75		2		
					49.61		1.2		At edge of ⊙ line 49.65 (*Fe*)
04.24		2—							
04.73		1.2			50.75		1	?	
05.19		1+	?		51.48		2		
07.56		1			53.75		1		
08.52		2+			54.73		2	?	Diffuse. *Mo?* ⊙ has 54.81 *Fe* .74 *Ti* 54.6 *Cu*. Very diffuse
10.80		1—							
12.15		2—							
12.62		2—							
13.55		2—			55.21		1.2		Diffuse. *Mo?*
13.80		1.2		Overlie the diffuse *Cu* band 13.86	57.53		2.3		⊙ has 57.56 *Fe*
13.94		1.2			58.50		1		
					.....	3658.09			
					59.51		3.4		
.....	3614.26				61.08		2—		
14.42		3			61.24		1.2		
14.87		1			61.91		2+	?	
15.32		1.2		⊙ has 15.34 (*Fe*). No coinc.	63.14		2	?	
					63.83		1+	?	
					64.45		2—		
15.91		1.2			64.98		3	1	
17.01		2— 1—			66.87		2+		⊙ 66.91 *Fe*
23.36		2+ 2		At red edge of the *Fe* line 23.34. R gives 23.36	68.03		1.2	?	
					69.50		2.3	1	
					72.07		3		
					73.38		1.2		
24.60		3			75.54		2		
24.77		1.2			76.15		1.2		*Fe* has 76.11 separated
26.33		2 3 1+		Also *Fe*					
28.50		1+			76.40		2+		
28.80		1.2			.....	3676.70			
29.45		2.3			77.83		2—		⊙ has 77.83– ? .76 *Fe*
3035.30		2 2		*Mo?* But separated from the ⊙ line 35.34 *Ti, Fe*	79.30		1.2		
					80.36		1+		
					80.75				On each side of ⊙ line 80.80
					3680.85		3.4		

Mo λ	R.	Mo i	⊙	Remarks	Mo λ	R.	Mo i	⊙	Remarks
3681.69		1.2			3728.70		2		
81.88		2—			30.75		1		
82.12		1			32.91		3	2	Double. Center
84.48		1.2			33.22		2—		
86.27		2—			33.59		1+		
86.72		2—			34.56		1.2		
87.12		1			35.80		1.2	1—	Coinc?
88.12		1			36.36		1+		
88.45		2—			38.10		2+		
89.13		2			.....	3740.20			
90.30		1			40.97		1+		
90.72		2.3			42.48		2.3		
.....	3691.45				43.98		1+		
92.24		1.2	?		44.55		2—		
92.79		2—	2	*Mo?* ⊙92.79 *Fe*	45.12		1.2		
					47.37		2—	1	
93.52		2—			48.66		2+		
95.09		3.4			51.38		2+		
96.18		1.2	1	*Mo?* 96.17 *Fe*	52.12		1—		
3698.69		1.2			55.31		2		
3700.15		1.2			55.68		1		
01.67		1			56.02		1.2		
02.33		1+			58.70		2+		
02.67		2		By the *Fe* line 02.63	59.80		1.2	1	
.....	3704.60				.....	3760.68			
					61.07		2—		
05.57		1+			61.93		2—		
07.35		1.2			62.27		1.2		
08.73		2—		At edge of ⊙ line 08.79	63.52		2		Also *Mn.* (63.51) Coinc?
10.32		1.2			64.20		1		
11.68		1.2			64.60		1.2		
12.22		1.2	?	Coinc.?	65.21		1+		
13.64		2—			65.40		2—		
14.73		1.2			65.92		1.2		
15.83		2			66.58		1		
16.27		2	1		67.90		1+		
17.05		1+			68.78		1+		
18.66		1+			68.92		1.2		
19.71		1+			70.66		2.3		
19.87		1+			72.11		2+		
20.42		2—			72.99		2+		
22.50		1—			76.27		1+		
23.70		1.2			76.73		1+	1.2	
24.00		1+			77.90		1.2		
.....	3724.53				.....	3778.20			
25.75		2+			79.92		2+	?	⊙ has a faint pair. Coinc?
26.45		2		Dbl. Center. $\Delta\lambda$=0.09	80.78		1—		
27.86		3		R. gives ⊙ 27.78 (*Fe*). Wholly separated	81.75		2.3	2	
					82.35		1.2		
					82.86		1		
					85.19		2		
3728.50		1.2			3785.67		1.2		

Mo λ	R.	*Mo* i	⊙	Remarks	*Mo* λ	R.	*Mo* i	⊙	Remarks
3786.54		1			3831.25		1.2		
88.42		2			31.95		1+		
94.60		2—			32.26		2		
95.48		1+			33.92		3		
96.19		1.2			34.82		1.2		
96.45		1+			35.15		1.2		
97.20		1.2			35.49		2—	1.2	Coinc. ⊙ line diffuse
97.46		2—							
3798.39		10	1	Intense, broad, diffuse, reversed, with a fine absorption line; coincides with the ⊙ line	39.65		1+		
					40.72		1+		
					.....	3843.40			
					44.09		1.2		
					46.12		2+	2	Coinc.?
					46.36		1.2		
					47.41		2+	2	Faint ⊙ group. Coinc.?
3800.28		1+							
.....	3801.82				48.45		2+	2	Coinc.? R. gives ⊙ 48.43.— 48.48 (*Ti*) separated from *Mo*
02.00		2.3		⊙01.98. Coinc?					
02.35		1+							
04.70		2							
06.15		2							
07.82		1			49.95		1+		
08.04		1			51.57		1+		
08.79		1.2			52.17		2—		
10.31		1+			55.09		1		
10.99		1			56.15		1+	1.2	*Mo?*
11.56		1.2		Sharp	.....	3863.53			
12.63		2		Sharp. Faint companion to violet	3864.25		10	1	Broad, diffuse, rever'd, with fine absorption line. R. gives 64.25 *Mo—C*
14.64		1+							
15.24		1.2							
17.37		1—							
18.83		2—							
19.98		2.3			66.87		1+		
21.09		1.2			69.25		2.3	?	⊙ has a group here
21.82		1							
22.14		1			70.62		1.2		
23.17		2+		⊙ has a faint band here	70.77		1.2		
					73.30		1.2		Weak. At edge of ⊙ line 73.25 (*Co*)
.....	3823.65								
24.34		1.2							
24.94		1.2			74.34		1.2	?	Coinc.? 74.32 (*Ti*). Intensity variable? Foreign line?
25.50		1.2							
25.63		1+							
26.85		2.3							
27.33		2+							
29.04		3—			79.20		1+		
29.95		1.2			.....	3885.66			
30.08		1+			86.98		2.3		⊙ has 86.94 (*Cr*), separated
30.22		2—							
3830.98		2—		Between ⊙ lines	87.87		1		
				31.00 } *Fe*	3888.15		1—		
				30.90 }					

Mo λ	R.	i Mo	⊙	Remarks	Mo λ	R.	i Mo	⊙	Remarks
3888.36		2—			3938.88		1.2		
89.06		2—	?	Coinc ?	39.30		1+		
90.88		1.2			39.65		1+		
93.50		1+		Between the ⊙ lines 93.54—*Fe* .45 *Co?*	40.50		1		
					.....	3941.02			
					43.19		3		
					43.66		2—	1+	*Al.* 44.10 distinctly fainter than 61.57
96.55		1.2							
97.05		1.2							
3897.68		1+			45.41		2—		⊙ has 45.47 (*Co*) separated from *Mo*
3900.40		1—							
00.87		1							
01.95		2.3							
.....	3902.40				47.00		1		
03.07		10	5	Extremely strong, diffused, reversed with fine absorp'n line	47.33		2—		
					50.40		1		
					51.14		2—		
					51.49		1+		
					51.70		1		
07.10		2—	1	*Mo?*	54.08		2		
08.42		1.2	1.2	*Mo?*	55.66		2—		
09.92		1.2			58.76		2—		
11.24		1.2		Diffuse	59.03		1		
12.10		1.2	1.2	*Mo?* 12.13 *Cr?*	59.83		1+		
					60.12		1+		*Al.* 61.57 strong
13.52		1.2							
15.60		1			.....	3960.42			
16.62		1+			63.68		1.2		
17.09		2—			64.14		2—		
17.70		2+	?	Coinc ?	65.89		1.2		
17.95		2			66.40		1.2		
20.25		1—			68.91		2—		
21.09		1+			69.17		1		
22.49		2—			71.54		1.2		
23.91		2			73.10		1+	?	
.....	3924.67				73.92		2+		
24.78		1			74.09		2		
26.00		1			78.08		2—		
28.45		1.2	?	Coinc.?	79.40		2—		
28.86		1.2			80.37		2		
28.95		1.2			80.87		1.2		
30.35		1.2			81.80		1.2		
31.57		1.2			.....	3981.92			
34.41		1.2		The *Ca* lines H and K occur with slight intensity	82.22		2		
					84.92		1+		
					85.88		1.2		
					86.45		2+	1	
					91.55		2—	1	
35.13		2		Centr. of close double	92.02		2—		
					93.22		1.2	1.2	Coinc. *Mo?*
35.33		1.2			94.06		2—	?	Perhaps Coinc.
36.30		1			94.79		1		
3936.89		1+		Also *Mn* Foreign line ?	95.66		1+		Weak
					3998.45		2—		

Mo λ	R.	i Mo	⊙	Remarks
				⊙ line 00.61 (*Fe*) between these two *Mo* lines
4000.55		2	−	
00.67		2		
03.62		1	+	
.....	4003.91			
05.86		1	2	Coinc. *Mo?* Also *V*. Other strong *V* lines are lacking
06.23		2		
06.85		1		
07.62		1		
08.21		1	+	
09.53		2		
12.12		2	−	Diffuse. *Mo?*
12.42		1		
12.68		1		
12.97		1	−	
16.86		1	+	
17.55		1.2		
.....	4018.42			
19.32		1		
20.59		1.2		⊙ has 20.64 − *Fe* .55 *Sc*
21.19		2		
25.64		1.2		
27.07		1	+	
28.80		1.2		
31.06		1		
31.60		1		
32.65		1.2		At red edge of ⊙ line 32.61 (*Fe* by R.; *V* by H.)
34.11		1	1 −	
36.83		1	+	
37.95		2	−	
38.20		2	1 −	
.....	4040.79			
41.30		1.2		
43.05		2		
43.44		1		
43.91		1.2		
47.07		1		
47.56		1	−	
47.75		1	−	
49.78		1		
50.27		1	+	
4051.35		1	+	

Mo λ	R.	i Mo	⊙	Remarks
4056.18		2		56.22 *Cr* 56.13 *Fe*
57.61		1	+	
57.77		1.2		Also *Ti* (57.76)
.....	4059.08			
59.79		2	−	
62.24		2.3		
66.52		2	− 2	Coinc. *Mo?* 66.52 *Co*
67.88		1	+	
70.05		3	+	Hard to separate. Total intensity = 4
70.17		2		
75.43		2	1	
75.72		2	−	
76.35		2		
76.69		1.2		Between the ⊙ lines 76.80 .64
78.25		1	+	
.....	4080.00			
81.02		3		
81.94		2	−	
84.54		3	?	Perhaps Coinc
86.16		2		At red edge of ⊙ line 86.13
89.90		1.2		
93.32		1	+	
94.63		1		
96.98		2	+	
4098.91		2	+	
4102.33		2.3	1	
03.94		1.2		
.....	4104.29			
05.27		2	+	At edge of ⊙ line 05.31 *V*
05.72		2	−	
07.03		3	3	⊙ has 07.65 *Ce*, *Fe*, *Zr* separated from *Mo*. *V* has 07.64
08.30		1.2	1.2	R. gives ⊙ 08.29. *Mo?*
10.46		1		
10.88		1	+	
12.29		1		
13.77		1	+	
15.08		2	− 2	*Mo?*
19.12		2		At edge of ⊙ line 19.05 (*Fe*)
4119.81		1	+ 1	

Mo λ	R.	i Mo ⊙	Remarks
4120.26		3	
22.55		1	
23.83		2	
24.72		2	
.....	4126.34		
28.46		2—	
29.02		2—	
32.07		2—	
32.41		2—	Intensity variable. Is it *Mo?*
32.90		1	
33.18		1+	
35.37		1	
35.55		1+	
37.10		1+	
38.35		1.2	
38.72		1.2	
39.72		1+	
42.28		1+	
43.73		4	
.....	4147.84		
48.88		1+	
49.14		2.3	
49.90		1	
52.07		2—	At edge of ⊙ line 52.11
55.47		2.3	
55.77		2.3	
57.59		2.3	
58.27		1+	Overlies a diffuse band at 58.10. *Cu?*
60.44		1+	
62.85		2.3	Does not coincide with ⊙ 62.83
64.26		1.2	
65.94		1	
66.47		1.2	
68.68		1.2	
70.01		2—	⊙ has 69.93 (*Fe?*)
70.55		1	
.....	4171.07		
71.27		1.2	At edge of ⊙ line 71.21 (*Ti*)
71.65		1+	
75.32		1 1.2	
77.00		1.2.1+	
77.45		2	
78.45		2	
4178.72		1	
4180.12		1.2	
80.69		1.2	
81.24		2	Center of close double
84.33		1	
84.59		1..	
85.98		3 1	
86.97		2— 1+	
88.49		4	
.....	4192.73		
94.20		1	
94.74		2.3	
4199.82		1	
4200.02		1	
00.76		1.2 1.2	
01.35		1+	
01.50		1.2	
02.42		1	
04.80		1.2	
06.00		2	
07.42		1+	
07.75		1+	
08.97		1	
09.84		1 1	
10.39		1	
11.23		2—	
.....	4213.81		
14.24		1.2	
17.02		1	
19.20		1+	
19.55		2	Appears to lie between the ⊙ lines 19.52 and 19.58. These lines belong to *Fe*
20.17		1	
22.59		1.2	
23.15		1.2	
24.10		1	
24.93		1	
25.10		1	
26.44		1.2	
32.75		3+	
33.68		1.2	
35.23		1.2	
....	4236.11		
39.25		2—	
39.37		2—	
40.26		2	Also *V*
40.48		2	
41.03		2.3	
4242.97		1.2	

Mo λ	R.	i Mo	i ⊙	Remarks	Mo λ	R.	i Mo	i ⊙	Remarks
4244.95		1.2		Diffuse. Double?	4308.85		1		
					10.58		2−	2.3	⊙ has 10.63 .54
46.19		2.3		At edge of ⊙ line 46.25 (*Fe*). K. R. give 46.20	12.98		1.2		
					13.16		1.2		
					13.74		1+		
					15.60		1		
					18.13		2.3		
50.87		1.2			18.46		1		
51.58		1			.....	4318.82			
52.03		2.3			22.17		2−		
52.69		1+			22.66		1+		
53.77		1+			24.72		1		
.....	4257.82				25.44		1		
58.85		1+		Between two ⊙ lines	26.33		3		
					29.50		1+		
60.52		1.2			29.82		1.2		
60.85		1.2		⊙ has 60.89. Separated	30.27		1+		
					32.68		1+		
61.17		1			33.40		1		
61.63		1.2			34.65		1		
64.81		1+			35.00		2	1	⊙ has a faint triple; coincidence is with the middle component
66.27		2−							
68.25		2−	2	Coinc. *Mo?*					
69.44		2.3	1						
72.24		1.2							
73.23		1.2		Sharp					
74.22		1+			36.38		1+		
.....	4274.96				38.73		1		
75.86		1+	?		38.90		2−		
77.08		3			39.42		1+		
77.38		3	?	R. gives ⊙ 77.38.	40.02		1.2		
					40.93		2−		
77.58		2	?	R. gives ⊙ 77.54 *Zr.* Coinc?	41.61		2+		
					42.16		1		
					.....	4344.67			
79.19		1+			44.86		1.2		
80.17		1+	1.2	Coinc. *Mo?*	46.40		1+		
82.00		2−			49.41		1−		
84.77		3		Diffuse. *Mo?*	50.53		3−		
87.26		2−			53.48		2−		
88.82		3+			54.88		1−		
89.56		2+		At edge of ⊙ line 89.50 (*Ca*)	57.50		1.2		
					62.20		1.2		
					62.87		1+	1.2	Coinc. *Mo?*
91.39		2−			63.21		1−		
92.34		3			63.82		1.2		
93.42		3			64.65		1.2		
94.07		3			64.76		1+		
.....	4295.91				64.00		1+		
4296.35		1.2			66.73		2		
4301.45		1.2			.....	4368.07			
04.20		1.2			69.23		2.3		
4305.10		2−			4370.33		1		

Mo λ	R.	*i* *Mo* ⊙	Remarks	*Mo* λ	R.	*i* *Mo* ⊙	Remarks
4372.31		1+		4449.92		3	
73.52		1+		52.77		1.2	
75.07		1+		.....	4454.95		
75.21		1.2		57.55		3.4	
76.87		1+		58.84		1.2	
80.47		2+		60.80		2—	
80.80		1.2	Sharp	64.96		3—	⊙ has 64.94 *Fe* separated
81.36		1+					
81.82		4		68.28		1+	
82.61		2—		68.46		3	
86.10		1+		71.85		1.2	
.....	4388.06			72.23		1.2	
88.49		1		73.37		2.3	
89.76		1		74.78		4	
91.71		1.2		75.82		2+	
92.32		1.2		.....	4476.22*		
94.49		1.2		85.16		2.3	
94.67		1.2		87.23		2+	
96.55		1		89.17		1.2	
96.83		2	Sharp	90.37		2	
97.02		1		91.46		3	
97.48		2+ ?	Perhaps coinc.	92.00		1	
98.68		1		92.24		1	
4402.67		2—		94.27		1+	
03.07		2+		4499.62		2—	
04.71		1.2	Sharp	4501.44		2	⊙ 01.42 *Ti* separated
07.04		1+					
09.61		1		06.13		{ 3	
10.15		2— 1	Coinc.?	06.22			
.....	4410.68			06.86		2	
11.76		2.3		.....	4508.45		
11.90		3		12.32		2.3	
12.96		2+		15.20		1.2	
17.40		1		15.36		2— 1	Coinc.?
20.91		1		17.30		3+ 1.2	Coinc. 17.28 *Co*
22.23		1.2					
23.24		1+	A line here also in *Ni*. Not given by R.	17.58		2	
				18.61		1+	
				22.37		2+	
23.79		2.3		24.53		3	
24.40		1		25.50		1—	
26.86		2.3		25.56		2+	
28.39		1		28.77		2.3 4	Diffuse. Coinc. *Mo?* 28.80 *Fe*
29.32		1					
.....	4433.39			29.59		2.3	Sharp
33.68		1.2	Sharp	.....	4533.42		
37.06		2—		34.63		2	Diffuse. *Mo?*
37.35		1+		35.00		2+	Diffuse; broad. *Mo?*
39.15		1+	Sharp				
42.37		2.3	Sharp	35.56		2 ?	Probably a ⊙ line
43.25		2	Sharp				
44.21		1+	Diffuse. *Mo?*	37.00		3+	
46.62		2—	Sharp	38.60		1	
4447.41		1.2		4539.84		1	

*Double 76.253, 76.185. Center.

Mo λ	R.	i Mo	i ⊙	Remarks
4541.75		2	—	
53.00		1		
53.40		1.2		
53.52		1.2		
54.00		2	3.4	*Mo?*
.....	4554.21			
58.30		2.3	1	Coinc.
58.92		1.2		
59.94		1+		
60.32		2+		Sharp. At edge of ⊙ line 60.27 (*Fe*). K. R.: 60.33
67.57		1	—	
67.87		2+		
69.21		1+		
70.30		2		Sharp
70.78		1+		Sharp
74.66		1+		
74.80		1		
75.36		1	—	
76.05		1+		
76.70		3	—	Sharp
77.97		1+		Sharp
78.06		1		
.....	4578.73			
79.02		1		
82.52		1+		
82.69		1+		
86.25		1+		
86.75		1+		
86.98		1+		
87.61		1		
88.33		1.2		Diffuse. *Mo?*
90.55		2		Diffuse. *Mo?*
92.40		1.2		Sharp
93.84		1		
95.35		3		
98.07		1+	1.2	Coinc. *Mo?*
98.44		1+		
4599.35		2	— ?	Probably a ⊙ line
.....	4603.13			
4603.78		1	—	
08.32		1		
08.90		1		Sharp
10.07		3	1	Coinc.
11.03		1+		
11.36		2	—	Sharp
14.94		1		
16.81		1+	1.2	Probably not *Mo*
17.82		1		
18.15		1+		Sharp
21.57		2.3		Sharp
4623.66		1.2		Sharp
4624.44		2		Diffuse. *Mo?*
.....	4625.23			
26.67		3.4		26.74 *Mn* separated. *I* 26.67 similarly
27.70		2.3	1	⊙ line extremely faint. Coinc.?
30.20		2	—	Sharp
32.75		1		
35.22		1		
41.12		1	—	
41.78		1	—	
42.90		1.2		
.....	4646.35			
48.02		2		Sharp
49.28		1.2		
51.25		2	—	
52.47		2		Diffuse, probably not *Mo*
56.57		1		
57.67		1		
62.11		2.3		Does not coincide with ⊙ 62.15
62.95		3	1	
63.31		1+		
65.59		1+		
.....	4667.77			
69.00		1+		
72.11		3	—	
73.24		1		
75.91		1	—	
.....	4679.03			
81.24		1+		
81.82		1		
82.44		1	—	
84.04		1.2		
84.54		1+		
86.01		2	—	
86.28		2	—	
88.41		2.3		Separated from 88.46 (*Fe*). R. gives 88.36 (*Fe*)
91.05		2	—	
92.10		1+		
92.89		1	—	
93.55		1	—	
96.06		1+		
4696.71		1.2		
.....	4700.34			
4700.71		2	—	

Mo λ	R.	i Mo	i ⊙	Remarks
4706.25		2		Also *Cr*
06.40		1+		
07.44		3.4	2.3	Rowland gives 07.46 (*Fe*). $\lambda_{Fe} > 07.46$? K. R. 07.52
08.43		3−		
10.16		1		
14.69		2−		
16.88		1+		
18.13		2.3		
19.08		2		
.....	4722.34			
23.27		1.2		
23.50		1		
25.55		1+		
29.36		3.3		
31.64		3.4		
34.34		1+		
35.51		1		
36.84		1+		
40.36		1		
40.58		1		
.....	4741.72			
49.06		1−		
49.35		1−		
49.61		1−		
50.60		2.3		Sharp
51.31		1		
53.56		1		
56.06		1		
58.71		2.3		
60.39		4		Faint lines near by. *Mo?*
.....	4764.11			
64.64		2		Sharp
73.47		1.2		
73.64		2+		
74.42		2−		
75.87		2.3		
76.54		3	1.2	R. gives ⊙ 76.55 (*Co*). Also in *V* as strong line, *i*=3
78.09		1		
83.16		2.3	1	Coinc.?
.....	4783.61			
84.64		1		
85.34		2.3		Sharp
4786.68		2		At edge of ⊙ line 86.73 (*Ni*). *V* has 86.70
4787.83		1		
88.39		1+		
92.96		2		
93.60		2		Sharp
94.03		1.2		
94.81		1+		
96.75		2.3		Sharp
.....	4798.45			
4805.13		1+		
05.78		2−		
08.29		2−		At violet edge of the ⊙ line 08.32 (*Fe*)
08.68		1+		
11.28		2.3		
14.68		1+		
17.92		2−		
.....	4817.99			
19.47		3		
22.62		1+		
23.16		1−		
28.67		2−		Sharp
30.15		1+		
30.73		3		Very sharp
33.13		1.2		
34.16		2−		
35.98		1−		
38.35		1		
.....	4839.73			
39.82		1		
45.38		1+		
50.05		1−		
51.92		1		
58.44		1.2		
.....	4859.93			
60.28		1.2		
60.99		1		Faint companion to red
66.07		1−		
68.23		3		
69.43		2		
.....	4871.00			
75.73		1+		At edge of the ⊙ line 75.67 (*V*)
78.59		1.2		
86.70		1+		
89.44		1+		
94.65		1−		
.....	4896.62			
97.50		1+		
4899.81		1−		
4904.03		2.3		
4907.65		1+		

Mo λ	R.	i Mo ⊙	Remarks	Mo λ	R.	i Mo ⊙	Remarks
4909.41		1+		5091.17		1.2	
.....	4917.41			91.56		1	
25.08		1−		92.40		1+	
26.42		2−		92.96		1	
26.65		2−		96.11		1.2	
31.42		1−		96.85		2+	At edge of the
33.30		2		97.71		2.3	⊙ line 97.67
33.99		1					
.....	4936.02			5098.27		1.2	
41.90		2+		5100.58		1−	
50.83		2.3		. ...	5105.72		At edge of the
52.20		1−		09.90		2+	⊙ line 09.83
56.83		1−		15.21		2	(*Fe*)
.....	4957.78			15.86		1−	Sharp
57.78		3	R. gives 57.78 *Fe*. The lines are separated, however, and λ > λ *Mo*. Probably it should be λ *Fe* =57.88. Kayser and Runge have 57.87	17.18		1.2	
				22.00		1	
				.....	5123.90		
				24.03		1+	
				26.94		1	
				35.17		1	
				41.47		1+	
				.....	5146.66		
				48.65		1	
				55.48		1−	
				63.40		2+	
64.42		1.2		.....	5165.59		
64.63		2−		67.08		2	
75.58		1		71.33		3	
76.23		1−		73.14		3	In the shade of
79.32		2.3					b_2
.....	4980.35			74.35		3	
4995.55		1−		5180.44		1.2	
5000.13		2+		.....	5186.07		
.....	5002.98			5200.37		2−	
14.80		1+		00.07		1	
16.00		2.3		.....	5204.72		
20.07		1−		12.08		1	
.....	5025.03			19.62		1.2	
20.21		2		.....	5225.70		
30.06		2−		31.27		1+	
39.12		1−		32.58		1−	
.....	5044.30			34.47		2−	
46.73		1		38.41		3	Companion to
47.90		2					violet
55.22		1.2		41.09		3	
58.30		1		.....	5242.66		
60.07		2.3		43.01		2	
62.76		1+		45.71		2−	
.....	5064.84			50.23		2+	
80.23		2.3		01.35		2−	
81.49		1		.....	5266.73		
.....	5084.28			72.00		1−	
84.47		1−		76.50		1−	
5090.80		1		5279.85		2−	

Mo λ	R.	*Mo* i	⊙	Remarks
5281.07		2+		
.....	5288.71			
92.30		1.2		
93.05		1		
5295.67		1.2		
5306.49		1—		
.....	5307.54			
14.13		2—		
18.20		1		
20.14		1		
24.70		1—		
27.35		1		
.....	5328.75			
55.12		2—		
55.76		1+		
56.70		2—		
60.76		4.5		
64.50		3.4		
67.30		2—		Diffuse
.....	5370.17			
72.63		1+		
88.94		1		
.....	5389.68			
94.75		2		Sharp. ⊙ has 94.91 } 84 } *Mn*
5397.63		1.2		
5400.64		1.2		
11.31		1—		
14.95		1—		
.....	5415.42			
17.64		1.2		
26.24		1		
27.14		1.2		
27.80		1		
31.27		1+		
.....	5434.74			
35.91		2+		
37.97		2.3		
39.95		1+		
47.86		1—		
48.78		1—		
50.73		2.3		
53.27				
....	5455.75			
56.71		2+		
65.83		2—		
73.64		3		
76.18		2		
.....	5477.12			
88.91		1—		
5490.54		2		

Mo λ	R.	*Mo* i	⊙	Remarks
5492.43		2—		
94.06		2		
97.18		1.2		Diffuse
98.76		2—		Sharp
5499.77		1+		Diffuse
5501.78		2		Sharp
02.18		2—		
03.82		1.2		
06.75		6		
.....	5507.00			
11.77		1+		
17.73		1		
20.32		1.2		
20.93		1.2		
.....	5525.76			
26.81		2		
27.27		2		
32.00		1+		
33.26		6		
34.85		1		
39.67		2		Sharp
41.93		1+		Sharp
43.38		2+		Sharp at [illegible] edge of ⊙ line 43.41 *Fe*
.....	5544.16			
44.78		2		Sharp
52.47		1+		
.....	5555.12			
56.55		2.3		
57.02		2		
62.74		1		
63.65		1		
64.34		2—		
68.88		2.3		
69.75		2		
70.69		6		
75.47		2+		
.....	5576.32			
89.02		2	2	Coinc.? At edge of ⊙ line [illegible] *Cr*
91.84		2—		
.....	5594.66			
5596.62		1.2		
5601.31		1.2		
08.90		2		
09.53		2—		
09.80		1		
11.20		3		
13.37		2		
.....	5615.88			
18.69		2—		
5619.03		1.2		

Mo λ	R.	i Mo ⊙	Remarks	Mo λ	R.	i Mo ⊙	Remarks
5619.63		2—		5751.67		4.5	
32.74		4		.....	5754.88		
.....	5634.17			57.80		1	
35.14		2.3		65.57		1—	
42.05		1—	Near the ⊙ line 42.11 (*Ni*)	66.79		1—	
				67.63		1—	
43.47		1—		70.02		1+	
50.40		4		71.33		1+	
51.54		1+		.....	5772.36		
52.12		1.2		74.85		1.2	
52.47		1+		78.46		1	
.....	5657.72			79.65		2	
64.65		1.2		80.38		1	
67.57		1.2		80.06		1—	
72.35		1+		83.54		2—	
73.92		2+		85.90		1	
74.77		2.3		.....	5788.14		
.....	5675.65			5792.10		4	
78.18		2.3		5800.72		2—	
83.20		2+		02.05		2+	
87.93		1+		06.40		1—	
89.39		1 5	Companion to red	08.54		1+	
				09.30		1	
.....	5693.86			.....	5809.44		
94.64		1+		14.14		1	
95.10		1		15.76		1+	
95.66		1—		16.00		1	
96.30		1.2		21.00		1—	
98.53		2—		25.28		1	
5699.57		2+		25.50		1.2	
5702.30		1.2		.....	5831.82		
05.97		3		35.87		1—	
08.28		1+		40.25		1—	
.....	5711.31			.....	5848.34	...	
12.05		2	Near the ⊙ line 12.09 (*Ti*)	49.10		1.2	
				49.99		2	
19.55		1—		51.80		2	
20.45		1—		58.52		4	
22.98		3.4		61.66		1—	
29.03		2—		.....	5866.67		
29.77		2		66.05		1—	
30.17		2—		69.57		2	
31.58		1—		76.00		1	
.....	5730.08			81.85		1—	
34.32		2—		.....	5883.07		
35.55		1—		83.11		1—	
38.40		1—		88.61		4	
39.93		1		91.89		1	
41.06		1		.....	5893.10		
47.08		1—		5893.67		2	
5747.93		1+ 1+					

SYSTEMATIC ERRORS IN THE MEASUREMENT OF SPECTRUM PLATES.

In the treatment of the photographic plates I have always followed the procedure of dividing the metallic lines into small groups, of from fifteen to twenty tenth-meters' range, and referring them to two lines of Rowland's list, simple if possible, from which the wave-lengths could be obtained with sufficient precision by linear interpolation; but, for the sake of controlling still further the values so obtained, a solar line is commonly also measured in every group, and its wave-length determined with the metallic lines must then also agree with the value given by Rowland. To show the accuracy with which this agreement was attained in these measures, I give the following table containing a series of such solar lines together with a comparison with Rowland's values:

H.	R.	H.− R.	H.	R.	H.− R.	H.	R.	H.− R.
3717.523	17.539	−0.016	4070.912	70.930	−0.018	4374.621	74.628	−0.007
29.954	29.952	+ 002	71.893	71.908	− 015	4395.198	95.201	− 003
44.253	44.251	+ 002	76.102	76.101	+ 001	4425.598	25.608	− 010
66.799	66.801	− 002	4098.327	98.335	− 008	4449.325	49.313	+ 012
86.300	86.314	− 014	4100.878	00.901	− 023	4571.261	71.275	− 014
3792.472	92.482	− 010	14.605	14.606	− 001	4598.286	98.303	− 017
3852.710	52.714	− 004	18.696	18.708	− 012	4636.025	36.027	− 002
73.900	73.903	− 003	36.675	36.678	− 003	4691.573	91.602	− 029
3892.073	92.069	− 004	56.962	56.970	− 008	4772.998	73.007	− 009
3916.868	16.879	− 011	63.802	63.818	− 016	4810.710	10.724	− 014
33.822	33.825	− 003	4185.040	85.058	− 018	5234.787	34.791	− 004
41.847	41.878	+ 019	4202.197	02.198	− 001	5250.808	50.817	− 009
48.227	48.246	− 019	04.138	04.132	+ 006	5322.234	22.227	+ 007
52.969	52.103	− 004	22.363	22.382	− 019	5353.575	53.571	+ 004
68.611	68.625	− 014	25.605	25.619	− 014	5445.255	45.259	− 004
3996.118	96.140	− 022	48.381	48.384	− 003	66.582	66.609	− 027
4097.409	97.429	− 020	67.108	67.122	− 014	5497.723	97.735	− 012
10.298	10.327	− 029	4288.319	88.310	+ 009	5560.430	60.434	− 004
30.323	30.339	− 016	4307.914	07.907	+ 007	5618.477	38.488	− 011
45.977	45.975	+ 002	08.075	08.081	− 006	5662.737	62.744	− 007
50.820	50.830	− 010	37.193	37 216	− 023	5701.766	01.772	− 006
66.508	66.524	− 016	51.200	51.216	− 016	5763.200	63.218	− 018
			58.657	58.670	− 013	5816.558	16.601	− 043

The first fact which attracts attention in this table, aside from the generally very small amount of the differences H.− R., is that with few exceptions they are negative, and hence my wave-lengths are throughout too small. Accordingly, there is a systematic difference or a personal equation between myself and Rowland, the amount of which, −0.009 tenth-meters, is the mean of the above differences. If this is added, with the reversed sign, as a systematic correction to the differences H.− R., they

then take the character of accidental errors of observation, from which the probable error of a wave-length measured by me, referred to Rowland's system, comes out 0.007 tenth-meters, an accuracy which must be regarded as in fact very satisfactory.

It may be regarded as certain that in general an equal precision is not attained for the metallic line; but if we should in this case double the probable error, it would not then reach the value 0.02, as already remarked; or, in other words, in all probability the measurements are not inconsiderably more accurate than I have hitherto considered myself justified in assuming.

In regard to the origin of the above-mentioned systematic difference between my measures of the solar lines and those of Rowland — which difference, by the way, appears of exactly the same magnitude in the measures of the arc-spectrum of tungsten which I have just concluded — we can hardly assume that it is to be sought in other than purely physiological peculiarities, which cause a certain dependence of the setting of the thread of the microscope on the spectral line upon the direction from which the settings are made. In my measurements this direction is always apparently from left to right in the field of view of the microscope, which is also the direction of decreasing wave-lengths. In order to get more accurate knowledge as to how far this suspicion is justifiable, I measured a number of solar lines used in connection with the tungsten spectrum after reversing the direction of the plate, hence from violet toward red; I thus obtained values which differ from the previous ones only slightly, but nevertheless systematically. These measures are contained in the first two columns of the following table, which gives also the differences of the two series of measures, their mean, and a comparison with the corresponding determinations by Rowland.

An examination of the third column of this table leaves us hardly room to doubt that there is a difference, distinct although very small, between the two series, amounting in the mean to 0.006 tenth-meters. In the first position of the plate the wave-lengths are accordingly too small by 0.003 tenth-meters, in the second position just as much too large. We have therefore

I	II	II − I	$\frac{I+II}{2}$	R.	H. − R.
4015.753	763	+ 0.010	758	760	− 0.002
25.962	972	+ 010	967	972	− 005
52.078	083	+ 005	080	070	+ 010
70.409	419	+ 010	414	431	− 017
4087.243	245	+ 002	244	252	− 008
4106.583	576	− 007	580	583	− 003
25.771	781	+ 010	776	776	000
42.017	025	+ 008	021	025	− 004
63.796	815	+ 019	806	818	− 012
4182.916	914	− 002	915	922	− 007
4201.078	075	− 003	077	089	− 012
20.493	506	+ 013	500·	509	− 009
39.501	511	+ 010	506	525	− 019
58.475	490	+ 015	482	477	+ 005
74.332	341	+ 009	336	348	− 012
4299.140	147	+ 007	143	149	− 006
4307.908	917	+ 009	912	907	+ 005
08.078	080	+ 002	079	081	− 002
13.029	038	+ 009	033	034	− 001
27.269	263	− 006	266	274	− 008
44.439	451	+ 012	445	451	− 006
68.054	067	+ 013	060	071	− 011
4379.377	394	+ 017	385	396	− 011
4400.548	546	− 002	547	555	− 008
17.858	868	+ 010	863	884	− 021
43.358	356	− 002	357	365	− 008
56.777	780	+ 003	778	794	− 016

obtained only a partial explanation of the systematic difference of my wave-lengths from Rowland's, since a systematic difference of −0.006 still remains after the values have been freed from the effect of the direction of measurement. If we could assume a personal error, of a similar sort but in opposite direction, for the measures of Rowland, everything systematic in the mutual differences of our measures would be eliminated, and they could be regarded purely as errors of observation. There seems to me to be nothing *a priori* against such an assumption.

Inasmuch as the probable error of the wave-lengths I determined in the solar spectrum amounts, as above stated, to ± 0.007 tenth-meters, or almost exactly the mean difference of series I and II, we might question the validity of the assumption of the slight effect of the direction in which the settings were made on the solar lines. The entirely similar appearance of the objects set upon—the line to be measured and the normal line—would

seem to exclude every sort of systematic difference in the mode of making the setting. However, since the differences of the two series are, with few exceptions, in the same sense, as appears from the third column, it seems to me we can hardly deny the actual presence of such an effect, although for one I am unable to produce any sufficiently satisfactory explanation for it.

Accordingly, if the observation of such similar objects as the solar lines appearing on a dark background gives evidence of an effect, however small, of the direction in the which the setting is made, we should certainly expect in advance a similar and more marked effect when the lines of the metallic spectrum were to be referred to those of the Sun. We should much sooner assume a difference in the mode of setting according to the position of the image in the case of setting the thread first on the bright solar line on a dark field and then upon the dark metallic line in a bright field, and this has been fully confirmed. In order to investigate the question further, I repeated the measures with the plate reversed for certain groups of lines of tungsten, which I had already measured in the usual direction of decreasing wave-lengths from left to right in the field of view of the microscope. The following table, in which the wave-lengths obtained in the two positions of the plate are given under the heading I and II, shows the results of these measurements:

I	II	II—I	I	II	II—I
4411.871	.834	0.037	4425.004	.030	—0.034
12.347	.343	.004	26.087	.052	— .035
13.173	.142	— .031	27.533	.508	- .025
14.042	13.087	- .055	28.663	.007	— .056
15.233	.246	+ .013	32.379	.314	— .065
15.891	.845	- .046	35.003	.874	— .020
18.000	.584	.025	37.070	.070	.000
18.067	.034	.033	38.400	.441	- - .028
19.429	.387	- .042	30.004	.845	— .059
20.027	.608	- .010	42.004	41.976	- - .028
21.168	.135	- .033	44.050	.592	— .058
22.010	21.958	- - .052	45.324	.297	— .027
			49.182	.164	— .018

We see that here also a systematic differance of the appreciably larger amount of 0.033 tenth-meters appears, the wave-lengths being measured too large by 0.016 in the first position,

and as much too small in the second. In the first position of the plate the left side in the field of view was toward larger wave-lengths, and hence it follows that in the two cases the setting of the thread on the metallic lines was systematically somewhat too far toward the left in respect to the settings on the solar lines. This physiological difference in the *Auffassung* of the two different kinds of spectral lines is so pronounced as to leave no doubt as to its reality. This systematic correction or personal equation in my measures of metallic spectra is decidedly larger than that just discussed in the case of the solar lines, but it is also only of about the same magnitude as the probable error of the wave-lengths of the metallic lines, and consequently of no great significance. It has not been taken into account in the above catalogue of the lines of molybdenum, since in its determination the plates of tungsten and not those of molybdenum were employed.

The personal equation in the measurement of photographic plates of spectra here described has also been noticed elsewhere. In his measurements of stellar plates for the determination of radial velocities, Reese,[1] of the Lick Observatory, found that his settings on the dark lines of the metallic comparison spectrum were systematically somewhat too far toward the right in the field of view of the microscope in comparison with the settings on the light lines of the star, producing a systematic correction of about 1 km. This is evidently the same phenomenon that occurs in my measurements, except that it occurs in the opposite sense, which is not at all surprising in view of the wholly personal character of the phenomenon. We may remark, as an odd coincidence, that the magnitude of the personal equation is the same in the two cases. If the above measurements had been made for determining radial velocities, a variation in wave-length of 0.016 tenth-meters would easily be found to correspond in the region of spectrum considered to a change of velocity of 1.08 km.

COMPARISONS WITH THE INVESTIGATIONS OF THE SPARK SPECTRUM BY EXNER AND HASCHEK.

It has already been indicated above that, while the arc-spectrum of molybdenum has hitherto remained practically unknown,

[1] *Lick Observatory Bulletin* No. 15; ASTROPHYSICAL JOURNAL, 15, 142, 1902.

its spark spectrum has recently been subjected to a thorough examination in the portions affecting common photographic plates by Exner and Haschek in the course of their similar investigations of all the chemical elements. Since it was the primary purpose of these investigations to establish a basis for future mineralogical researches, the greatest accuracy of the determinations of the wave-lengths was less important than the most rapid and convenient possible method of operation. Accordingly the authors determined the wave-lengths by simply projecting a thirtyfold enlargement of the plate upon a scale graduated to half-centimeters, on which, after the adjustment of the standard lines, the wave-lengths of the remaining lines were obtained by a single reading. With the enlargement employed, the 5 mm scale divisions represent tenth-meters, and accordingly the reading had to be accurate only to 0.5 mm in order to attain the accuracy of 0.1 tenth-meters originally sought for. It might, however, seem questionable in how far this method of observation can be protected from systematic errors. In order to obtain definite data on this point, I have, for the 600 lines of molybdenum common to our catalogues, taken the means of the differences of the wave-lengths by tens, and have thus obtained the following mean deviations for the portions of spectrum given:

λ	H. - E.H.	λ	H. — E.H.	λ	H. - E.H.	λ	H. — E.H.
3510	+0.09	3740	+0.07	3904	+0.14	4318	+0.03
3530	+ .07	3757	- .02	4020	+ .16	4333	+ .01
3553	+ .09	3770	- .02	4040	+ .13	4357	+ .05
3508	+ .12	3785	+ .02	4056	+ .04	4381	+ .05
3585	+ .12	3800	- .01	4000	+ .01	4400	- .08
3603	+ .06	3812	+ .05	4115	+ .03	4425	- .00
3617	+ .02	3825	+ .03	4134	- .03	4450	- .07
3630	+ 02	3837	+ .02	4152	- .06	4475	- .10
3650	.00	3856	+ .02	4177	+ .12	4502	- .05
3662	+ .04	3875	+ .03	4200	+ .08	4529	- .02
3677	+ 07	3900	+ .06	4225	+ .05	4558	- .02
3692	+ .08	3918	+ .07	4248	+ .08	4505	.00
3707	+ .11	3935	+ .11	4265	.00	4607	+ .06
3720	+ .08	3955	+ .10	4280	- .05	4640	+ .15
3730	+0.11	3980	+0.13	4295	0.05	4684	+ .10

We may safely conclude from these figures that the accuracy sought is in general attained; but a certain periodicity is revealed

in the way the figures run, which would confirm the suspicion expressed above if there were not other circumstances which would appear to throw the systematic element of the deviation rather upon the plates themselves than upon the adjustments of the enlarged images on the scale and the reading. In the course of their further investigations Exner and Haschek have, in fact, found that in the method employed at first of photographing the comparison spectrum in juxtaposition to the spectrum under investigation by successive exposures, there occurred small displacements of the two spectra in respect to each other which were beyond checking, due to an insufficient stability in the mounting of the spectrograph. The injurious effect of these displacements could be avoided only by photographing the iron comparison spectrum simultaneously with the spectrum under investigation as an impurity of it. With the use of such plates new wave-lengths were obtained by projection in the same manner as before for the stronger lines of the metals previously investigated, including molybdenum. The following table gives the comparison of these new wave-lengths with mine:

H.	E.H.	Δ	H.	E.H.	Δ	H.	E.H.	Δ
3524.76	3524.70	+0.06	3786.54	3786.43	+0.11	4269.44	4269.43	+0.01
37.41	37.40	+ .01	98.39	3798.35	+ .04	77.08	77.00	+ .08
85.74	85.78	− .04	3864.25	3864.25	.00	77.38	77.43	− .05
3596.54	3596.51	+ .03	3903.07	3903.07	.00	79.19	79.15	+ .04
3612.15	3612.27	− .12	41.62	41.60	+ .02	88.82	88.78	+ .04
14.42	14.43	− .01	3986.45	3986.32	+ .13	92.34	92.25	+ .09
35.30	35.29	+ .01	4062.24	4062.20	+ .04	4293.42	4293.33	+ .09
51.48	51.30	+ .18	70.11	70.05	+ .06	4326.33	4326.30	+ .03
58.50	58.48	+ .02	81.62	81.60	+ .02	63.82	63.75	+ .07
80.80	80.82	− .02	4084.54	4084.51	+ .03	4381.82	4381.77	+ .05
84.48	84.32	+ .16	4107.63	4107.61	+ .02	4411.90	4411.82	+ .08
88.45	88.42	+ .03	19.81	19.75	+ .06	33.68	33.62	+ .06
3692.79	3692.82	− .03	20.26	20.25	+ .01	49.92	49.91	+ .01
3702.67	3702.70	− .03	22.55	22.45	+ .10	57.55	57.50	+ .05
17.05	17.10	− .05	43.73	43.70	+ .03	74.78	74.72	+ .06
42.48	42.45	+ .03	85.98	86.00	− .02	4491.46	4491.43	+ .03
44.55	44.55	.00	4188.49	4188.50	− .01	4517.30	4517.30	.00
55.68	55.63	+ .05	4209.84	4209.82	+ .02	17.58	17.50	+ .08
3732.35	3782.17	+ .18	32.75	32.80	− .05	24.53	24.50	+ .03
			44.95	44.90	+ .05	4537.00	4536.98	+ .02
			4250.87	4250.82	+ .05	4610.07	4610.00	+ .07

We see that, in fact, not only a far better agreement is attained, but that also the former periodic variation of the differences can no longer be recognized. It therefore appears probable that these variations were produced by the above-mentioned small mutual displacements of the spectra, which occurred

first toward one side and then toward the other in the plates of the different portions of the spectrum; and it would appear accordingly that no systematic errors of *this sort* are to be feared for the direct method of reading in itself. The prevalence of a positive sign of the differences indicates, however, a small systematic deviation averaging +0.03 tenth-meters,[1] which is doubtless of personal origin, and may be partially explained by the above-mentioned personal equation affecting my measures. If we apply this as a systematic correction we get as the probable deviation:

$$H. - E.H. = \pm 0.032 \text{ tenth-meters},$$

whence, on the assumption of a probable error of ± 0.015 in my wave-lengths, there would follow a probable error of ± 0.027 for the wave-lengths of Exner and Haschek. The accuracy is therefore almost the same. This result is not a little surprising, and if it should be confirmed by further comparisons of the tables of Exner and Haschek with measured wave-lengths, it would argue for the adoption of their method of direct reading, at least in cases where the occasional occurrence of isolated larger deviations was not of importance in comparison with the great importance of rapid work.

RELATIONS BETWEEN THE SPECTRUM OF MOLYBDENUM AND THAT OF THE SUN.

As a preliminary result of his comparisons of metallic spectra with the solar spectrum, Rowland published two lists of those metals whose presence in the absorbing stratum of the Sun may be regarded as certain. In these lists the metals are are arranged, first in the order of intensity of the corresponding solar lines, and second, in the order of the number of observed coincidences. In both cases molybdenum occupies a position near the middle. Since in the latter list molybdenum precedes magnesium, for which over twenty coincidences were observed, molybdenum should be represented by at least an equal number of coincidences in the general solar spectrum. But, if we search Rowland's list of solar spectrum wave-lengths for coincidences

[1] The lines at λ3651.30 and 3782.17, which showed the unusual difference of 0.18 tenth-meters, were excluded in taking the average.

with molybdenum, we find only seven such, whence it must be assumed that Rowland wished to include only the very strongest lines in this list, expressly designated as provisional, and that he considered a further examination of the reality of the coincidences necessary for the less conspicuous lines. This view seems the more probable on account of the fact that with few exceptions the ultimate decision of this question of coincidence is rendered extremely difficult by the very slight intensity of the solar lines concerned in comparison with the molybdenum lines.

The above-mentioned lines of molybdenum occurring in Rowland's list of solar lines are the following:

λ	Intensity in ⊙ (Rowland)	λ	Intensity in ⊙ (Rowland)
3132.75	1	3304.37	1
3170.45	2	3798.40	0
3194.09	000	3864.25	1
3264.53	0		

Only the first two of these lines fall within the limits of my previous observations of the arc-spectrum of molybdenum. They appear on my plates with a very conspicuous intensity, far surpassing all the other lines of the spectrum, and they are greatly broadened and contain narrow, sharp absorption lines which can be observed to coincide beyond a doubt with two fine solar lines. The line at λ 3903.07, which together with the two just-mentioned lines constitutes the most conspicuous element of the whole spectrum, is of almost the same intensity and otherwise of similar appearance. Rowland gives in the solar spectrum at λ 3903.09 a very strong line, ascribed to *Fe* and *Cr*, of which the solar analogue of the *Mo* line, if present, is probably a faint and very close companion not separable from it. The intensity of the solar line given by Rowland in itself makes improbable any relation to the molybdenum line, and this is further confirmed by the fact that on my double exposures to the molybdenum and iron spectra a separation of the two in the sense $\lambda_{Mo} < \lambda_{Fe}$ was suspected on account of the reversal of the two lines. If

the *Mo* line in question is represented in the solar spectrum like the two other lines, as can hardly be doubted, then the corresponding solar line must be so closely blended with the line at λ 3903.09 that they could not have been separated by Rowland. We have in this fact without doubt the reason why Rowland left this strong line of molybdenum unmentioned in his list of the solar lines.

I know nothing as to the intensity of the remaining lines attributed by Rowland to *Mo*. With the exception of the last, they are all lacking from Exner and Haschek's list[1] of the stronger lines of the arc-spectrum, while the three principal lines of the arc-spectrum also appear there as the strongest lines of the entire spectrum.

The catalogue of wave-lengths shows that the number of more or less certain coincidences with solar lines which I have been able to observe in this investigation of the *Mo* spectrum, aside from those already mentioned, is very limited. If we consider only those cases in which on account of the greater intensity of the *Mo* lines, the suspicion of contamination by a foreign metal may be regarded as excluded, we obtain the following summary:

λ	*i* Mo	*i* ☉	Sun λ	(Rowland) *i*	
3563.30	2.3	1	3563.30	000	
3026.33	2.3	1+	3026.33	1	*Fe*
3664.98	3	1	3664.97	000	
3669.50	2.3	1	3669.54	00	
3969.25	2.3	?	3969.29	0	*Cr, Co*
4084.54	3	?	4084.58	0	
4102.33	2.3	1	4102.32	0	*V*
4185.98	3	1	4185.94	0	
4269.44	2.3	1	4269.45	0	
4277.38	3	?	4277.38	1	
4517.30	3	1.2	4517.32	0	*Co*
4558.30	2.3	1	4558.29	0	
4610.07	3	1	4610.09	0	
4627.70	2.3	1	4627.73	0	
4662.95	3	1	4662.93	0	
4770.54	3	1.2	4770.55	0	*d? Co*
4783.16	2.3	1	4783.17	00	

With the exhibit of this table, in connection with what has been

[1] *Wien. Sitzungsber.*, 106, 1897.

said above as to the three principal lines of *Mo*, the presence of the metal in the general absorbing layer of the Sun may be regarded as proven. My investigations thus far do not permit a decision as to how far the remaining lines of *Mo* of greater intensity are represented in the general solar spectrum. The following comparison of these strong *Mo* lines with Rowland's solar spectrum indicates with considerable probability that in several cases this is true. In making this comparison I have assumed the possibility of a coincidence only in cases where the difference of wave-lengths does not exceed 0.05 tenth-meters.

Mo λ	*i*	Sun (Rowland) λ	*i*	Remarks
3508.26	2.3	3508.23	000	
21.56	2.3	.....		
37.41	3	37.44	0000	
42.32	2.3	....		
58.25	3	58.21	1	
70.82	2.3	70.83	0000	
3582.03	3.4	3582.08	1	
3603.10	2.3	3603.11	0000	
14.42	3	14.45	0000	
24.60	3	24.60	0000	
29.45	2.3	29.49	000	
38.35	2.3	38.38	1	
57.53	2.3	57.56	1	*Fe*
59.51	3.4	.....		
72.97	3	72.94	0000	
80.75 / 80.55	3.4	80.80	3	no coinc.
90.72	2.3	90.73	0000	
3695.09	3.4	3695.04	0000	
3727.86	3	3727.83	1	
32.91	3	32.89	2	
42.48	2.3	.....		
70.66	2.3	70.67	000	
3781.75	2.3	3781.75	1	
3802.00	2.3	02.05	00	*Mn*
19.98	2.3	19.94	00	
26.55	2.3	26.84	0	
29.04	3—	.....		
33.92	3	33.92	0	
3869.25	2.3	.....		
3901.95	2.3	.....		
3913.19	3	.....		
4062.24	2.3	4062.20	00	
70.05	3+	70.05	000	
4081.62	3	4081.58	00	
4107.63	3	4107.65	5	*Ce, Fe, Zr*
20.26	3	.....		
43.73	4	.....		

Mo λ	*i*	Sun (Rowland) λ	*i*	Remarks
4149.14	2.3	....		
55.47	2.3	55.48	00	
55.77	2.3	55.80	00	
57.59	2.3	57.59	00	
62.85	2.3	62.83	1	no coinc.
88.49	4	88.48	00	
4194.74	2.3	4194.78	00	
4232.75	3+	4232.76	00	*V*
41.03	2.3	.....		
46.19	2.3	46.18	0	
52.03	2.3	52.04	000	
77.08	3	.....		
88.82	3+	.....		
92.34	3	.....		
93.42	3	.....		
4294.07	3	4294.08	000	
4318.13	2.3	.....		
26.33	3	.....		
50.53	3—	4350.55	00	*Ba?*
69.23	2.3	4369.25	00	
4381.82	4	.....		
4411.76	2.3	4411.75	00	
11.90	3	11.88	00	
23.79	2.3	23.75	000	
26.86	2.3	26.84	000	
42.37	2.3	.....		
57.55	3.4	.....		
68.46	3	68.46	00	
73.37	2.3	73.38	000	
74.78	4	74.74	00	
4485.16	2.3	85.12	000	
4491.46	3	.....		
4506.13 / 06.22	3	4506.09 / 06.26	000 / 00	*Ba*
12.32	2.3	.....		
24.53	3	24.57	1	*Mn*
29.59	2.3	.....		
37.00	3+	.....		

Mo λ	*i*	Sun (Rowland) λ	*i*	Remarks	*Mo* λ	*i*	Sun (Rowland) λ	*i*	Remarks
4576.70	3—	76.69	00		5097.71	2.3	5097.67	0	no coinc.
4595.35	3	4595.39	00		5171.33	3	.....		
4621.57	2.3	.....			73.14	3	.....		
27.70	2.3	4627.73	0		5174.35	3	.....		
62.11	2.3	62.15	1	*Fe?*	5238.41	3	5238.42	000	
62.95	3	62.93	0		5241.09	3	5241.04	000	
72.11	3—	72.09	000		5360.76	4.5	.....		
4688.41	2.3	.....			5364.50	3.4	.....		
4707.44	3.4	4707.46	5	*d? Fe*	5437.97	2.3	5438.00		
08.43	3—	08.46	000		50.73	2.3	.....		
18.13	2.3	.....			5473.64	4	.....		
29.36	2.3	29.38	0000		5506.75	6	5506.72	000	
31.64	3.4	31.65	4	*Fe?*	33.26	6	33.25	000	
50.60	2.3	.....			56.55	2.3	.....		
58.71	2.3	.....			68.88	2.3	68.92	0000	
60.39	4	60.40	0000		5570.69	6	.....		
75.87	2.3	.....			5611.20	3	.....		
85.34	2.3	.....			32.74	4	.....		
4796.75	2.3	.....			35.14	2.3	.....		
4811.28	2.3	4811.24	00	*Ti*	50.40	4	5050.42	0000	
19.47	3	.....			74.77	2.3	.....		
30.73	3	30.71	0000		5078.18	2.3	.....		
4868.23	3	.....			5680.39	4.5	.....		
4904.03	2.3	.....			5705.07	3	.....		
50.83	2.3	4950.80	0000		22.98	3.4	.....		
57.78	3	?			51.67	4.5	5723.00	0000	
4970.32	2.3	.....			5702.10	4	5702.14	00	
5016.90	2.3	.....			5858.52	4	5858.50	00	
60.07	2.3	5060.11	0000		5888.61	4	5888.65	0000	
80.23	2.3	.....							

A close examination of this table hardly permits a doubt that numerous cases of actual identity occur among these approximately equal wave-lengths. Nevertheless the extraordinary faintness which almost universally characterizes the solar lines here in question renders exceedingly difficult the final decision of the question of coincidence, even with the use of the greatest possible dispersion; while, on the other hand, this faintness gives a sufficient explanation of the fact that these solar lines appear in only the rarest cases on my plates taken hitherto. The beautiful plates which I have recently obtained with the above-mentioned concave grating will, it is hoped, assist in the further solution of this special question.

The great contrast in the intensity of the solar and *Mo* lines, which are to be regarded as belonging to the general solar

spectrum, from what has preceded directly suggests the analogous, though less pronounced, relations which I have previously observed in connection with the spectrum of vanadium. In this metal also only the principal lines of the arc-spectrum are represented in the general solar spectrum, but the lines offer considerably less difficulty in observation on account of a less excessive faintness. Since these vanadium lines have a considerable intensity in Sun-spots according to Young's observations, it would be natural to suspect a similar behavior of the molybdenum lines. To my knowledge nothing is known on this point, however, and the decision of the question will necessitate investigations of the spot spectra of much greater completeness than those which are at present available.

THE VARIABLE STAR 7582 *X CEPHEI*.

By J. A. PARKHURST.

THE variability of this star (R. A. $21^h\ 3^m\ 38^s.5$; Dec. $+82^\circ\ 39'\ 50''.3$ (1900) was noticed by Madame Ceraski in 1898 from an examination of photographs taken at the Moscow Observatory by M. Blajko).[1] Brief notices in regard to the star have been published in various journals, to which reference will be made later, but so far no report approaching completeness has appeared.

INSTRUMENTS.

The visual and photometric observations were made with a 6-inch (157 mm) Brashear reflector and the 12 (305 mm) and 40-inch (101 cm) refractors of the Yerkes Observatory; the photograph of the field reproduced in Plate I was made with the 24-inch reflector, the original negative having a field three inches square, covering $1^\circ\ 52'$. The central part of this field, enlarged four and one-half diameters, appears in the plate.

The photometric observations were made with the equalizing wedge photometer devised by Professor E. C. Pickering and described in this JOURNAL, **13**, 249;[2] the constant of Wedge II there found, **0.130** magnitude, being used. This value of the constant has been confirmed by Mr. Edward S. King at the Harvard College Observatory.[3] In 1902 a different wedge (V) was used, whose constant proved to be somewhat different from Wedge II. A new determination of the constant of Wedge V, as yet unpublished, was made in the same manner as that of Wedge II, the resulting value, **0.110** magnitude, being used in the reductions.

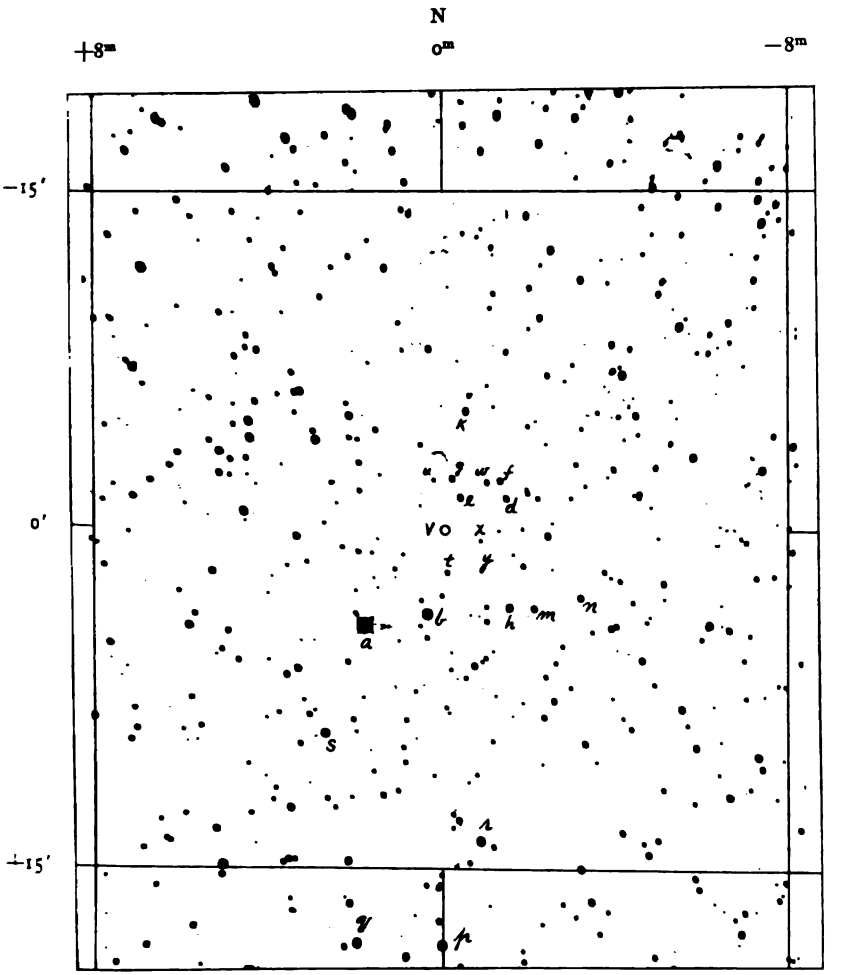
POSITION OF THE VARIABLE.

The variable was connected with the comparison stars *a* and *b* on October 11, 1898, with the filar micrometer on the 6-inch

[1] *Astronomische Nachrichten*, **147**, 141.

[2] See sectional drawing, Fig. 1, *loc. cit.* [3] *H. C. O. Annals*, XLI, 242.

PLATE I.



Scale: 1 mm=20″

7582 *X CEPHEI.*

(21h 3m 38ṣ5; +82° 39′ 50″.)

reflector, but, as it was thirteenth magnitude at that time, the position found was not very precise, differing nearly 13″ from the final place. The measures were repeated on October 3, 1899, when the variable had risen to tenth magnitude; with the following results:

b = *Carrington* 3222	21h	06m	59.5	+82° 25′	05.″7 (1855)
Precession		—02	57.5	+10	53.2
b	21	04	02.0	82 35	58.9 (1900)
Δ(*bv*)		—00	23.1	+03	51.4
v from *b*	21	03	38.9	82 39	50.3 (1900)
a = *Carrington* 3225	21	08	22.7	82 24	28.9 (1855)
Precession		—02	55.0	+10	57.0
a	21	05	27.7	82 35	25.9 (1900)
Δ(*av*)		—01	49.6	+04	24.3
v from *a*	21	03	38.1	82 39	50.2 (1900)
v mean	21	03	38.5	82 39	50.3 (1900)
Precession		+03	00.7	—10	52.2
v	21	06	39.2	82 28	58.1 (1855)

COMPARISON STARS.

When the variable had faded below the limit of the 40-inch refractor I sent a request to Professor Keeler, the late director of the Lick Observatory, to have the field photographed with the Crossley reflector, and he kindly sent me a plate taken July 24, 1900, with an exposure of two hours. The co-ordinates of the comparison stars were measured on this plate by Dr. Caroline E. Furness, with the Repsold measuring machine of Vassar College Observatory, for which courtesy I wish to record in this place my thanks. The results of these measures, with the magnitudes described later, are given in Table I.

The measurement of the light of these comparison stars, one of the principal objects of this investigation, was carried on in the following manner:

Five *Meridian Photometer* stars, lying near the field, were

TABLE I.

Comparison Stars for *7582 X Cephei.*

Letter	Co-ordinates from V		Light		Letter	Co-ordinates from V		Light	
	R. A.	Dec.	Mag.	Steps		R. A.	Dec.	Mag.	Steps
	s	′ ″				s	′ ″		
n	−190	− 3 11	12.68	25.8	*k*	− 30	+ 5 11	12.29	29.0
l	−144	− 0 23	12.79	25.8	*e*	− 23	+ 1 20	12.43	24.4
m	−125	− 3 39	12.0	29.0	*g*	− 10	+ 2 11	12.66	22.9
h	− 91	− 3 35	11.99	29.0	*t*	− 4	− 2 1	14.18	17.8
d	− 87	+ 1 19	12.88	25.8	*p*	+ 1	−18 35	10.2	37.4
f	− 78	+ 2 5	13.23	22.5	*u*	+ 16	+ 2 7	14.88	13.5
w	− 60	+ 2 0	13.98	17.0	*b*	+ 23	− 3 51	9.12	42.3
r	− 51	−13 57	10.08	37.8	*a*	+109	− 4 24	8.24	....
y	− 55	− 0 56	17.0	0.0	*q*	+115	−18 29	10.6	36.5
x	− 50	− 0 36	15.63	7.5	*s*	+160	− 9 11	10.86	32.8

b = *B. D.* + 82° 635 = *Carrington* 3222.

a = *B. D.* + 82 636 = *Carrington* 3225.

TABLE II.

Standard Magnitude Stars from *Harvard Annals*, XXIV.

Letter	*B. D.*		1855		H. C. O.
	No.	Mag.	R. A.	Dec.	Mag.
			h. m.	′	
A	+82 648	8.0	21 30.2	+82 21	7.99
B	+81 737	7.5	21 25.2	+81 54	7.83
C	+81 736	7.9	21 24.7	+81 24	7.47
D	+81 735	7.8	21 23.5	+81 8	7.73
E	+81 742	8.0	21 32.4	+81 6	8.07

selected from *Harvard Annals*, XXIV; their notation, positions, and photometric magnitudes being given in Table II. With these stars as standards, the magnitude of four comparison stars between eighth and eleventh magnitudes, *a*, *b*, *s*, and *r*, were measured with the photometer attached to the 6-inch reflector. From these as standards, four stars between the eleventh and thirteenth magnitudes were measured with the 12-inch; and finally the remaining stars were measured with the 40-inch. That the reader may be able to form an independent estimate of the merits of the methods and instruments for this work, the measures are given in detail in Table III, which is nearly self-explanatory.

There are three main dangers to be guarded against in this work: (1) the lack of close resemblance between the real and artificial stars; (2) variation in the light of the artificial star; and (3) variation in the transparency of the atmosphere, from changing zenith distance or other causes. To insure resemblance in the star images the movable diaphragm close to the lamp was provided with five holes, 0.10, 0.15, 0.20, 0.25, and 0.30 mm in diameter, the choice of the hole used being governed by the telescope and atmospheric conditions of each evening. To eliminate the effect of variation in the light of either star, the series of scale readings was repeated in inverse order (given under "Second" in Table III). An inspection of the scale readings and their means will therefore show any variations, either irregular or progressive, in the light of either star, and the final means will tend to eliminate their effects.

In each division of Table III the stars used as standards are given first, with their magnitudes in bold-faced type. The magnitudes sought for the remaining stars were deduced in the following manner: Means were formed of the scale readings (d_o) and magnitudes (M_o) of all the standards. The difference between the mean scale reading for each star and d (d–d_o), was converted into magnitudes by multiplication into the wedge constant, and its value (ΔM) added to M_o.

An idea of the errors in the settings and the maximum effect of irregular changes in the light of the stars can be gained from the average deviations of the separate settings from the means. The values are:

For the 6-inch	0.06 mag.
For the 12-inch	0.09 mag.
For the 40-inch	0.10 mag.

The agreement of the final magnitude results is as follows:

Average Deviation of One Night's Measure from Mean		
6-inch	0.10 mag.	4 stars
12-inch	0.04 mag.	4 stars
40-inch	0.08 mag.	7 stars

TABLE III.

Photometric Measures of Comparison Stars.

1900, October 19; 6-inch. Wedge II; seeing very good.

Star	Scale Readings						Mean Scale Reading	Mag.
	First			Second				
A	16.5	16.4	17.2	18.1	19.1	18.0	17.55	**7.99**
B	15.4	15.1	15.8	16.2	16.8	14.7	15.67	**7.83**
C	12.0	11.0	11.2	12.5	12.6	11.6	11.82	**7.47**
D	14.2	13.3	14.6	13.4	13.5	12.9	13.65	**7.73**
E	18.0	17.5	17.2	14.9	16.1	15.0	16.45	**8.07**
a	17.9	18.8	16.9	17.4	17.4	17.9	17.72	8.24
b	24.0	23.9	22.9	23.2	24.0	23.1	23.52	9.12
s	36.4	36.9	38.4	38.9	37.1	38.8	37.85	10.86
r	34.2	33.4	33.2	32.0	32.0	30.8	32.60	10.08
p	32.4	34.5	33.8				33.57	10.02
q	37.0	35.9	37.2				36.70	10.6
h	44.0	46.3	44.9				45.07	12.0

1900, November 2; 6-inch. Wedge II; Moonlight.

E	16.0	16.0	15.8	16.1	16.8	15.2	15.82	**8.07**
D	12.7	13.0	12.3	13.9	13.2	14.6	13.29	**7.73**
C	11.0	11.7	11.4	11.0	11.6	10.4	11.19	**7.47**
B	15.5	16.2	15.8	15.4	16.1	16.0	15.83	**7.83**
A	16.7	17.4	17.4	17.9	17.4	17.0	17.30	**7.99**
a	18.4	18.9	18.2	18.1	18.8	18.2	18.44	8.31
b	25.8	26.3	24.8	25.8	26.0	24.4	25.52	9.23
s	38.3	38.1	38.2	36.8	37.3	36.2	37.49	10.78
r	34.3	32.9	33.9	33.7	32.6	32.0	33.24	10.23

1900, December 12; 6 inch. Wedge II.

E	19.8	20.8	20.8	20.7	20.8	20.0	20.49	**8.07**
D	18.8	19.4	18.3	16.9	16.8	16.2	17.73	**7.73**
C	16.5	17.6	17.1	15.7	16.1	15.8	16.45	**7.47**
B	19.4	20.3	20.0	18.0	18.6	17.0	18.87	**7.83**
A	21.9	21.0	21.2	20.8	20.1	20.6	20.94	**7.99**
b	29.9	29.8	29.0	28.8	31.0	29.8	29.72	9.22
a	22.8	22.0	22.4	22.5	20.8	22.1	22.10	8.24
h	51.2	53.8	52.4	54.4	53.5	52.7	53.00	12.2
r	35.2	36.0	36.0	35.1	34.0	34.0	35.05	9.92
s	42.7	44.2	43.0	44.4	43.8	42.2	43.39	11.00

TABLE III.—*Continued.*

RESULTING MAGNITUDES FROM MEASURES WITH 6-INCH.

Star	Oct. 19	Nov. 2	Dec. 12	Mean
a	8.17	8.31	8.24	8.24
b	8.92	9.23	9.22	9.12
s	10.79	10.78	11.00	10.86
r	10.10	10.23	9.92	10.08
p	10.23		.....	10.2
q	10.64		.....	10.6

1900, Sept. 12; 12-inch. Wedge II; moonlight, clear.

Star	Scale Readings						Mean Scale Reading	Mag.
	First			Second				
a	12.7	13.2	13.2	13.5	13.1	13.1	13.13	**8.24**
b	20.5	20.0	20.6	21.6	20.9	20.8	20.74	**9.12**
h	45.0	41.5	43.9	40.3	42.0	43.1	42.30	11.98
n	51.1	46.5	48.6	47.4	47.9	46.4	47.98	12.71
l	49.0	51.8	48.8	48.2	49.3	47.0	49.02	12.84
d	47.2	49.6	48.0	48.2	49.9	49.9	48.90	12.84

1900, October 25; 12 inch. Wedge II; seeing good.

a	12.5	12.9	13.7	13.0	13.0	11.9	12.83	**8.24**
b	20.9	21.0	20.9	21.1	20.9	21.0	21.61	**9.12**
s	29.2	30.1	30.0	30.0	30.9	29.9	30.22	**10.86**
d	49.0	48.7	48.3	48.0	47.0	47.5	48.09	12.92
h	41.3	42.3	41.0	41.2	42.1	40.8	41.45	12.00
n	45.0	47.8	47.7	47.4	46.5	44.1	46.42	12.64
l	46.1	46.1	44.2	49.0	47.0	45.0	47.09	12.73
				49.1	49.5	50.8		

RESULTING MAGNITUDES FROM MEASURES WITH 12-INCH.

Star	Sept. 12	Oct. 25	Mean
h	11.98	12.00	11.99
d	12.84	12.92	12.88
l	12.84	12.73	12.79
n	12.71	12.64	12.68

TABLE III.— *Continued.*

1900, September 13; 40-inch. Wedge II; seeing good.

Star	Scale Readings						Mean Scale Reading	Mag.
	First			Second				
s	14.2	18.2	15.9	16.9	15.1	14.1	15.74	**10.86**
h	22.4	22.2	22.8	22.2	23.3	22.0	22.49	**11.99**
d	25.8	28.1	26.8	.....	....	....	26.90	**12.88**
l	29.8	27.2	29.2	28.5	27.0	30.0	28.62	**12.79**
t	38.8	40.8	38.8	37.0	37.4	36.9	38.29	14.06
x	49.3	49.3	47.1	49.3	49.4	50.0	49.07	15.47
f	31.1	29.9	32.1	....	....	....	31.10	13.13
w	36.2	37.2	36.2	....	....	....	36.53	13.82
e	26.1	26.2	26.0	25.9	24.0	27.5	25.95	12.46
g	26.5	29.5	27.2				27.73	12.69
u	43.8	43.3	44.4				43.83	14.78
k	26.7	22.8	24.6				24.70	12.29

1902, January 7; 40-inch. Wedge V; seeing good.

d	21.9	25.7	24.7	24.8	23.6	24.7	25.4	24.1	24.42	**12.88**
l	29.2	27.9	29.9	28.2	22.5	23.5	21.8	21.8	25.60	**12.79**
n	22.2	24.0	21.7	23.2	20.4	20.4	18.8	19.3	21.18	**12.68**
e	17.0	20.3	20.4	20.9	22.0	19.7	21.8	20.1	20.15	12.39
x	49.8	51.7	48.8	50.7	51.1	52.8	52.0	50.6	50.94	15.77
g	20.0	23.8	21.8	23.2	23.2	22.5	21.8	21.8	22.27	12.62
w	38.1	39.2	38.6	38.2	32.9	33.1	33.8	33.3	35.01	14.12
f	28.7	29.5	30.6	28.9	28.2	28.8	28.0	26.2	28.62	13.32
t	33.2	35.0	33.7	36.0	37.2	37.2	37.4	37.8	35.94	14.12
u	42.9	44.3	42.7	43.7	43.5	43.4	44.2	43.8	43.57	14.96

RESULTING MAGNITUDES FROM FIRST TWO SETS WITH 40-INCH.

Star	Sept. 13	Jan. 7	Mean
e	12.46	12.39	12.43
k	12.29	.....	12.29
g	12.69	12.62	12.66
f	13.13	13.32	13.23

TABLE III.—*Continued.*

1900, September 6; 40-inch. Wedge II; bright Moon.

Star	Scale Readings						Mean Scale Reading	Mag.
	First			Second				
d	22.0	21.5	23.0	29.4	27.8	25.1	24.80	12.88
l	28.2	29.5	29.2	33.0	30.5	29.9	30.05	12.79
e	25.4	24.8	27.0	25.4	25.0	23.6	26.20	12.43
g	27.8	28.0	28.1	26.3	30.3	27.0	27.92	12.66
f	30.6	31.8	32.8	30.0	32.7	33.0	31.82	13.23
w	38.0	35.9	35.3	36.8	39.0	39.0	37.34	13.99
t	38.7	40.8	40.6	....	....	....	40.05	13.35
u	41.9	44.2	43.1	46.8	45.0	45.5	44.42	14.91

1900, May 8; 40-inch. Wedge II; seeing poor.

e	28.8	31.2	28.2	27.2	30.4	27.7	28.5	31.8	29.23	12.43
g	27.8	27.6	31.2	28.5	32.2	31.8	33.2	26.5	29.86	12.66
t	42.2	41.8							42.0	14.2
u	48.9	48.9							48.9	15.1

RESULTING MAGNITUDES FOR *t*, *u*, and *w* WITH 40-INCH.

Star	Sept. 13	Jan. 7	Sept. 6	May 8	Mean
t	14.06	14.12	14.35	(14.2)	14.18
u	14.78	14.96	14.91	(15.1)	14.88
w	13.82	14.12	13.99	.....	13.98

1902, May 29; 40-inch. Wedge V; seeing fairly good.

Star	Scale Readings								Mean Scale Reading	Mag.
	First				Second					
t	39.0	39.2	38.4	39.8	39.5	39.4	40.1	39.0	39.30	14.18
u	44.7	45.4	45.6	45.8	45.0	45.0	46.0	45.5	45.38	14.88
x	52.4	52.6	52.5	54.0	52.2	50.5	52.0	53.5	52.47	15.66

RESULTS FOR ALL MEASURES OF *x* WITH 40 INCH.

Star	Sept 13	Jan. 7	May 29	Mean
x	15.47	15.77	15.66	15.63

MAGNITUDE CURVE.

The visual observations of the variable were made by Argelander's method, and a light-scale formed from the resulting intervals between the comparison stars, expressed in steps. Fig. 1 shows the correspondence between the light of the stars expressed in steps and the magnitude found by the photometer.

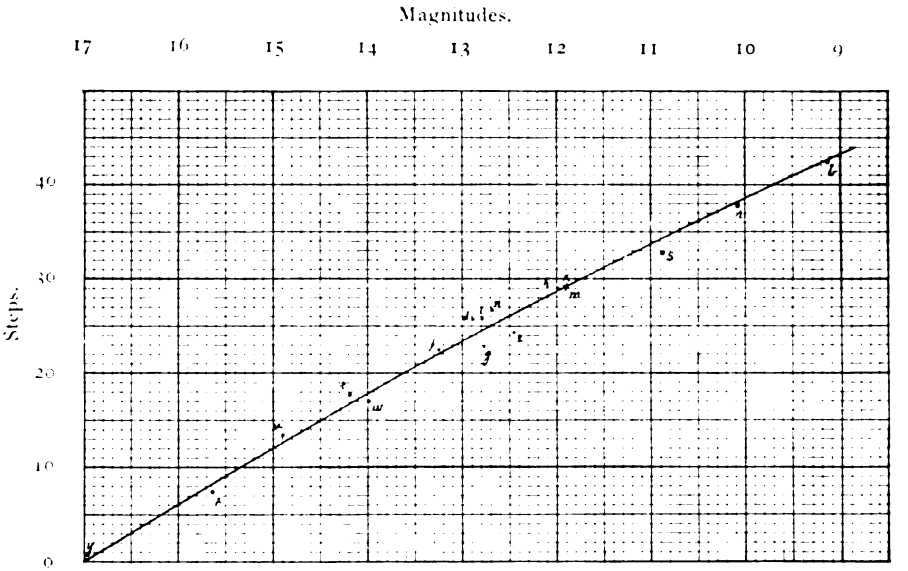


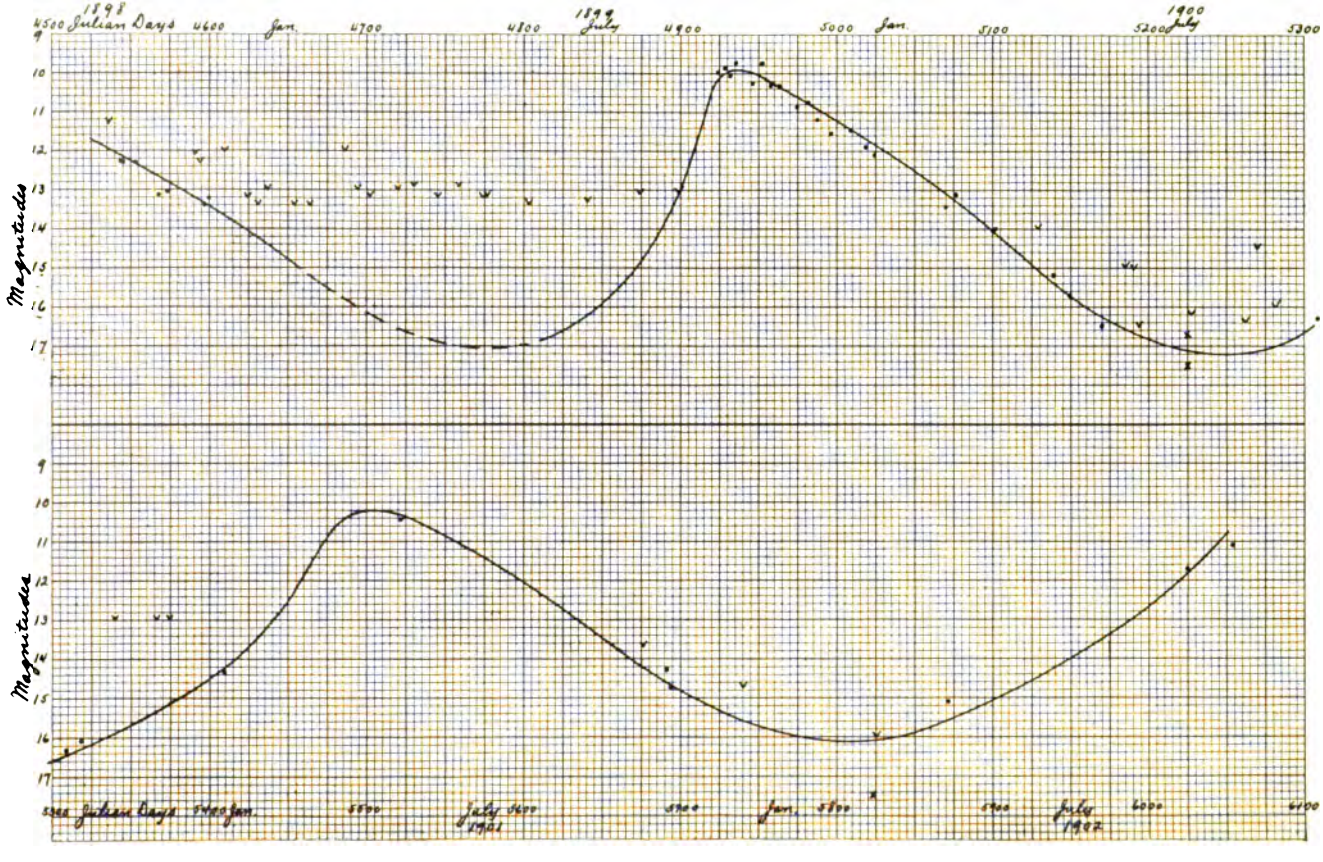
FIG. 1.—Magnitude Curve of *X Cephei*.

It is platted with the photometric magnitudes as abscissæ and the step values in the light-scale as ordinates. The average distance between the platted points and the smooth curve drawn through them is about 0.10 magnitude. This curve serves to fix the magnitudes of the three stars, *j*, *k*, and *m*, which were not measured with the photometer, and also to translate the brightness of the variable, expressed in steps in the comparisons, into magnitudes.

VISUAL OBSERVATIONS OF THE VARIABLE.

These observations are given in detail in Table IV, the headings of the columns leaving little explanation necessary. In the

PLATE II.



Light Curve of *X Cephei.*

column headed "Details in Steps" the light of the variable is deduced from the position of each comparison star in the light-scale as given in the last column of Table I. When the variable was invisible, the brightness of the faintest comparison star which could be seen is given, preceded by the inequality sign, $<$, which should be read "fainter than," instead of in its usual significance, "numerically less than," the quantity following. In the comparisons, v stands for the variable.

THE LIGHT CURVE OF THE VARIABLE.

Plate II gives a graphic representation of the star's variations, platted with time as abscissæ and magnitudes as ordinates. The visual magnitudes are shown in dots and the photographic magnitudes from the two plates, July 24, 1900, and March 13, 1902, in small crosses. These two latter are not directly comparable with the visual results, a difference of one-half magnitude or more being indicated. On dates when the variable could not be seen the limiting magnitude visible is indicated by the sign v. The curve drawn through the platted points shows the maximum magnitude to be 9 or 10. This agrees sufficiently well with the visual and photographic observations of Ceraski and Blajko,[1] who found it between ninth and tenth magnitude in May and June, 1898. The magnitude at minimum is not far from 17, being below the limit of the 40-inch with a low power.

The Julian dates of the two maxima are about 241 4935 and 241 5500; of the minima, 241 5250 and 241 5810; the intervals being 565 and 560 days, respectively. The sudden rise to maximum in 1899 is not entirely confirmed by the other parts of the curve, but the observations seem definite on that point.

LIMITING MAGNITUDES.

A point of interest in regard to the photometry of faint stars is touched by the limiting magnitudes of the different apertures

[1] *A. N.*, 153, 297.

TABLE IV.

7582 X Cephei

Comparisons of the Variable by Argelander's Method.

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1898		C. S. T.	G. M. T.			
1	Sept.	3	9	4536.6	80	6	*a* and *b* seen, nothing else near
2		6	9	4539.6	150	6	*v* 1-2 *d*, *d* 2 *e*, *e* 1 *f*, *e* 1 *g*
3		7	9	4540.6		6	No change
4		20	8	4553.6	150	6	*h* 2 *v*. *v* limit, is this *v*?
5	Oct.	5	8	4568.6	150	6	*e* 2 *v*, *e* 1 *g*, *e d*, *d* 3 *f*
6		11	8	4574.6	80	6	*g* 1 *v*, *e* 1-2 *v*, *d* 2 *v*, *v* 2 *f*
7		28	6	4591.5	150	6	*v*, *d*, *f*, *e* and *g* not seen
8		31	6	4594.5	80	6	*v*, *d*, *f*, *e* and *g* not held
9	Nov.	2	7	4596.5	200	6	*e* 3 *v*, *g* 1-2 *v*, *f* 1-2 *v*
10		6	7	4600.5	80	6	*e*, *g* and *d* seen, *f and v* glimpsed
11		16	7	4610.5	200	6	Nothing seen near *v*
12		30	6	4624.5	150	6	*e*, *d*, *f* and *g* seen, *v* not seen
13	Dec.	7	7	4631.5	200	6	*v* not seen
14		13	6	4637.5	150	6	*v* not seen
15		30	6	4654.5	200	6	*d*, *e*, *f* and *g* seen, *v* not seen
	1899						
16	Jan.	8	6	4663.5	200	6	*d*, *e*, *f* and *g* seen, *v* not seen
17	Feb.	1	6	4686.5		6	*d*, *e*, *f*, *g* and *v* not seen
18		7	7	4692.5		6	*d*, *e* and *g* seen, *v* not seen
19		15	18	4702.0	200	6	*d*, *e*, *f* and *g* seen, *v* not seen
20	Mar.	6	7	4720.5	150	6	*d*, *e*, *f* and *g* seen, *v* not seen
21		15	8	4729.5		6	*e* seen, *v* not seen
22		31	8	4745.5	150	6	*d*, *e*, *f* and *g* seen, *v* suspected
23	Apr.	12	8	4757.0	150	6	*e* and *d* seen, *v* not seen
24		28	9	4773.6	200	6	*d*, *e*, *f* and *g* seen, *v* not seen
25	May	1	9	4775.6	200	6	*d*, *e*, *f* and *g* well seen, *v* not seen
26		29	9	4804.6	200	6	*d*, *e*, *f* and *g* well seen, *v* not seen
27	July	6	10	4842.7	150	6	*d*, *e*, *f* and *g* well seen, *v* not seen
28	Aug.	5	9	4872.6	150	6	*d*, *e*, *f* and *g* seen, *v* not seen
29		30	9	4897.6	150	6	*d*, *e* and *g* seen, *v* not seen
30	Sept.	26	7	4924.5	150	6	*h* 4 *v*, *v* 0-1 *r*, *v* 1-2 *p*
31		30	7	4928.5	150	6	*h* 2-3 *v*, *v* 1-2 *r*, *v* 1 *p*
32	Oct.	3	8	4931.6	80	6	*h* 3-4 *v*, *v r*
33		6	7	4934.5	40	6	*h* 2-3 *v*, *v* 2 *r*
					150	6	*h* 3-4 *v*, *v* 2-3 *r*
34		18	7	4946.5	80	6	*h* 4 *v*, *r* 1 *v*, *v p*, *v* 1 *q*
35		24	7	4952.5	40	6	*h* 3-4 *v*, *v* 2-3 *r*, *v* 2-3 *p*
36		29	7	4957.5	150	6	*h* 4-5 *v*, *r* 2-3 *v*
					40	6	*h* 4 *v*, *r* 1 *v*, *v* 4 *s*
37	Nov.	4	7	4963.5	150	6	*h* 5 *v*, *v* 3 *s*
					40	6	*h* 4-5 *v*, *v r*, *v* 3 *s*
38		15	7	4974.5	150	6	*h* 10 *v* ±, *r* 2 *v*, *v* 2 *s*
39		22	7	4981.5	40	6	*h* 6-8 *v*, *r* 3 *v*, *v* 2 *s*
40		28	7	4987.5	150	6	*r* 4 *v*, *s* 1 *v*, *v* 4 *h*
41	Dec.	6	7	4995.5	150	6	*s* 2 *v*, *v* 2 *h*
42		19	6	5008.5	150	6	*s v*, *v* 1-2 *h*

TABLE IV.

7582 X Cephei

Reductions of Observations.

	Details in Steps	Means		Seeing	Remarks
		Steps	Mag.		
1		<27	<11.3	moon	limit 10<*b*
2	27.3	27.3	12.27	fair	limit *g*
3			12.3±		
4	27.0	27.0	12.30	fair	limit *v*, is it *v*?
5	22.4	22.4	13.16	good	
6	21.9, 22.9, 23.8, 24.5	23.2	13.01	good	limit *f*
7		<28	<12.1	full moon	limit 1<*h*
8		<27	<12.3	fair	limit 2<*h*
9	21.4, 21.4, 21.0	21.2	13.40	good	
10		22±	13.2±	good	
11		<29	<12	fair	limit *h*
12		<22	<13.2	good	limit *f*=3<*e*
13		<21	<13.4	good	limit 1<*f*
14		<23	<13.0	fair	limit *g*
15		<21	<13.4	good	limit 1<*f*
16		<21	<13.4	good	limit 1<*f*
17		<29	<12	poor	limit *h*
18		<23	<13.0		
19		<22	<13.2	good	
20		<23	<13.0		
21		<24	<12.9	clouds	
22		<22	<13.2	good	
23		<24	<12.9	good	
24		<22	<13.2	good	
25		<22	<13.2	good	
26		<22	<13.4	good	
27		<22	<13.3	good	
28		<23	<13.1		
29		<23	<13.1	fair	
30	38.3, 38.3, 38.9	38.5	10.00	good	
31	39.8, 39.3, 38.4	39.1	9.88	good	
32	38.8, 37.8	38.3	10.05	good	
33	39.8, 39.8 38.8, 40.3	39.9	9.74	fair to good	
34	38.3, 36.8, 37.4, 37.5	37.5	10.22	moon, fair	
35	38.8, 40.3, 39.9	39.7	9.76	good	
36	37.8, 34.8 38.3, 36.8, 36.8	36.9	10.33	good	
37	37.3, 35.8 37.8, 37.8, 35.8	36.9	10.33	good	
38	32.3, 35.8, 34.8	34.2	10.87	moon, fair	
39	35.3, 34.8, 34.8	34.9	10.78	fair to good	
40	33.8, 31.8, 33.0	32.8	11.20	fair to good	
41	30.8, 31.0	30.9	11.57	fair	
42	32.8, 30.5	31.6	11.44	fair	

TABLE IV—*Continued.*

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1899		C. S. T.	G. M. T.			
43		29	7	5018.5	150	6	*s* 3 *v*, *h* 1 *v*, *v* 4 *l*
	1900						
44	Jan.	4	7	5024.5	150	6	*s* 4-5 *v*, *h* 2 *v*, *v* 3 *l*
45		21	6	5041.5	350	40	
46	Feb.	18	14	5069.8	350	40	*e* 8 *v*, *v* 8 *t*, *v* 2 *e*
47		25	9	5076.6	275	12	*e* 1 *v*, *v f*, *v* 3 *l*
48	Mar.	21	8	5100.6	275	12	*t* 1 *v*, *v* 1 *w*
49		22	10	5101.7	350	40	*v l*, *v* 1 *w*
50	Apr.	18	13	5128.8	350	40	*v* not seen, *t* glimpsed
51		27	12	5137.8	237	40	*w* 6 *v*, *l* 6 *v*, *u* 3 *v*, *v* 3 *x*
					350	40	*u* 5 *v*
52	May	8	14	5148.8	460	40	*u* 6 *v*, *x* 0-1 *v*
					460	40	*u* 5 *v*, *x* 1 *v*, *w* 8 *v*, repeated
53		29	13	5169.8	237	40	*x* 4-5 *v*
54	June	13	12	5184.8	350	40	*v*, *u* or *x* not seen, *w* seen
55		19	14	5190.8	350	40	*v* and *x* not held
56		21	12	5193.0	237	40	*v* not seen
57	July	24		5225			
58		25	15	5226.0	460	40	*v* not seen
59	Aug.	29	15	5261.0	460	40	*v* not seen
60	Sept.	6	9	5269.6	237	40	*v* or *x* not seen
61		13	8	5276.6	237	40	*v* not seen
62	Oct.	16	9	5309.6	460	40	*x* 3 4 *v*, *v* 4 *y*
63		25	11	5318.7	460	40	*x* 2 *v*
64	Nov.	15	6	5339.5	150	6	*v* not seen
65	Dec.	11	6	5365.5	150	6	*v* not seen
66		19	6	5373.5	150	6	*v* not seen
	1901						
67	Jan.	24	7	5409.5	350	40	*t* 2 *v*, *v* 2 *u*
68	May	16	8	5521.6	80	12	*b* 5 *v*, *r* 2 *v*, *v* 4 *s*
69	Oct.	18	8	5676.6	80	12	*v* not seen
70	Nov.	1	8	5690.6	275	12	*t* 2 *v*, *v* 1 *w*, *v* 1 *u*
71		4	9	5693.6	460	40	*t* 4 *v*, *w* 4 *v*, *v u*
72	Dec.	21	7	5740.5	350	40	*v* not seen
	1902						
73	Mar.	13	11	5827.0			24 inch reflector, *v* $\frac{1}{2}^{m}$ ± < *y*
74		15	14	5824.9	460	40	*v* or *y* not seen
75	May	2		5872	237	40	*t* 6 *v*, *u* 4 *v*, *v* 4 *x*
76		29	9	5899.6	237	40	photometer
77	Oct.	1	9	6024.6	237	40	*s* 2-3 *v*, *b* 10 *v*, *v* 10 *h* ±
78		31	9	6053.6	237	40	*s v* ±

TABLE IV — *Continued*

	Details in Steps	Means		Seeing	Remarks
		Steps	Mag.		
43	29.8, 28.0, 29.8	29.2	11.89	good	
44	28.3, 27.0, 28.8	28.0	12.11	fair	
45				good	Nothing as bright as 16 *m* within 1′.5 of *v*
46	16.4, 25.8, 20.9	21.0	13.42	fair, moon	
47	23.4, 22.5, 20.8	22.8	13.11	good	limit *t*
48	16.8, 18.0	17.4	14.05	good	limit *w*
49	17.8, 18.0	17.9	14.00	good	
50		<18	<14.0	moon, haze	
51	11.1, 11.8, 10.5, 10.5	10.9	15.20	good	limit, 2–3 < *x*
52	8.5, 7.5, 7.0 7.5, 7.0, 9.0 8.5, 6.5, 9.0	7.8	15.71	fair	limit *v*
53	3.0	3.0	16.50	fair	limit *v*
54			<15	moon	limit 1 < *t*
55			<15	fair	limit 1 < *t*
56		< 3.0	<16.5		limit 4–5 < *x*
57			<16.5		Crossley plate
58		< 4.5	<16.2	good	limit 3 < *x*
59		< 3.5	<16.4	good	limit 4 < *x*
60		<13.5	<14.7	moon	limit *u*
61			<16		limit ½ *m* < *x*
62	4.0, 4.0	4.0	16.33		limit *y*
63	5.5	5.5	16.09	thro' clouds	
64		<22	<13	good	
65		<22	<13	good	
66		<23	<13	good	
67	15.8, 15.5	15.6	14.39	fair	
68	37.3, 35.8, 36.8	36.6	10.42	good	
69		<19.4	<13.7	fair	limit 3–4 < *g*
70	15.8, 18.0, 14.5	16.1	14.30	good	limit 1 < *w*
71	13.8, 13.0, 13.5	13.4	14.76	poor	limit 1 < *u*
72		<13.5	<14.7	fair, moon	limit *u*
73			17½ ±	good	
74		< 5	<16	unsteady	limit 2–3 < *x*
75	11.8, 9.5, 11.5	11.3	15.10	good	
76			15.21	fairly good	
77	30.3, 32.3, 39.0	30	11.7 ±	poor	
78		33	11.1 ±	poor	

used in this work. If we assume 13.0 magnitude as the limit of 6 inches aperture, we have the following table:

Aperture		6	12	40
Magnifying power		200	275	460
		M	M	M
Limit	Observed	13.2	14.2	17.0
	Calculated	13.0	14.5	17.

The visibility of stars fainter than the theoretical limit of a 6-inch reflector seems due to the use of a (proportionally) high power, while the lack in the 12-inch seems due to the comparatively low power.

I wish in this place to acknowledge my indebtedness for assistance in the work to Miss Kate Bloodgood, who has formed the light-scale and reduced the visual observations; to Mr. Ferdinand Ellerman for preparing the photograph of the field for the engraver, and to Mr. Frank Sullivan, who recorded the photometric measures with the 40-inch.

YERKES OBSERVATORY,
December 1902.

THE PROPER MOTIONS OF THE STARS.

By GAVIN J. BURNS.

OF all methods of investigating the problem of the constitution of the stellar universe, none is more promising than the study of the proper motions of the stars. Some of the inferences that may be drawn from what is already known of the proper motions of the stars will be considered in this paper.

The material for this investigation has been obtained from the catalogue of the proper motions of 2,641 stars by M. J. Bossert, published in the *Annales de l'observatoire de Paris* in 1896. This catalogue is confined to proper motions of over $0^s.01$ in R.A. or $0''.1$ in declination which have been deduced from observations extending over at least fifty years. The following table gives a summary of the stars contained in this catalogue, arranged according to magnitudes. The numbers in the first column are proper motions measured on the arc of a great circle:

PROPER MOTION	MAGNITUDE									TOTAL
	1 to 1.9	2 to 2.9	3 to 3.9	4 to 4.9	5 to 5.9	6 to 6.9	7 to 7.9	8 to 8.9	9 and over	
Sec. per annum.....	No.	No.	No.	No.	No.	No.	No.	No.	No.	
0.1 to 0.2.....	4	10	33	68	128	218	265	236	70	1,032
0.2 to 0.4.....	4	12	23	50	91	213	271	292	67	1,023
0.4 to 0.8.....	3	7	9	22	24	64	88	113	24	354
Over 0.8.....	4	0	8	11	17	30	30	29	14	143
Total.....	15	29	73	151	260	525	654	670	175	2,552

On examining the above table it will be found that little connection can be traced between the magnitude of a star and its proper motion. The first two lines, for instance, consist of a nearly equal number of stars distributed throughout the various magnitudes in nearly the same proportion. The numbers

also in columns 6 to 9 show much the same distribution of proper motions. Thus we find that

The average magnitude of a star having a p. m. of 0".1 to 0".2 is 7.0.
The average magnitude of a star having a p. m. of 0".2 to 0".4 is 7.1.
The average magnitude of a star having a p. m. of 0".4 to 0".8 is 7.2.
The average magnitude of a star having a p. m. of over 0".8 is 6.9.

I believe it is now generally admitted by astronomers that the proper motion of the stars is, on an average, proportional to their parallax. It follows that a very large proportion of the fainter stars out of the 2,552 above enumerated must be at nearly the same distance as the brighter stars. Each number, in fact, in any horizontal line represents a group of stars at nearly the same average distance from us. Now, if we leave out of consideration stars of the ninth magnitude and over, it will be seen that the numbers increase from left to right with only three exceptions. Hence, if we agree to measure the "size" of a star by the quantity of light it emits, it follows that, so far as the stars actually enumerated are concerned, they become more numerous as they decrease in "size." The apparent exception in the case of stars of the ninth magnitude is doubtless due to lack of material for calculating the proper motion of the fainter stars.

The question here arises as to how far the 2,552 stars under consideration may be taken as representative of the stars in general. Now, it is much more likely that, for any given proper motion and for any two given magnitudes, the number of stars of the fainter magnitude will hereafter be augmented by further observation and research than the number of stars of the brighter magnitude. Taking this into consideration, it appears very probable that the stars do, in fact, increase in number as they decrease in size. Doubtless, there is some particular "size" of star which is more numerous than any other, larger or smaller, but there are no data for determining what it is.

Let us next see what light may be gained on the question of the distribution of stars in space.

The following list shows the number of stars under the seventh magnitude:

Under magnitude 2.00 there are 40 stars.
From 2.00 to 2.99 there are 99 stars = 2.5×40.
From 3.00 to 3.99 there are 317 stars = 3.2×99.
From 4.00 to 4.99 there are 1,020 stars = 3.2×317.
From 5.00 to 5.99 there are 2,865 stars = 2.8×1,010.
From 6.00 to 6.99 there are 9,554 stars = 3.4×2,865.

The above numbers, except the last, have been obtained by actual enumeration from the Harvard *Photometries*. The 9,554 stars is an estimate made by Mr. J. E. Gore from the numbers in the Harvard *Photometric Durchmusterung*. Professor Newcomb calls the ratio of the numbers of stars of two successive magnitudes (given above) the "star-ratio." It will be noted that the star-ratio to the seventh magnitude has an average value of 3.

Now, suppose that at the distance x_1 there are n stars of such a size as to appear as stars of the first magnitude; that at the distance x_2 a star of the same size appears as a star of the *second* magnitude, and so on. Suppose, further, that the number of stars at the distance x_1 of the second magnitude is $n m_2$, of the third magnitude $n m_3$, etc. Then, on the hypothesis that stars of all sizes are distributed in the ratio $1, r, r^2, r^3 \ldots$ at the distances $x_1, x_2, x_3, x_4 \ldots$, we find that, if stars nearer than x_1 are neglected,

The number of stars of first magnitude $= n$
The number of stars of second magnitude $= nm_2 + nr$
The number of stars of third magnitude $= nm_3 + nm_2 r + nr^2$
The number of stars of fourth magnitude $= nm_4 + nm_3 r + nm_2 r^2 + nr^3$.

Consequently, if R_1, R_2, R_3 are the star ratios,

$$R_1 = m_2 + r$$

$$R_2 = \frac{m_3}{m_2 + r} + r$$

$$R_3 = \frac{m_4}{m_3 + m_2 r + r^2} + r .$$

We see from the above formulæ that r is always less than the star-ratio, unless $m = 0$, in which case $R = r$. If all the m's be equal, the star-ratio will constantly diminish for higher magnitude. If, as appears to be the case in reality, the m's increase with the magnitude, r might be much less than the star-ratio. Now, it can be readily shown that for a uniform distribution of

the stars in space $r = 4$. But $R = 3$. Consequently $r < 3$, and probably r is considerably less than 3. This implies that the stars thin out as their distance from us increases. It is true that this reasoning is not demonstrative, for we have assumed r to be constant. Nevertheless it is difficult to avoid the conclusion that with a star-ratio of 3 the stars do in reality thin out as they recede from our system.

The truth of this may be shown in another way. If the stars were all moving at the same average velocity at right angles to the line of sight, the number of stars having angular proper motions lying between a and $2a$ would be eight times the number of those having proper motions between $2a$ and $4a$, upon the hypothesis of uniform distribution. A glance at the table already given will show how greatly the smaller proper motions fall short of the number we should expect. It is true that this defect of small proper motions may be largely due to insufficiency of data, as large proper motions are much more likely to be discovered than small ones. If this were the only cause, we should expect that, as our knowledge extended, the known number of small proper motions would show a relative increase. But this does not seem to be the case. The following table gives the number of stars in Dunkin's list[1] published in 1863 and Bossert's catalogue published in 1896:

Proper Motion in Seconds per Annum.	Dunkin		Bossert	
	No. of stars	Per cent.	No. of stars	Per cent.
0.1 to 0.2	222	60	1,032	40
0.2 to 0.4	90	27	1,023	40
0.4 to 0.8	30	10	354	14
Over 0.8	12	3	143	6
Total	300	100	2,552	100

It will be remembered that Bossert's catalogue omits proper motions under 0".01 in R. A. Consequently it is deficient in proper motions between 0".1 and 0".2 of arc. When allowance is made for this deficiency, the two lists will be found to contain nearly the same proportion of proper motions of different mag-

[1] *Memoirs of R. A. S.*, 32.

nitudes, except for those over 0″.8, in which there is a decided increase. This is exactly the opposite of what might have been anticipated. The inference is that there is in reality a much greater number of stars with large proper motions than there would be if they were uniformly distributed throughout the stellar universe.

Upon the whole we appear to be justified in coming to the following conclusions:

1. The stars increase in number as they decrease in size. In other words, smaller stars are more numerous than larger ones.

2. The stars thin out as their distance from the solar system increases.

Another remarkable fact connected with the distribution of proper motions is that double stars have frequently large proper motions. The following comparison illustrates this:

Average p. m. of 778 stars from first to fifth magnitude (average magnitude = 4) contained in Dunkin's list, 0″.15.

Average p. m. of 54 double stars from first to seventh magnitude (average magnitude = 4.5) from Struve's catalogue, 0″.37.

The following table exemplifies the peculiarity still further:

Magnitude	Average p. m. according to Auwers	Average p. m. according to Kapteyn	Average p. m. of southern double stars (Innes)		Average p. m. of Burnham's double stars	
1	0″.263	0″.196	(7)	0″.71	(3)	0″.07
2	.137		(11)	.09	(6)	.05
3	.090		(26)	.18	(12)	.18
4	.075	.147	(44)	.22	(23)	.07
5	.063	.101	(69)	.18	(58)	.08
6	.055	.079	(39)	.12	(71)	.15
7	.049	.066	(24)	.23	(15)	.12
8	.045	....	(12)	.37	(18)	.21

The numbers in brackets denote the number averaged. An allowance of 0.01 has been made for each star described as having a "small" proper motion in Innes' catalogue. It will be noted that it is the fainter stars that have the proper motions above the average.

This apparent large proper motion of double stars is worthy of more attention than seems to have been given to it.

WEYMOUTH, ENGLAND,
December 1, 1902.

THE ORBIT OF THE SPECTROSCOPIC BINARY η *ORIONIS.*

By WALTER S. ADAMS.

THE variation in the radial velocity of η *Orionis* was discovered at the Yerkes Observatory in December of 1901, and announced at the Washington meeting of the Astronomical and Astrophysical Society during that month. The star has been on the regular observing list since that time, and sufficient material has been secured to make an accurate determination of its orbit possible.

The star belongs to the class of binaries in which one component is relatively dark, as no certain evidence of superposed spectra has been found on any of the photographs. The spectrum is that of the *Orion* type, but contains, in addition to the regular helium and hydrogen lines, three silicon lines, and a number of lines due to oxygen and nitrogen which have proved of great value in determining the star's velocity. In general the lines may be said to be slightly better for purposes of measurement than in the case of the average star of this type, though still rather ill-defined and diffuse. At some points in the star's orbit the change in its radial velocity is so rapid as to amount to several kilometers in the course of the exposure required to photograph the spectrum, and this, no doubt, influences to a considerable degree the character of the lines upon the plates taken at such times.

The number of lines measured upon the different plates varies considerably, but in the great majority of cases is from eight to ten. A careful examination of the cases in which more or less lines have been measured has led to the conclusion that any attempt to make a distinction in the weight given to a plate according to the number of its lines would be fully as liable to introduce error as to eliminate it, and accordingly unit weight has been assigned in each case. So far as the individual lines are concerned, however, the method has been adopted of assign-

ing a weight to each line at the time of measurement, representing in the judgment of the observer the accuracy with which the line is measured. As the weights are assigned previous to any knowledge of the velocity values given by the different lines, this procedure is free from any tendency to be influenced by these values in giving the weights.

To illustrate the amount of range shown by the different lines, the results of the measures of two of the plates are given in full below. These plates are taken at random, one from among the earlier observations, and one from among the observations of the past few months, and are of an average quality.

B 269 1902, JANUARY 9, G. M. T. 13h 38m			B 448 1902, NOVEMBER 6, G. M. T. 18h 28m		
Line	Velocity	Weight	Line	Velocity	Weight
4340.634	+188.9km	2	4345.677	−50.7km	2
4349.541	187.8	1	4367.012	45.5	2
4388.100	199.3	2	4415.076	57.4	3
4415.076	190.0	2	4417.121	52.0	3
4417.121	191.4	2	4471.676	54 2	3
4471.676	195.1	2	4481.400	55.6	2
4552.750	198.4	3	4552.750	57.6	2
4574.900	191.6	1	4567.950	49.0	2
4591.066	192.2	1	4574.900	52.2	2
4649.250	188.8	2	4591.066	56.6	2
4713.308	189.3	3	4596.291	55.3	2
Weighted Mean	+192.4		Weighted Mean	−53.4	
Reduction to Sun	−13.4		Reduction to Sun	+15.3	
Radial Velocity	+179.0 km		Radial Velocity	−38.1 km	

The following table contains the series numbers, dates, and results of measurement of the twenty-eight plates which have been obtained, and which are used in the determination of the orbit. The range of velocity is very great, amounting to over 285 kilometers, and is the largest which has hitherto been found among binaries of this class.

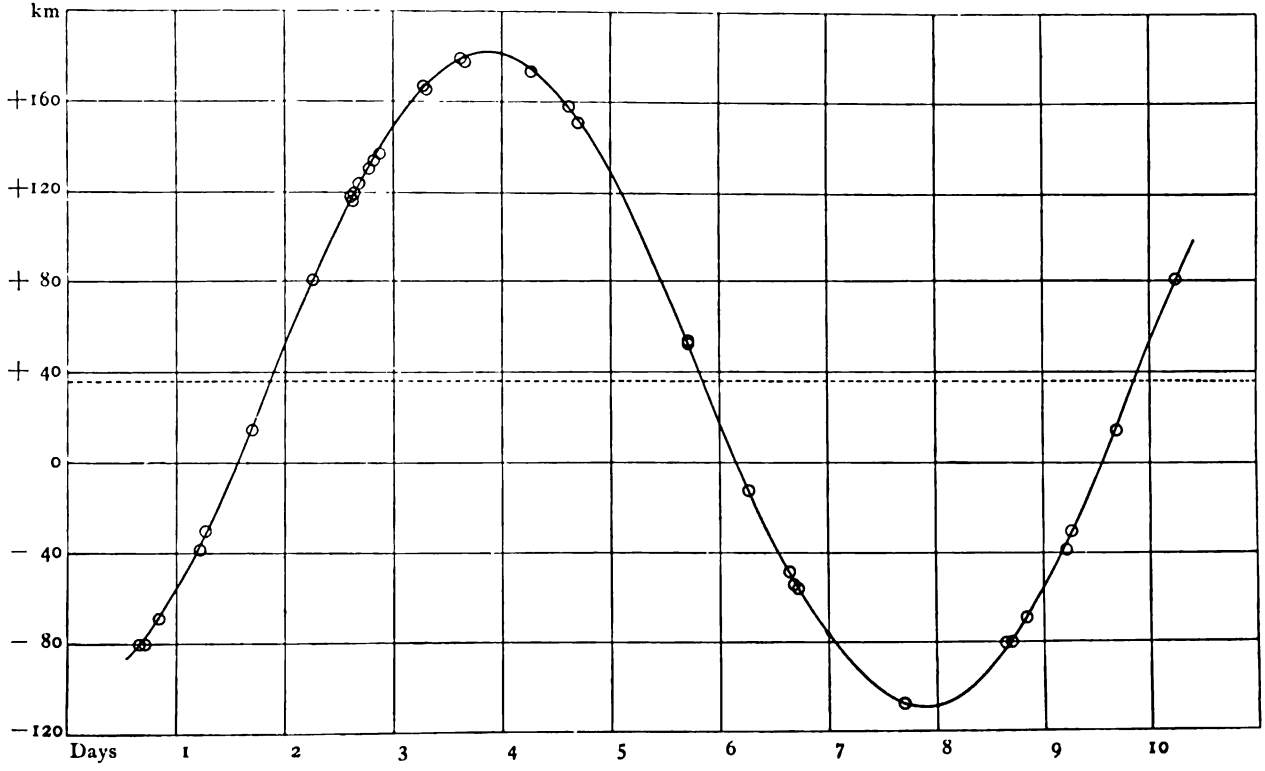
The spectrograms were all taken by the writer, with the exception of the first two and one of the last, which were obtained by Professor Frost.

Plate No.	Date G. M. T.	Velocity	O.–C.
		km.	km.
B 251	1901, November 27.846	− 68.9	−0.6
A 294	December 6.691	+ 14.6	−1.8
B 256	December 18.703	− 54.4	+1.9
A 296	December 19.676	− 53.7	−0.5
B 260	December 31.585	+115.9	−1.0
B 265	December 31.835	+136.6	−2.4
A 299	1902, January 4.691	− 55.1	+1.2
A 305	January 8.722	+130.0	0.0
B 269	January 9.568	+179.0	+0.1
B 274	January 16.547	+117.6	+2.2
B 279	January 16.762	+133.1	−1.6
A 309	January 22.590	− 80.5	+0.6
B 283	January 24.569	+119.8	+1.3
A 310	February 10.565	+177.8	−1.9
A 315	February 11.525	+158.5	+0.1
A 316	February 12.629	+ 53.1	+1.1
A 321	February 19.598	+151.0	−2.0
A 326	February 21.549	− 47.9	+1.3
B 296	March 13.564	+123.9	+0.2
A 339	March 27.563	− 80.0	−2.6
B 309	April 3.542	−106.5	0.0
B 405	September 3.908	− 29.5	+0.4
A 372	September 4.892	+ 80.5	+1.6
B 411	September 13.901	+166.6	+0.8
B 416	October 8.860	+173.6	−1.9
B 426	October 15.888	+165.0	−2.2
B 448	November 6.769	− 38.1	−2.6
B 460	November 19.822	− 11.8	0.0

All of the above measures were made by Adams with the exception of those of B 269 and B 309, which are the means of measures by Frost and Adams. Plate B 279 was measured twice, and the value given is the mean of the two determinations.

The method used in the derivation of the orbit was that of Lehmann-Filhés, although the very low value of the eccentricity would' no doubt, have made some of the methods using developments in power series equally applicable. The observations were plotted on millimeter paper with a period of $U = 7.9896$ days, and a smooth curve was drawn through them subject to the condition that, after an abscissa had been constructed, making the areas above and below it equal, the adjacent areas included between the maximum and minimum ordinates, the abscissa, and the curve should be equal. The areas in question were adjusted by means of a planimeter, and the following quan-

PLATE III.



Light Curve of η *Orionis.*

tities were obtained as a basis for the computation of the elements:

Velocity of system $V = +35.5$ km.
$A = 146.5$ km; $B = 143.0$ km; $z_1 = +1.35$; $z_2 = -1.38$.

The unit in the cases of z_1 and z_2 is arbitrary, as their ratio alone is required. The notation is that of Lehmann-Filhés.

The elements derived from these quantities are as follows:

$$u_1 = 90^\circ\ 41'.6$$
$$\omega = 42^\circ\ 16'$$
$$e = 0.016$$
$$\log \mu = 9.89566 \text{ or,}$$
$$\mu = 45^\circ.059$$
$$T = 1901, \text{ December } 1.821$$
$$a \sin i = 15{,}901{,}000 \text{ km.}$$

Owing to the very low value of the eccentricity, ω and T are the most uncertain of these elements, but even in these cases the errors should not be large.

An ephemeris was computed with these elements, and the differences between the observed and the computed velocities are given in the column O.−C. of the table above. In view of the character of the spectrum measured, these residuals are entirely satisfactory. A least squares solution might reduce them slightly, but as no significance is to be attached to fractions of a kilometer in measures of stars having this type of spectrum, it has not seemed desirable to undertake it. The accompanying diagram shows the curve derived from the set of elements, and the positions of the observed velocities in reference to it. Owing to the high velocities involved, the scale is necessarily small, but the general agreement of the values is well shown.

In conclusion attention may be called to the fact that η *Orionis* is also a visual double star with components of the fourth and sixth magnitudes. It is, of course, with the brighter of these that we are dealing here.

YERKES OBSERVATORY,
December 20, 1902.

HERSCHEL'S NEBULOUS REGIONS.[1]

By ISAAC ROBERTS.

WILLIAM HERSCHEL'S observed nebulous regions, fifty-two in number, are in this paper compared with the writer's photographs of the same regions, taken, simultaneously, with the 20-inch reflector and the 5-inch Cooke lens.

The nebulous regions referred to in the above title were described by William Herschel in a paper which he communicated to the Royal Society in the year 1811, and which was published in the *Philosophical Transactions*, Vol. **74,** under the heading "Construction of the Heavens."

So far as I can gather, no systematic efforts were made to verify Herschel's observations of these fifty-two regions until six years ago, when the work of photographing them was commenced at my Observatory, using for the purpose the 20-inch reflector and the 5-inch Cooke lens.

The photographs were taken in duplicate, simultaneously, with exposures of ninety minutes' duration, and at times when the objects were as near as practicable to the meridian and the sky clear during the exposure. The plates were selected and tested for sensitiveness, so that with the reflector the images of stars to about the seventeenth magnitude would appear on the plates and stars of about the fifteenth magnitude would appear on the plates exposed with the Cooke lens.

My long previous experience in photographing the heavens enabled me to judge that under these conditions nebulosity of at least the degree of faintness that could be seen by Herschel with his two- and four-foot reflectors would be shown on the photographic plates.

The tabular method adopted by Herschel in publishing the results of his telescopic observations enables me to give the photographic results in a concise and intelligible form, coincid ing line by line with his, by comparing the headings and reading the descriptive matter relating to each object respectively.

[1]The manuscript of this article was accepted by the editors on the supposition that it was not to be published in other current journals.

TABLE OF WILLIAM HERSCHEL'S FIFTY-TWO REGIONS.

Herschel's Nos.	R. A. (1900)	Dec. (1900)	Herschel's description in *Phil. Trans.*, 1811	Dates when photographs were taken	Isaac Roberts' descriptions of his photographs
1	$0^h\ 10^m\ 8^s$	$9^\circ\ 20'$	Much affected with nebulosity	1900, Nov. 22	Sky clear ; stars small and faint, and few in number ; large areas void of stars ; no nebulosity on plate.
2	0 17 37	3 59	Much affected	1899, Sept. 5	Sky clear ; stars small and faint, and not very numerous ; large areas void of stars ; no nebulosity on plate ; film dark.
3	0 22 23	29 9	Affected	1899, Sept. 9	Sky clear ; stars small and very numerous ; one star of 5.9 mag., *B. D.* $28^\circ 75$, on plate ; small areas void of stars ; no nebulosity.
4	0 25 37	3 59	Much affected	1900, Nov. 22	Sky clear ; stars few and faint ; large areas void of stars ; no nebulosity on plate.
5	0 30 11	23 25	Much affected	1900, Oct. 27	Sky clear ; stars faint and numerous ; nebulæ H *III*, 476, and *N. G. C.* 169, d'Arrest and Ld. R., together with other fainter ones on plate ; many areas void of stars ; no diffused nebulosity.
6	0 36 28	0 29	Appeared to be affected with very faint nebulosity	1899, Oct. 28	Sky very clear ; stars small and very few in number ; large areas void of stars ; some small nebulæ on plate ; no diffused nebulosity.
7	0 38 0	41 10	Affected with nebulosity	1895, Oct. 17	Sky very clear ; stars crowded on plate ; many small areas void of stars ; several photographs have been taken of this region, which includes the great *Andromeda* nebula *M.* 31, part of the *n. f.* end of which would cross Herschel's field of view in this sweep.
8	0 39 27	39 16	Unequally affected	1900, Oct. 17	Sky clear ; stars crowded on plate ; many small areas void of stars ; part of *s. p.* end of *M.* 31 on plate ; no other diffused nebulosity.
9	0 41 19	43 30	Suspected faint nebulosity	1900, Oct. 26	Sky clear ; stars small and crowded on plate ; many small areas void of stars ; no diffused nebulosity.
10	0 48 38	43 35	Suspected faint nebulosity	1900, Oct. 26	Sky clear ; stars small and crowded on plate ; numerous areas void of stars ; nebula *N. G. C.* 317 on plate ; no diffused nebulosity.
11	1 41 8	29 48	Suspected to be tinged with milky nebulosity	1900, Nov. 27	Sky clear ; stars small and numerous ; large areas void of stars ; no nebulosity.
12	2 27 55	19 0	Much affected with nebulosity	1900, Dec. 13	Sky clear ; stars small and not very numerous ; large areas void of stars ; some very small and faint nebulæ on plate ; no diffused nebulosity.
13	4 2 14	25 11	Much affected	1901, Feb. 13	Sky very clear ; stars small and numerous ; large areas void of stars ; no nebulosity.
14	4 23 51	35 7	Suspected pretty strong nebulosity	1901, Feb. 13	Sky very clear ; stars small and crowded on *s.* and *s. p.* sides, but few on the rest of the plate ; large areas void of stars ; nebula H *I* 217 and also a tenth mag. star surrounded by very faint nebulosity 11′ *s. f.* H *I* 217 on plate ; no nebulous region.
15	4 24 51	35 8	Suspected nebulosity		
16	4 26 29	−7 30	Strong milky nebulosity	1901, Feb. 14	Sky clear ; stars small and very few on plate ; large areas void of stars ; no nebulosity.

TABLE OF WILLIAM HERSCHEL'S FIFTY-TWO REGIONS.—*Continued.*

Herschel's Nos.	R. A. (1900)	Dec. (1900)	Herschel's description in *Phil. Trans.*, 1811	Dates when photographs were taken	Isaac Roberts' descriptions of his photographs
17	4 29 2	20 50	Much affected	1901, Feb. 15	Sky very clear; stars small and very numerous; small areas void of stars; no nebulosity.
18	4 44 5	20 50	Much affected	1901, Feb. 15	Sky very clear; stars small and crowded on plate; small areas void of stars; no nebulosity.
19	4 52 17	26 45	Strong suspicion of very faint milky nebulosity	1901, March 9	Sky clear; stars small and very few; large areas void of stars; no nebulosity.
20	5 15 50	25 1	Very much affected	1901, M'ch 12	Sky clear; stars small and very few on plate; large areas void of stars; no nebulosity.
21	5 19 20	25 1	Affected		
22	5 28 53	−6 56	Affected with milky nebulosity	1901, M'ch 13	Sky clear; stars not very numerous; large areas void of stars; H *IV* 33 *Orionis* on plate; no nebulosity.
23	5 30 10	−2 43	Affected	1901, M'ch 12	Sky clear; stars small and very few; large areas void of stars; no nebulosity.
24	5 31 56	−4 18	Visible and unequally bright nebulosity. I am pretty sure that this joins to the great nebula in *Orion*	1902, M'ch 5	Sky very clear; stars small and not very numerous; areas void of stars; no nebulosity on plate.
25	5 35 34	−2 31	Diffused milky nebulosity	1900, Jan. 25	Sky clear; stars very numerous on *p.* half of plate, but few on *f.* half, where there are large areas void of stars; large cloud of nebulosity *n. f.* ζ *Orionis* with broad division void of stars, but with some nebulosity in *s. f.* to *n. p.* direction; other divisions break up the cloud into separate masses. To the *s.* of ζ is a stream of nebulosity, 54 minutes of arc in length, with an embayment free from nebulosity dividing it in halves. Another faint nebulosity extends from ζ 27 minutes of arc toward the *s., s. p.* and *n. p.* The star *B. D.* — 1°1001 is in the midst of nebulosity, and it has a companion on the *s. p.* side. The star *B. D.* — 1°1005 is involved in a large cloud of streaky nebulosity, and it has a companion on the *p.* side. The star *B. D.* — 2°1345 is H *IV* 24, *N. G. C.* 2023; it is in the midst of a large, dense streaky cloud of nebulosity which has in it condensations and remarkable rifts free from nebulosity; near the *s.* end of one of these rifts is a twelfth magnitude star. The star *B. D.* — 2°1350 is in the midst of a cloud of nebulosity with some faint structure in it, and it has a companion on the *n. p.* side. The region here referred to, which covers four square degrees of the sky, has so many remarkable features that it is necessary, in order to make it intelligible to the reader, to present the photograph annexed along with the above description.

TABLE OF WILLIAM HERSCHEL'S FIFTY-TWO REGIONS.—*Continued.*

Herschel's Nos.	R. A. (1960)	Dec. (1900)	Herschel's description in *Phil. Trans.*, 1811	Dates when photographs were taken	Isaac Roberts' descriptions of his photographs
26	$5^h\ 36^m\ 52^s$	$-6°\ 57'$	A pretty strong suspicion of nebulosity	1901, Mar. 22	Sky clear; stars small and few; large areas void of stars; no nebulosity.
27	5 43 11	$+1\ 8$	Affected with milky nebulosity	1901, Mar. 13	Sky clear; stars very few in number; large areas void of stars; no nebulosity.
28	6 1 1	$+3\ 44$	Much affected.	1902, Jan. 29	Sky clear; stars crowded on *n. p.* and *s. p.* sides; large areas void of stars; no nebulosity.
29	6 0 54	$-20\ 27$	Affected	1902, Mar. 6	Sky clear; stars small and very numerous; many areas void of stars; no nebulosity.
30	6 40 7	$41\ 16$	Affected	1901, Mar. 22	Sky clear; stars few in number; large areas void of stars; cluster H. *VIII*, 71 on plate; no nebulosity.
31	9 27 32	$-18\ 27$	Affected	1902, Mar. 6	Sky clear; stars small and few in number; large areas void of stars; no nebulosity
32	9 36 43	$71\ 13$	Much affected with very faint whitish nebulosity	1901, Aprl. 12	Sky clear; stars small and numerous; several large areas void of stars: no nebulosity.
33	10 11 50	$-9\ 3$	Very faint whitish nebulosity	1901, Aprl. 15	Sky clear; stars small and numerous; large areas void of stars; no nebulosity.
34	10 22 25	$51\ 32$	Much affected	1901, Aprl. 13	Sky clear; stars small and not numerous; large areas void of stars; no nebulosity.
35	10 40 59	$62\ 45$	Affected with very faint nebulosity	1901, Aprl. 14	Sky clear; stars small and not very numerous; large areas void of stars; no nebulosity.
36	11 4 30	$62\ 44$	Affected	1901, Aprl. 15	Sky clear; stars small and numerous: areas void of stars; several small, faint nebulæ on plate; no diffused nebulosity.
37	12 2 5	$30\ 37$	Affected with whitish nebulosity	1901, Aprl. 17	Sky clear; stars small and few in number; large areas void of stars; H. *II*, 321, and H. *II*, 802, on plate; no nebulosity.
38	12 12 40	$30\ 37$	Affected with whitish nebulosity	1901, Aprl. 18	Sky clear; stars few in number; large areas void of stars; four small prominent nebulæ on plate; no diffused nebulosity.
39	13 12 15	$34\ 8$	Much affected	1901, Aprl. 17	Sky clear; stars not very numerous; large areas void of stars; no nebulosity.
40	14 2 20	34	Very much affected and many faint nebulæ suspected	1899, June 2	Sky clear; stars small and not numerous; areas void of stars; no nebulosity.
41	15 9 37	$18\ 57$	Affected with very faint nebulosity	1899, June 12	Sky clear; stars small and not very numerous; areas void of stars; no nebulosity.
42	21 3 26	$-1\ 53$	Much affected with whitish nebulosity	1902, Nov. 4	Sky clear; stars very numerous; no nebulosity.
					Herschel's sweep 42, as given in the *Phil. Trans.* (*R. A.* (1800), $20^h\ 58^m\ 20^s$, *N. P. D.* (1800) = 92 17) is not in sequence; as this may be due to a typographical error in one of the co-ordinates, a plate corresponding to *R. A.* (1800) = $20^h\ 38^m\ 20^s$, *N. P. D.* (1800) = 92 17 was taken on Aug. 28, 1897, as follows:
				1897, Aug. 28	Sky very clear; stars crowded on plate; no nebulosity.
43	20 53 15	$16\ 44$	A good deal affected	1897. Oct. 20	Sky clear; stars crowded on plate; no nebulosity.

TABLE OF WILLIAM HERSCHEL'S FIFTY-TWO REGIONS.—*Continued.*

Herschel's Nos.	R. A. (1900)	Dec. (1900)	Herschel's description in *Phil. Trans.* 1811	Dates when photographs were taken.	Isaac Roberts' descriptions of his photographs
44	20 54 34	43 32	Faint milky nebulosity scattered over this space, in some places pretty bright	1896, Oct. 10	Sky very clear; stars crowded on parts of plate; large areas void of stars on others; Nebula, H. *V* 37, *N.G. C.* 7000 forms part of this region; the photograph shows it as a magnificent object. I have published a photograph of this region in Vol. II of *Stars, Star-Clusters* and *Nebulæ*, pl. 24. p. 155. and also in *Knowledge*, Nov. 1, 1898; a copy is also annexed to this paper.
45	20 57 34	−1 34	Much affected with whitish nebulosity	1897, Sept. 21	Sky clear; stars small and numerous; no nebulosity.
46	20 56 55	43 16	Suspected nebulosity joining to plainly visible diffused nebulosity	1896, Oct. 10	Regions 44 and 46 are on the same plate; see description given above, No. 44.
47	21 5 8	14 21	Affected	1899, Aug. 6	Sky clear; stars small and crowded on plate; no nebulosity.
48	21 34 15	10 19	Much affected	1898, Oct. 12	Sky clear; stars small and numerous; areas void of stars; no nebulosity.
49	21 46 52	21 31	Affected	1899, Aug. 9	Sky clear; stars small and crowded; areas void of stars; no nebulosity.
50	22 57 24	25 45	Much affected	1898, Sept. 20	Sky clear; stars very numerous; areas void of stars; no nebulosity.
51	22 57 54	25 45	Affected		
52	23 0 17	29 17	A little affected	1902, Oct. 27	Sky clear; stars small and very numerous; areas void of stars; H. *II.* 212, on plate; no diffused nebulosity.

CONCLUSION.

The final results of the correlation of Herschel's nebulous regions and my photographs can be given in a few words as follows:

Of the fifty-two nebulous regions described by Herschel, the photographs show diffused nebulosity on four of them only; there is no visible trace of diffused nebulosity on forty-eight of the areas, but on the remaining four, which are Nos. 7, 25, 44, and 46, respectively, in the table, there is nebulosity with remarkable characteristic features, and these are delineated upon three of the photographs, regions Nos. 44 and 46 being on one plate.

STAKEFIELD, CROWBOROUGH, SUSSEX,
November 1902.

PLATE IV.

NEBULÆ AROUND ζ *ORIONIS* (January 25, 1900).

PLATE V.

NEBULA HERSCHEL V 37 *CYGNI* (October 10, 1896).

DIFFUSED NEBULOSITIES IN THE HEAVENS.

By E. E. BARNARD.

IN the present number of this JOURNAL there is a paper by Dr. Isaac Roberts containing an account of a photographic investigation of certain regions of the sky supposed by Herschel to be affected with diffused nebulosity. This investigation was made simultaneously with Dr. Roberts' 20-inch (51 cm) reflector and 5-inch (12.7 cm) Cooke portrait lens. The exposures given by Dr. Roberts to test the existence of these diffused nebulosities was ninety minutes, which, it is stated, showed stars of the sixteenth and seventeenth magnitudes. Out of these fifty-two regions he found only four that showed any traces of nebulosity. These four regions, however, had already been shown by numerous photographs to be nebulous. One of these, the great nebula north of *α Cygni*, was first photographed by Dr. Max Wolf some twelve years ago and has lately been called by him the "America Nebula"[1] from its striking resemblance to North America as shown on maps and globes.

The curious nebulous ribbon extending southward from *ζ Orionis* seems to have been first photographed by Professor W. H. Pickering and others as far back as 1889.

This question of large areas of diffused nebulosity in the sky is a very important one, not yet fully appreciated, but which must sooner or later have the highest bearing on a proper understanding of the physical condition of the universe. Dr. Roberts' negative results are so sweeping in character that it is highly important that anything tending to prove the existence of any of these questioned regions of nebulosity should be brought forward at once.

First, I do not think 90 minutes a sufficient exposure to test the existence of some of these nebulosities with Dr. Roberts'

[1] The "North America Nebula" would perhaps be more definite, for it is North America to which Dr. Max Wolf intends the compliment.

outfit. The photographing of sixteenth or seventeenth magnitude stars does not necessarily prove that the same exposure ought to show diffused nebulosities, for photographing faint stars often stands on an entirely different footing from photographing faint nebulosities.

Second, it is a little unreasonable to suppose that Herschel, who made so few blunders compared with the wonderful and varied work that he accomplished, should be so palpably mistaken in forty-eight out of fifty-two observations of this kind.

I have myself been very much interested in the diffused nebulosities of the sky and have independently come across some of these very regions of Herschel, besides others not noted by him. It has been a long-cherished desire of mine to investigate them further photographically, and I now hope to be able to put this desire into a practical reality within the next twelve months. Some of these regions I have already shown to be extraordinary features of the sky—as instances, the nebulous regions of 15 *Monocerotis*, of ρ *Ophiuchi*, the region surrounding the *Pleiades*, etc. All of these were known to me previous to their really being proved by the photographic plate and the portrait lens to be true nebulosities.

As far back as January 1892, in *Knowledge*, **15**, 14–16, I called special attention to these nebulous regions of Herschel and gave the table contained in Dr. Roberts' present article. Attention was called to these objects as being suitable for photographic investigation in these words:

> It would appear that this table of diffused nebulosities will just now be of extremely great value, as it at once points out to those interested in photographing such objects, the proper pointings of their exposures. I have taken the liberty to copy the foregoing table in full for the benefit of those not familiar with it and who may wish to try exposures on these objects.

What leads me to hope that more of these regions given by Herschel may yet be shown to be nebulous with photographic plates, is that one of these very objects, which the photographs of Dr. Roberts show to be free from nebulosity, is really the brightest portion of one of the most extraordinary nebulæ in the sky, as shown by photographs made by two independent observers with

three different photographic telescopes on several different occasions. I refer to region 27 of the list. This is described by Herschel as being "affected with milky nebulosity" and its position given for 1800.0 as α 5^h 38^m 5^s; P. D. 88° 55′.

The right ascension and declination for 1900.0 would be closely

$$\alpha = 5^h\ 43^m\ 13^s;\ \delta = +1^\circ\ 8'.$$

Dr. Roberts' note on this is: "Sky clear; stars very few in number; large areas void of stars; no nebulosity."

In *Popular Astronomy*, **2**, 151–154, December 1894, the writer has given some experiments with a very small magic lantern lens in photographing diffused nebulosities. In this an account is given of the finding of a great nebula extending in a curved form over the entire body of *Orion*. The brightest portion of the nebula is near 56 and 60 *Orionis*. From the photographs, the position of this brightest portion is in

$$1900.0\ \ \alpha = 5^h\ 43.7^m;\ \delta = +1^\circ\ 0'.$$

This would make it identical with Herschel's No. 27. These pictures were made with an ordinary child's magic lantern lens, of 1.5 inches diameter and 4.9 inches equivalent focus. The exposures were 1894, October 3, for 2^h 0^m, and October 24, for 1^h 15^m. The shortest exposure, 1^h 15^m, showed it best. I suppose a half-hour's exposure would have shown traces of it. Unknown to me at the time, this nebula had already been photographed in 1889 by Professor W. H. Pickering on Mount Wilson (altitude 6,250 feet) in southern California, with a Voigtlander portrait lens of 2.6 inches aperture and 8.6 inches equivalent focus, with an exposure of three hours.[1]

Besides the photographs made with the magic-lantern lens, I have two made with the six-inch Willard portrait lens that show portions of the nebula distinctly. They were made 1893, October 17, with 3^h 0^m exposure; and 1894, October 3, with 2^h 0^m exposure. In both the photographs with the Willard lens, the region of the nebulosity falls near the edge of the plate and hence the stars are deformed. At the time of making these

[1] See *Sidereal Messenger*, **9**, 1, 1889.

pictures I knew nothing of the existence of this object and therefore had no choice as to its location on the plate. One of these photographs has been selected for reproduction, though it will necessarily be unsatisfactory because of the position of the object close to the edge of the plate.

[Through the courtesy of the editor of *Popular Astronomy*, the map of *Orion* showing the location of the nebula is here reproduced. (Plate VI, A).]

To further show the reality of the nebulous region No. 27 of Herschel's list, I made an exposure of $2^h\ 10^m$ on the night of January 17, 1903, on the region with a small, cheap lantern lens belonging to Professor Hale. This lens is 1.6 inches in diameter and has an equivalent focus of 6.3 inches or $\frac{a}{f}=\frac{1}{4}$.

The sky was clear and the conditions fair. The resulting negative showed a fairly good field of nearly 25°.

Most of the great curved nebula is clearly shown, especially the region described by Herschel, which is now in question and which, as I have said, is the brightest portion of the nebula.

This plate does not show the nebula as strongly as those made with the lantern lens used by me at the Lick Observatory, because, for one reason, the present lens is relatively of smaller angular aperture, $\frac{1}{4}$ as compared with $\frac{1}{32}$; and for another reason the sky was not perhaps as pure; but the main portion of the nebula is conspicuous. There is therefore no question but that this nebulosity exists where Herschel saw it.

Just what the faintest stars are that appear on this plate I am not prepared to say, not having had any chance to make a comparison with the sky; but they are certainly several magnitudes brighter than the fifteenth or sixteenth magnitude.

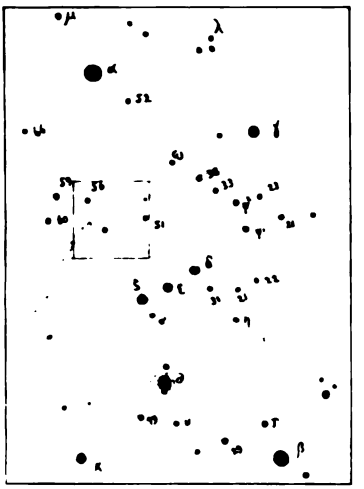
It was with the same instruments described in his present paper that Dr. Roberts failed to get any traces of the exterior nebulosities of the *Pleiades*, which have been shown by four observers with four different instruments not only to exist, but to be not at all difficult objects.

YERKES OBSERVATORY,
January 1903.

PLATE VI.

N.

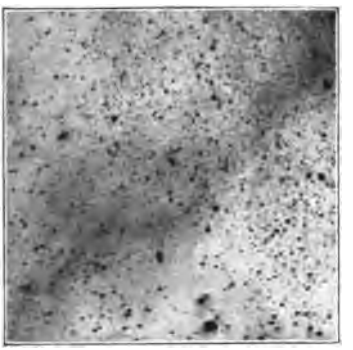
E. W.



A.

N.

E. W.



B.

THE GREAT CURVED NEBULA IN ORION.

A.—Drawing from two lantern lens photographs (Oct. 3 and 24, 1894).

B.—Photograph, with Willard lens, of region inclosed in the square in the drawing above (Oct. 17, 1893).

Minor Contributions and Notes.

NEW COLLABORATORS OF THE ASTROPHYSICAL JOURNAL.

THE ASTROPHYSICAL JOURNAL has from the outset enjoyed many advantages from the cordial co-operation of eminent investigators at home and abroad. The list of collaborators includes many of the ablest representatives of the branches of science with which the JOURNAL is most directly concerned. On the death of M. Cornu, who had assisted the JOURNAL in many ways, Professor Bélopolsky kindly accepted an invitation to fill the vacant place. As the editors have always desired to maintain a fairly even balance between the physical and the astronomical sides of the subject, they now take special pleasure in announcing that Professor Kayser and Professor Runge have signified their willingness to serve as collaborators. With this added strength on the laboratory side, the JOURNAL will continue to emphasize the importance of treating celestial and terrestrial problems of radiation from a single point of view.

A CONTRIBUTION TO THE KNOWLEDGE OF SPECTROSCOPIC METHODS.[1]

THE investigation undertaken by Dr. Konen proposes to test the method of the electric discharge under liquids as a means of solving the important problem of the origin of band-spectra. His paper touches upon the difficulties which presented themselves to former investigators — notably the transitory nature of band-spectra, their faintness, and the difficulty of ascertaining just what element or elements were essential to the production of any given spectrum; it mentions the methods and apparatus previously employed — namely, Geissler tubes and a flame or a carbon arc in closed receptacles; and states that we should expect this method of discharge under liquids to present fewer difficulties owing to the presence of fewer impurities.

The discussion of the *arc* discharge is preceded by a brief résumé

[1]Abstract, by Norton A. Kent, from advance proofs (furnished by the author) of a paper to appear in *Annalen der Physik.*

of the work of Davy, Masson, and Liveing and Dewar; and attention is called to the fact that very few investigations other than visual have been made upon this subject. The apparatus used by Konen consisted of a glass vessel silvered within, blackened without, and fitted with a quartz window. The pole pieces were held in brass clamps. The current employed varied from 15 to 20 amperes (below 15 the arc ran poorly, while above 20 the liquid was unnecessarily heated). The light passed through a quartz window and was focused by a quartz lens upon the slit of (1) a Steinheil prism (flint glass) spectroscope, in the first part of the investigation, and (2) later and in most of his work, a Rowland concave grating of one meter radius of curvature, rigidly mounted and used in the first spectrum — the extent of region covered being from $\lambda 5500$ to $\lambda 1800$. The spectrum was photographed on films.

As a result of the tearing asunder of the electrodes, the immersing water is rendered turbid, especially so where carbon or iron poles are used; all liquids are befouled sooner or later; water, alcohol, and electrolytes last the longest; some liquids admit of filtration; all clear by standing.

Besides a continuous spectrum there are always present (1) the H and K lines of calcium, also generally other lines of the calcium spectrum; (2) the two chief aluminum lines; and (3) the D lines of sodium. With metal poles *Fe*, *Cu*, *Ba*, *Na*, *K*, *Li*, *Tl*, and brass, the characteristic line spectra appear — but band spectra never. The liquid-arc lines are generally fully as sharp as those of the air-arc; many are sharper. The relative intensity varies and some air-arc lines are wholly absent in the liquid-arc spectrum. No displacement is apparent. The nature of the surrounding liquid has no influence upon the spectrum given by metal poles — this being true for alcohol, distilled water, and glycerine — and, with certain reservations, for salt solutions. In concentrated $BaCl_2$ and $CaCl_2$ solutions the strongest barium and calcium lines are present, but they are incomparably weaker than in air, and only the strongest calcium lines are reversed. The effect is not a function of decrease of temperature, nor of the behavior of the poles, *i. e.*, whether merely a single point or the whole pole glows. The probable physical and chemical processes which take place when carbon poles are used in a $NaCl$ solution is as follows: Near the poles the liquid is in the spheroidal shape; the $NaCl$ remains in solution; the glowing carbon and the water vapor react to produce CO and H, etc.; and these gases form the medium in which the true arc burns.

Vaporized liquid surrounds these gases, being forced aside by the latter, which are better conductors by reason of their hotter and ionized condition. Only the gases of the poles enter. This is further shown by the use of cored carbons filled with salt — an arrangement which gives the sodium spectrum.

The problem of the origin of the so-called carbon and cyanogen bands is then attacked. The carbon arc in air gives bands whose edges lie at λ 4606.33, 4216.12, 3883.55, 3590.48. These are generally ascribed to cyanogen. The band at λ 3883 photographs most quickly, and is therefore used as a test of the presence of the cyanogen spectrum. By using boiled water and boiled electrodes Konen succeeded in causing all four of these bands to disappear. Small amounts of air suffice to produce weak traces of them; therefore they will appear if unboiled carbons or conducting water be used, or if an excess of air be blown into the liquid, or, further, if the arc be submerged in strong water solutions of ammonia. In alcohol and CCl_4 the "cyanogen" bands are lacking; in petroleum a trace of λ 3883 appears when the poles used have been made from carbon rods glowing under aniline. In KNO_3 only traces appear — even weaker than those given by water containing air. But in view of what has been said of the part played by salt solutions in the development of the arc, this negative result is not surprising. *In every case the presence of nitrogen* is necessary to the production of the bands in question. This forms a new proof that these four bands belong to cyanogen.

As to the origin of the Swan spectrum — a much-contested question for over fifty years — Stokes advanced the hypothesis that the so-called "carbon" band spectrum belonged to carbonic oxide — an hypothesis the truth of which Konen considers not yet rigorously proven.

To facilitate the discussion of this question six classes of spectra are distinguished:

1. A continuous spectrum without bands or lines.
2. A line spectrum.
3. The cyanogen spectrum with edges at λ 3590, 3883, 4216, and 4606.
4. The so-called "carbon" band, or Swan spectrum, with principal heads at λ 4382, 4738, 5164, 5633, and 6187.
5. The so-called "carbonic oxide" spectrum with edges at λ 4509, 4834, 5196, 5608, 6078, 6399, 6622.
6. The carburetted hydrogen flame spectrum with edges λ 4315, 4368, 3872, and 3627.

The condition under which the first three appear are briefly given. No. 4 appears with No. 6 in the carburetted hydrogen flame; in the arc in air and different gases; in the liquid arc under certain conditions; in vacuum tubes filled with carburetted hydrogen; in the cyanogen flame; in carbon-dioxide gas at moderate pressures; in tubes during the first part of the discharge in carbonic-oxide gas; in the brush discharge in alcohol, glycerine, ether, acetic acid, etc.; in the Sun; and, doubtfully, in numerous other instances. The Swan spectrum is absent in the *CO* and CS_2 flames. No. 5 is obtained from an impurity in Geissler tubes; in *CO* and CO_2 tubes; also with oxygen or air in various carbon compounds. It is absent in the *CO* and CS_2 flame, and in the spark of hydrogen or water vapor. No. 6 is obtained only by burning carburetted hydrogen with oxygen in, *e. g.*, a Bunsen flame. It is absent in the spark discharge between carbon electrodes in air, CO_2 and *H;* also when No. 4 is strongly present. It is a remarkable fact that most observers have identified the flame or Swan spectrum with No. 4.

Spectrum No. 4 is then discussed. The question of its origin is connected closely with that of No. 5. The points to be settled are: Does the spectrum belong to (1) carbon itself; (2) to the positive ion of carbon in an ionized state; (3) to a hydrogen compound of carbon; or (4) to an oxygen compound — *i. e.*, to *CO ?* (1) and (2) may be treated together. The results obtained with cyanogen and water, etc., show that (3) is out of the question. Moreover as no rule has been found whereby a certain spectrum may be shown to be due to one of two components, or to the "completed compound," or to all these together, it is clear that some uncertainty must enter into the proof that a definite compound causes that spectrum. In the light of this reasoning Smithell's investigation, in which he sought to explain the behavior of *CO* and CO_2 by different degrees of dissociation, is very artificial and, at best, hypothetical. If it cannot be shown indubitably that the spectrum in question comes from a finished product, then the proof that it is caused by the presence of *CO*, for instance, means no more than that one or the other element is essential.

The real question is: "Is the presence of oxygen necessary to the formation of the 'carbon' band-spectrum?" Two experimental difficulties are present. An affirmative result — the presence of a certain band in a given case — admits of no conclusion; while the value of a negative result — the absence of that band — depends wholly upon the sensitiveness of the spectral reactions. If this limiting sensitiveness be passed it is practically and chemically impossible to exclude the

presence of a definite substance. But it is possible to establish one limit showing how much of an impurity must be present to produce a given spectrum, and another showing how much can be present without being active spectroscopically. Only within such bounds has the following experiment any meaning; and it *cannot presume to lead to a decision in the question of the origin of spectra 4 and 5*.

Metal and carbon poles were used. The latter were placed in a glass tube attached to a mercury pump; the air was exhausted, the tube steadily heated, and an electric discharge — as strong as possible — was passed between the pole pieces. Then the tube was sealed off and broken open beneath the surface of a testing liquid; the terminals were then placed in position, boiled in liquid, and covered with a layer of graphite by the action of the arc discharge — a result most easily accomplished by the use of aniline. The liquids used — water, salt solutions, alcohol, glycerine, carbon tetrachloride, aniline, carbon bisulphide, petroleum, and ammonia — were chemically pure and, as far as possible, were preserved over phosphorous pentoxide in the dark. In distilled and conducting water, spectrum No. 4 appeared complete. As was suspected, *CO* is developed in large amounts. An air-blast made no difference. The same is true for alcohol, glycerine, and ammonia. The bands are less intense than in air. Metal terminals cause a further weakening. In some liquids, *e. g.*, very strong salt solutions, the bands can be blotted out. It is well known that in air the introduction of metals weakens the carbon and cyanogen bands. In both cases it is quite appropriate to explain the phenomenon in accordance with the *CO* hypothesis, as a process of oxidation in which the metal seizes the oxygen. The temperature of the carbon poles under water plays a part. With metals this temperature is visibly less, as is clear from the smaller consumption of carbon and the shrinking of the heated area of the pole, and it is in precisely these instances that the carbon bands are lacking. Carbon bisulphide fogs quickly, permitting only visual observations. In all cases the carbon bands are unquestionably present. In aniline bands appear at λ 56, 51, and 47, but the decomposition produces such strong absorption near λ 3883 that that region of the plate is not affected at all, and the cyanogen band therefore does not appear. In carbon tetrachloride these three bands are stronger than in aniline; they are most intense in petroleum and here a trace of λ 3883 appears, while the bands of spectrum No. 6 as well as all lines are absent. Visual observation of carbon bisulphide, chloroform, and turpentine give the same result, apart from the continuous spectrum the green band alone is present, and it only faintly.

If the sensitiveness of the arc to traces of water or oxygen is small enough, then the deduction may be drawn that "the presence of oxygen is not necessary for the production of spectrum No. 4. The old theory is then confirmed — *the so-called carbon band-spectrum rightly deserves its name.*"

On the other hand, we must emphasize the impossibility of keeping the carbon rods and the liquids free from oxygen; and, if we rate higher the sensitiveness of the liquid arc, we must abstain from any conclusion. If the existence of impurities is assumed, an exceptional position must be conceded to oxygen and carbonic acid.

Dr. Konen discusses other forms of discharge in liquids, namely, the *glow*, *brush*, and *uncondensed* and *condensed sparks*. The points of greatest interest in connection with the subject in hand are that with the *brush* discharge in ether, alcohol, and solutions of iodine or bromine in ether, the Swan spectrum is obtained in an especially beautiful manner. In the arc the electrodes were the important element, in the brush discharge the liquid is. "The only deduction possible 'from brush discharge experiments' must take the form of a reiteration of the statement made regarding the results of the carbon arc."

With the *condensed spark* it is impossible with water, ether, alcohol, and salt solutions, and either carbon or graphite electrodes, to obtain traces of the carbon or cyanogen spectrum; and here, as with the arc, salt solutions have no effect. With a 9.5 per cent. $BaCl_2$ solution and *Fe* terminals Konen obtained a bright line spectrum where Hale had found an absorption spectrum.

H. KONEN.

FIRE AT THE YERKES OBSERVATORY.

DURING the past two years a horizontal reflecting telescope, to be used with a coelostat of 30 inches aperture, has been in process of construction at the Yerkes Observatory. In conjunction with the instrument a large concave grating spectrograph had been provided for photographing stellar spectra. On December 22, when the final adjustments of the spectrograph were being made, the building caught fire from the breaking down of the insulation of the wires connected with the comparison spark apparatus. Under ordinary circumstances the small blaze thus produced might easily have been extinguished. But it had been necessary to construct the constant-temperature spectrograph house and the building containing the telescope as cheaply as

possible as very little money could be obtained for the purpose. For this reason building paper and wood, with intervening air spaces, had been employed. This burned so rapidly that although the fire extinguishers were at once brought into action the flames were past control. In spite of the best efforts of the members of the Observatory staff, the buildings were destroyed, together with the new spectrograph and the 30-inch plane mirror, driving-clock, and all the smaller parts of the cœlostat. A second plane mirror of 24 inches aperture was saved, but the 24-inch concave mirror of 62 feet focal length was destroyed.

The house containing the horizontal telescope was situated at some distance from the main Observatory building, which was not damaged in the least.

The work of reconstructing the mechanical and optical parts is already under way, and it is hoped that funds can be secured to complete the instrument in a thoroughly satisfactory manner, and to provide a suitable house of permanent construction.

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In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to uniformly employ the metric units of length and mass; the English equivalents may be added if desired.

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THE "SOLAR CONSTANT" AND RELATED PROBLEMS.[1]

By S. P. LANGLEY.

PHYSICAL astronomers, armed with new methods and perfected appliances, are helping us to a view of the progress of creation, from its beginning in the nebula, which must interest every student of nature. But however much our attention is aroused by the purely scientific aspect of such general studies, we must, it seems to me, consider, in the case I have now to present, *utility*, even before abstract interest. I refer to the study of the Sun, for though the most unformed nebula may hold the germs of future worlds, yet for us these possibilities are but interesting conjectures. For, as I have said elsewhere, I recognize that every nebula might be wiped out of the sky tonight without affecting the price of a laborer's dinner, while a small change in the solar radiation may conceivably cause the deaths of numberless men in an Indian famine.

From the foundation of the Smithsonian Astrophysical Observatory until now, I have therefore directed its work

[1]Read before the Washington meeting of the American Association for the Advancement of Science, December 30, 1902.

toward solar study, with a view to its probable utilities as well as to its purely scientific value, while still regarding this last as of high importance.

While the Sun, then, can be viewed merely as the nearest and most accessible star, yet it is here considered in a more important aspect to us, as the source of the radiation on which all human life depends.

1. *Early work at the Allegheny Observatory and on Mount Whitney.*—Some twenty years ago it was my good fortune to devise a successful means of measuring those minute quantities of heat on which depends an accurate determination of the possible variation in the enormous quantities which the Sun sends the Earth. This instrument, the bolometer, I used for nearly a decade at the Allegheny Observatory in the investigation of the Sun. It early appeared that many questions of the gravest importance, especially that of the so-called "solar constant," depended for their solution on the observer's stationing himself as high above our lower air as practicable. Accordingly, when through public and private channels the necessary means were at length provided, I conducted an expedition to Mount Whitney, in southern California. This expedition was provided with such essentials of a solar observatory as could be gathered and transported with the limited means at command, and was fortunate in including in its personnel the youthful Keeler, whose untimely end we all of us regret.

The observations secured at Mount Whitney confirmed my conclusion that the earlier values of the "solar constant" by Pouillet and others were too low. They had been obtained by the aid of the actinometer or the pyrheliometer—instruments which were made to father results that they were not strictly responsible for, through the use of wrong methods of extrapolation. It had been tacitly assumed that all solar rays are equally absorbed, or at least that all are similarly absorbed, in passing through successive layers of the Earth's atmosphere. Nothing could be more incorrect. Some large gaps are left in the spectrum where, for a range of wave-lengths as great as from the violet to the red, almost no energy remains after the solar beam

has passed through half the layer of air between the Sun and sea level, and I have shown that the absence of the study of individual rays—a study since made possible by the bolometer—leads to an altogether too small value of the solar constant.

It was, in short, shown by these observations that the solar radiations must be analyzed and their amount measured wave-length by wave-length, and coefficients of atmospheric transmission worked out for narrow spectral regions, before any trustworthy value could be assigned to the solar radiation outside the Earth's atmosphere. It is satisfactory to see how thoroughly this view, novel enough then, has now been accepted. Its results[1] gave values of from 3 to 3.5 calories for the solar constant, thus nearly doubling the classical value, 1.76 calories, of Pouillet. They brought no certainty of the fixity of this so-called "constant," which, however, indications now lead us to consider as possibly variable, and one whose changes are not improbably of the deepest practical concern to us. Where the most elaborate determinations of a quantity by competent observers thus differ by nearly 100 per cent., it is evident that, if there be considerable real variations in this quantity, they may exist without detection.

Were there no intervening terrestrial atmosphere, we might determine these solar variations, if they exist, by observations made here; but the difficulties of such a research, when conducted anywhere through the Earth's actually intervening and changing atmosphere, are manifest, and in our actual station perhaps insuperable. I shall proceed to give an example of the best means at our command for overcoming them—means admittedly insufficient here, but which we may hope to employ later in a more favorable locality, at an elevated station. This done, it will remain to consider an independent method which, by direct study of the Sun's atmospheric envelope, holds out some possibility of success in detecting variations in the Sun's radiation, if these exist, even in our actually unfavorable conditions.

[1] See *Professional Papers, Signal Service*, XV, "Report of the Mount Whitney Expedition."

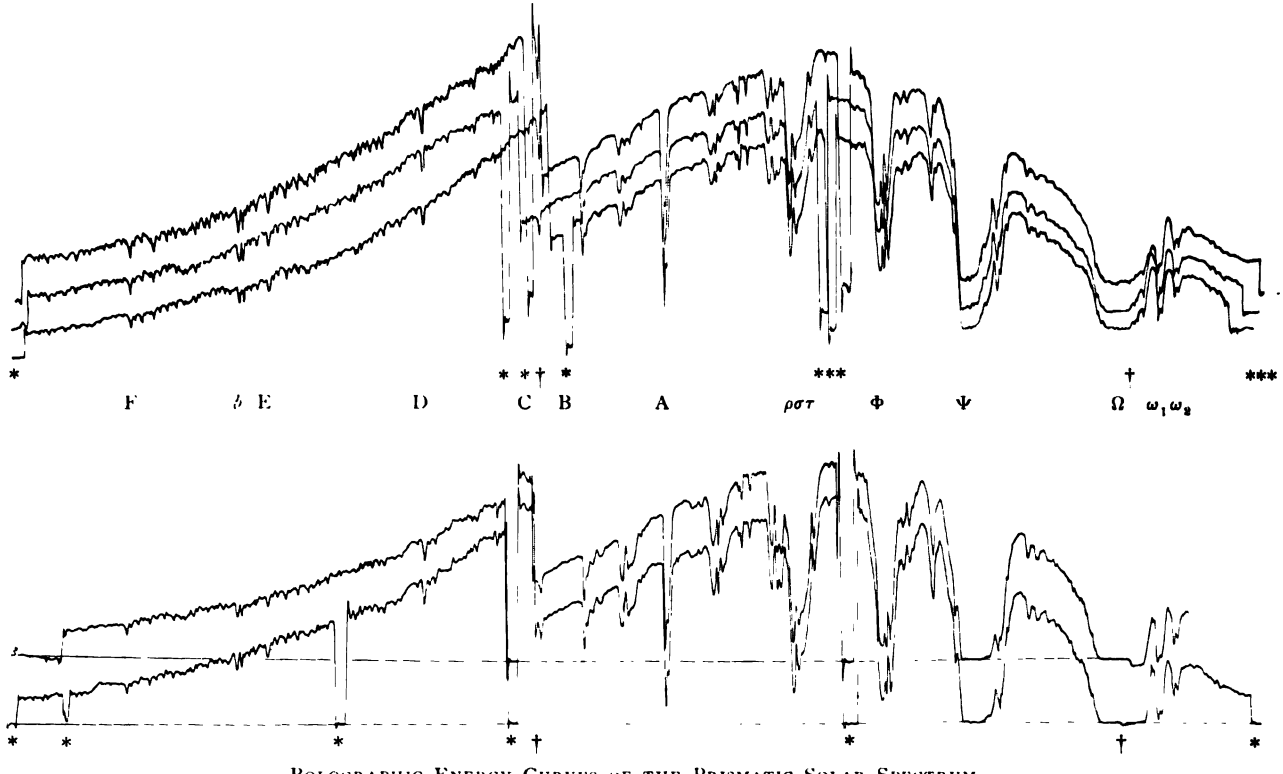
2. *Improvements in apparatus.*—Since the institution of the Smithsonian Astrophysical Observatory, almost every year has witnessed improvements of the apparatus employed for making solar energy spectrum observations, such as are necessary in the problems relating to the solar constant. The story of all these improvements is quite too long to tell here, but can be found in Vol. I of the Observatory's *Annals*.

In consequence of these changes, which have now occupied me so many years, with final success, where two years of my personal assiduous labor formerly passed (at Allegheny) in getting an imperfect picture of the distribution of the energy in the solar spectrum, extending from the violet down to a wave-length of 2.5μ—this can now be obtained with ease in fifteen minutes of time, by an almost automatic process.

This stretch of spectrum contains almost all the energy which reaches sea level, and includes many regions of powerful water vapor absorption. It extends into a portion of the infra-red entirely unknown before the time of the Mount Whitney expedition, and which was, in fact, discovered in my observations on Mount Whitney itself.

You see superposed upon this chart (Plate VII) before you five of the energy spectrum curves to which I have just referred. The curves were taken at intervals of about one hour in two different afternoons. You will note that, while the five are generally similar, the lower pair shows greater absorption by water vapor in the great infra-red bands.

3. *New work of the Smithsonian Observatory.*—Armed now with apparatus capable of such results as I have just noted, and with the ability to gather in a quarter of an hour observations which could not have been obtained with equal accuracy in years of former work, the problem of the solar constant and the absorption of the solar radiation within the solar envelope, and within the Earth's atmosphere, has, after long intermission, now been taken up again at the Smithsonian Astrophysical Observatory, not with the expectation that a final solution can be reached, in its unfavorable local conditions, but with the hope that we may reach a useful approximate one, and become

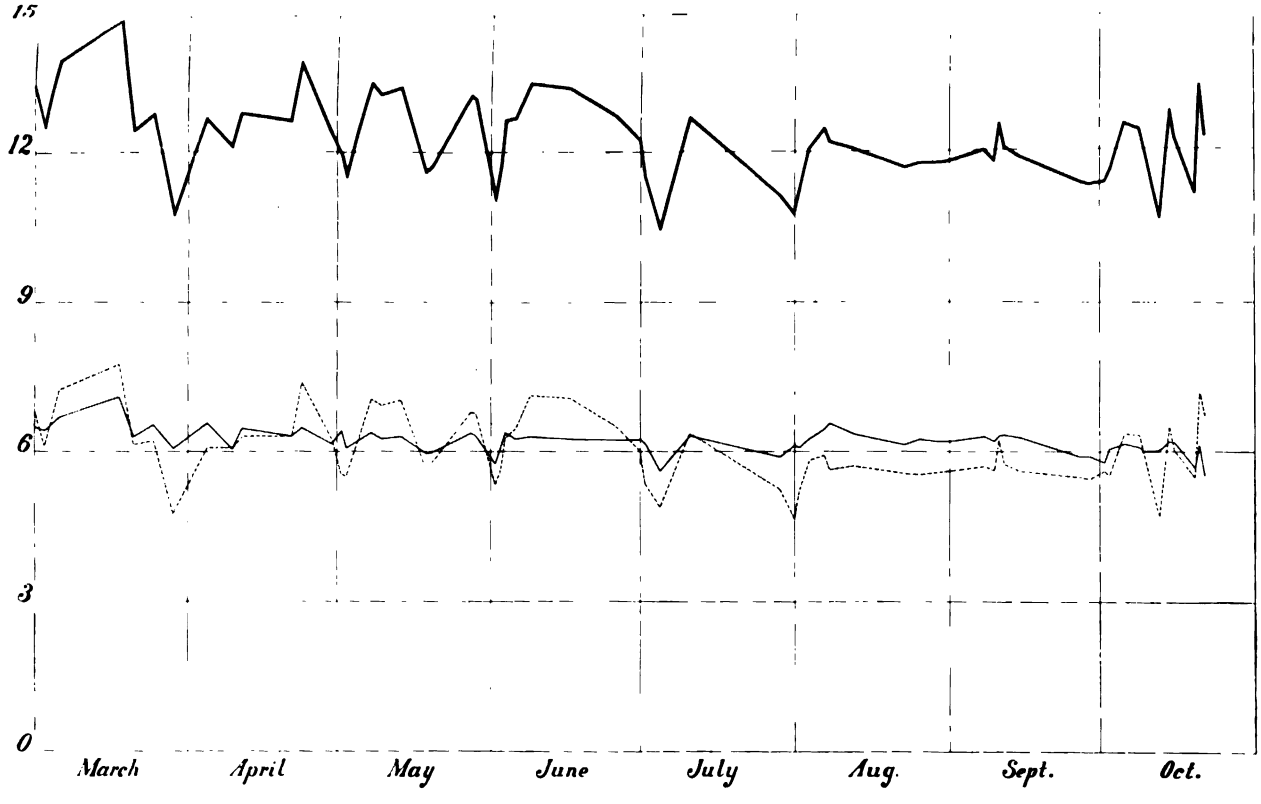


BOLOGRAPHIC ENERGY CURVES OF THE PRISMATIC SOLAR SPECTRUM.

Each curve made in fifteen minutes. The three upper curves taken on a day of moderate water vapor absorption; the two lower curves indicate much greater absorption. See bands $\rho\sigma\tau$, Φ, Ψ.

* Shutter closed at these points to give zero line.

† Height of slit altered at these points.



VARIATION OF SOLAR RADIATION CAUSED BY DIFFERENCES OF ATMOSPHERIC ABSORPTION.

(From Bolographic Studies of 1902.)

Upper curve represents total radiation between wave-lengths 0.76 μ and 2.0 μ.
Dotted curve represents portion of this affected by the great water absorption bands.
Lower full curve represents the remainder relatively unaffected by absorption of water vapor.

familiar in practice here with methods to be carried out later, it is hoped, under more favorable conditions.

4. *The method of study.*—The work now commenced here is planned to include the following parts:

a) A study of the yearly variations of the selective absorption of the Earth's atmosphere by the aid of a long series of bolographs.

b) The production of bolographic energy spectrum curves at different altitudes of the Sun for the purpose of obtaining coefficients of atmospheric transmission at any desired wave-lengths.

c) Experiments to determine the loss experienced by the radiations in passing through the various optical parts of the bolographic apparatus.

d) The standardization of the bolographic apparatus by actinometer observations simultaneous with the bolographs.

e) The reduction of these several kinds of data to furnish coefficients of atmospheric transmission, coefficients of transmission of the apparatus, and ultimately means of getting values of the solar constant.

These five classes of studies refer mostly to absorptions within the Earth's atmosphere, but finally, and of the greatest importance to the immediate study of the variations of solar radiation, comes a study of the absorption of the solar envelope for any desired wave-lengths by means of bolographs taken at selected points on a large solar image.

5. *Selective absorption of water vapor.*—Continuing our consideration of the absorptions in the Earth's atmosphere, we observe that this chart (Plate VIII) gives a summary of eight months of bolographic spectral actinometry. The bolographs taken near noon of all clear days since February, 1902, have been reduced to a uniform scale of ordinates by a method which need not here be described, and their areas between wave-lengths 0.76μ and 2.0μ have been obtained. These areas are proportional to the total solar radiation between wave-lengths 0.76μ and 2μ which reaches the sea level.

In this infra-red portion occur the great water vapor absorption bands $\rho\sigma\tau$, ϕ, ψ, and Ω, but there is some reason to suppose

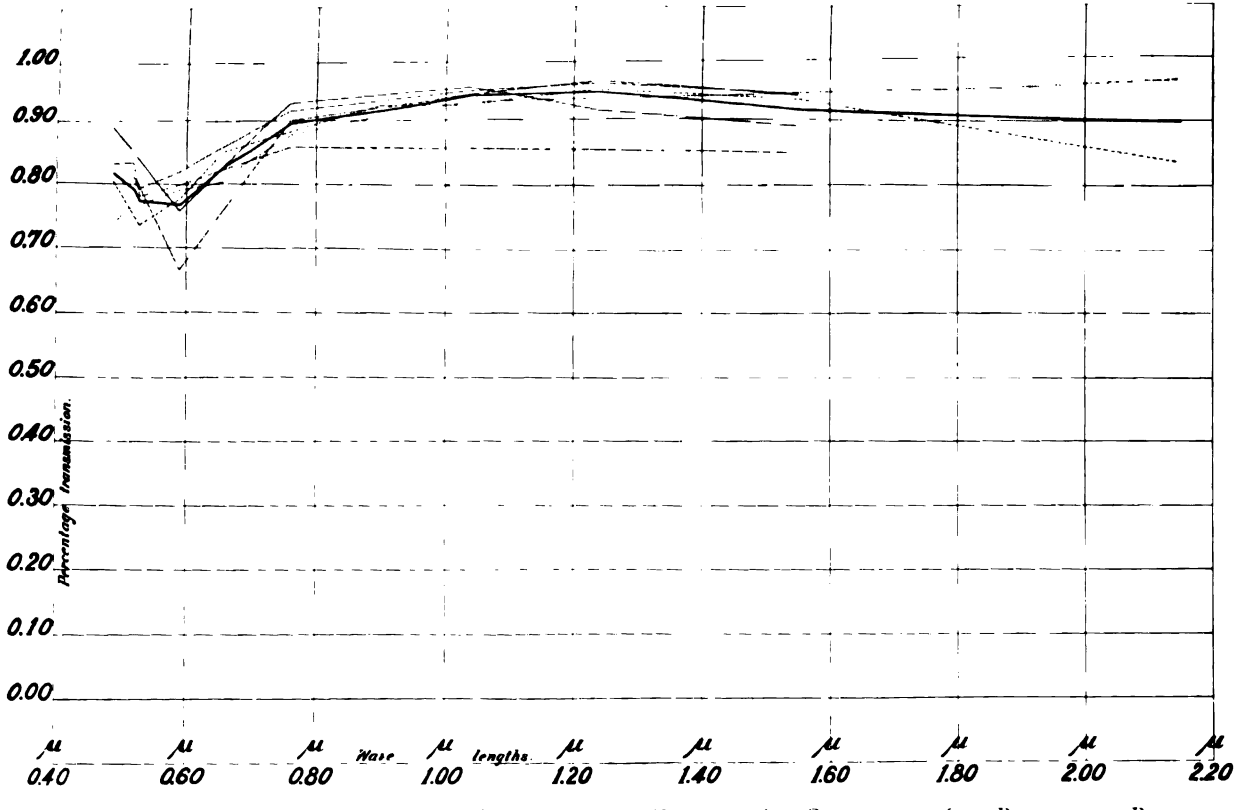
that the percentage absorption of water vapor is no greater here than in the visible spectrum as a whole.

The areas measured are plotted in the upper curve of Plate VIII. It will be seen how broken and variable the height is, but how the general slope indicates a considerable falling off in energy in the summer months. The reason of this appears in the next lower curves. Of these the dotted one gives the summation of the areas of those parts of the bolographs known to suffer greatly from water vapor absorption, and the full one gives the remainder not so much affected by it. You see how obviously our solar radiation is affected by the water vapor in the air. The difference between March and August amounts to about 20 per cent. It has been pointed out in Vol. I of the *Smithsonian Astrophysical Observatory Annals* that this water vapor absorption is annually periodic in its fluctuations, and is at a minimum in spring. A secondary minimum occurs in the autumn.

6. *The general absorption of the Earth's atmosphere.*— During the past year attempts have been made on each promising day to get a series of solar energy curves taken with different altitudes of the Sun, beginning in the morning and continuing every hour till late in the afternoon. Many of these series were stopped or rendered valueless by the appearance of clouds or haziness; for it is, indeed, surprising to find how little apparent change in the clearness of the air alters decidedly its transparency for some wave-lengths.

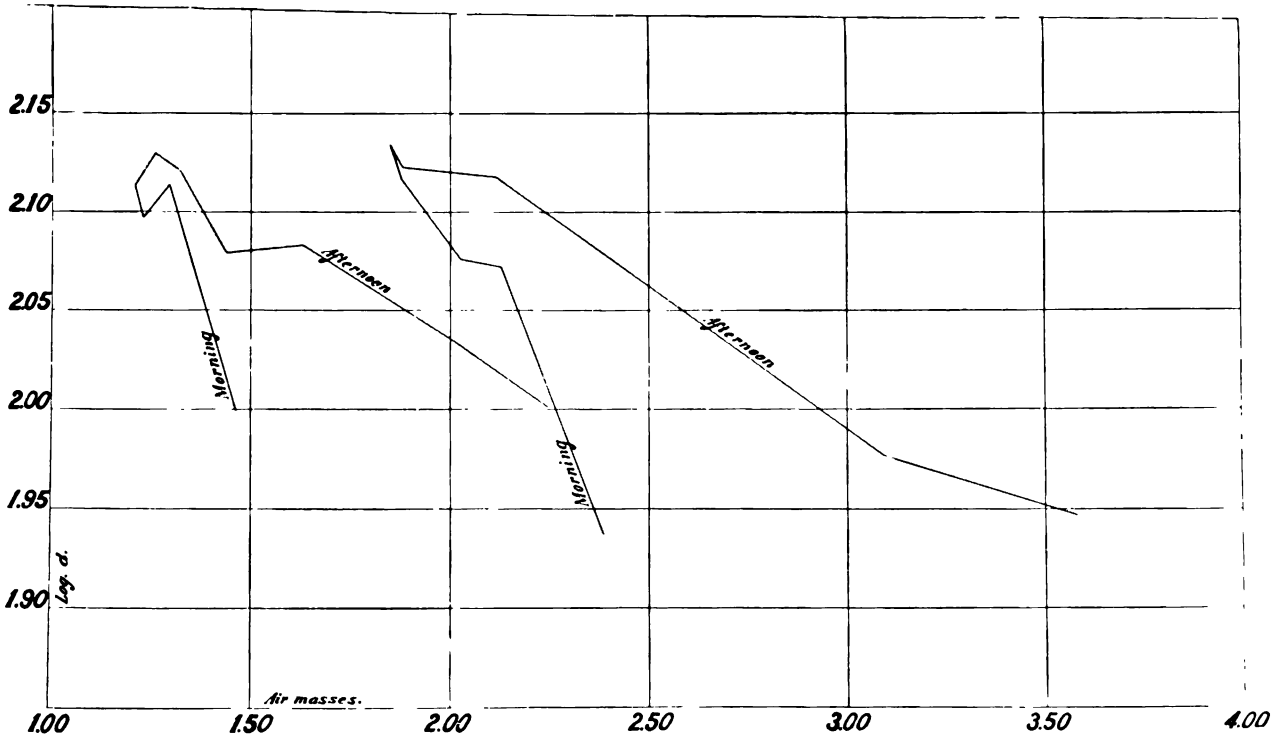
The method of reduction of the observations is as follows: A number of points on the bolographs are selected where there are no prominent bands of selective atmospheric absorption. At these points the heights are proportional to the deflections of the galvanometer and to the amounts of solar radiation of certain known wave-lengths which fall upon the bolometer. For each bolograph the time of passing these selected points is known, and therefore the altitude of the Sun.

We thus have all the data for fixing the mass of air traversed by the solar beam on several occasions, and the relative amounts of radiation of certain wave-lengths remaining after the passage through the air column. We assume now the validity of the



TRANSMISSION OF EARTH'S ATMOSPHERE AT WASHINGTON FOR VERTICAL AIR COLUMN AT 76 CM BAROMETRIC PRESSURE.

The heavy line is the mean of the results of six afternoons' bolographic observations.



VARIATION OF ATMOSPHERIC TRANSMISSION AT WASHINGTON FOR MORNING AND AFTERNOON HOURS.

The observations are for wave-length 0.51 μ and for two different good days. If the transparency of the atmosphere remained uniform, all the points plotted for each day should lie upon a straight line, the tangent of whose inclination is log a of Bouguer's formula.

familiar formula of Bouguer, which seems, as applied to individual wave-lengths, to express well the results of these experiments.[1]

Such a procedure as I have indicated, applied to the work of six of the best days, has yielded the results which appear on this chart (Plate IX) before you. The six thin broken lines show the values of atmospheric transmission coefficients determined for the twelve different wave-lengths as indicated by the abscissæ of the chart. The heavy line represents the mean result of these six different days of observation. You will see how well the results generally agree. They were taken from different seasons of the year and very different air masses. The significant depression of the curve as it ends in the infra-red and also near the D lines I suppose to be due to a general absorption of water vapor in these spectral regions.

Another interesting feature of the work is that in the morning hours the transparency of the air generally increases rapidly and sometimes irregularly, while in the afternoon it continues longer of nearly uniform and maximum transparency. This result is shown by this chart (Plate X), in which the lines are plotted from the values of log d and $m\frac{B}{B_0}$ (see footnote, p. 95)

[1] If we suppose e to represent the amount of radiant energy per square centimeter of the given wave-length which reaches the Earth's surface; e_0 the amount prior to absorption by the air; a the proportion transmitted by a unit air mass, represented by vertical transmission through an air column of barometric pressure B_0, then for the given barometric pressure B and for the air mass m we have by Bouguer's formula:

$$e = e_0 a^{m\frac{B}{B_0}} .$$

But since the height of the bolograph d at the given wave-length is proportional to e, we may write, by introducing an instrumental constant k,

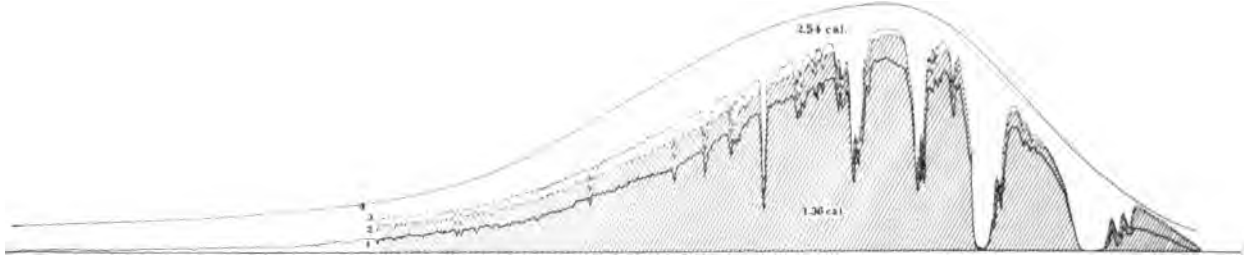
$$d = ke = ke_0 a^{m\frac{B}{B_0}} .$$

It is convenient for graphical computation to put this formula in logarithmic form as follows:

$$\log d = m\frac{B}{B_0}\log a + \log ke_0 .$$

The last member of this equation is supposedly constant, and hence we have here the equation of a straight line. Accordingly, if all the observed values of log d and $m\frac{B}{B_0}$ are plotted on any convenient scale, the quantity log a appears as the tangent of the inclination of the line determined by them.

PLATE XI.



BOLOGRAPHIC METHOD OF DETERMINING THE SOLAR RADIATION CONSTANT.

Curve 1 is the original bolograph.
Curve 2, same corrected for absorption of spectroscope, and representing by its shaded area a known actinometer reading.
Curve 3, same corrected for absorption of siderostat mirror.
Curve 4, same corrected for absorption of Earth's atmosphere.

for two different days at a wave-length of 0.51μ. As I have said, all the observed points should lie upon a straight line the tangent of whose inclination is the value of log a. It will be noted that the morning hours yield much steeper and more irregular lines than the afternoon hours, indicating the increase of transparency, which culminates shortly after noon.[1]

This general result of the preliminary experiments on the Earth's atmosphere may be of interest to meteorologists.

7. *The absorption of the apparatus and its standardization by the actinometer measures.*—Let us recall that before reaching the bolometer each ray of light, of whatever wave-length, is diminished in intensity, first by the absorption of the solar envelope, second by the absorption and diffuse reflection of the Earth's atmosphere, third by a similar loss at the siderostat mirror, and fourth within the spectroscopic train. These several losses are not the same for different wave-lengths, but all tend to increase as we pass from the infra-red into and through the visible spectrum.

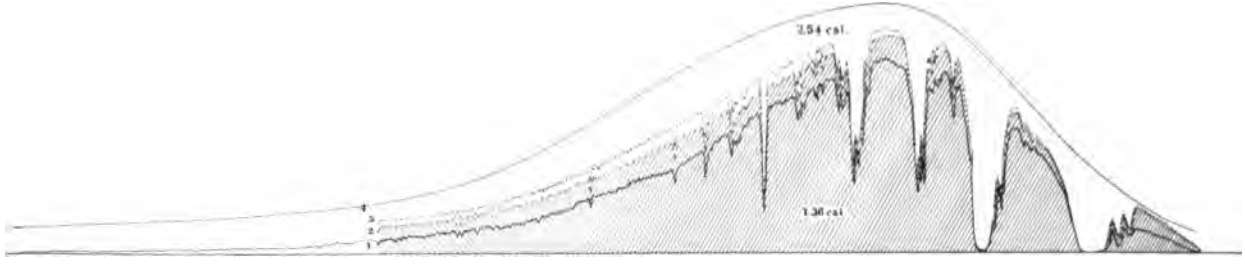
The atmospheric transmission has just been mentioned, and we will now consider the losses within the train of apparatus. In my earlier work at Allegheny, I determined the reflection coefficients for silvered glass mirrors, and more recently other investigators have done the same. I do not therefore stop to give more recent results obtained here, as they are in substantial agreement with these others, but it may be well to say that, inasmuch as the reflecting power of the silvered siderostat mirror rapidly deteriorates in the visible spectrum, it is necessary to determine this quantity frequently in solar constant work.

Similarly, the three mirrors of the spectroscope deteriorate, and require that frequent determinations of the relative transmission of the spectro-bolometer for all wave-lengths should be made. I do not here detail the elaborate procedure adopted for this purpose.

Referring to Plate VII again, you see before you five solar

[1] It is at the same time to be noted that conditions of "good seeing" are usually associated with the morning hours.

PLATE XI.



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Curve 3, same corrected for absorption of siderostat mirror.
Curve 4, same corrected for absorption of Earth's atmosphere.

spectrum energy curves. Take any one of these in illustration. Its ordinates represent intensities of radiation for all wave-lengths from 0.45μ to 2.5μ. The area bounded by this curve and the axis of abscissæ is therefore proportional to the total solar radiation between these wave-lengths which reaches the bolometer. If now all the ordinates should be increased by dividing their lengths by the corresponding transmission coefficients of the spectro-bolometer, we should obtain a new corrected energy curve. This curve may be extended by extrapolation to include the small amounts of solar radiation not included between wave-lengths 0.45μ and 2.5μ,[1] and its area will then be proportional to the total solar radiation in the beam reflected by the siderostat mirror.

8. *A provisional value of the solar constant.*— In the steps which follow, the necessary data are introduced to furnish a provisional value of the solar constant.

The energy curve is corrected for the loss at the siderostat mirror, and further, by the application of Bouguer's formula and the atmospheric transmission coefficients, the form of the energy curve outside the Earth's atmosphere is reached. Before this final step all the great terrestrial bands are smoothed over. Plate XI illustrates the several steps by means of which the bolograph is corrected to exhibit the distribution of the solar energy outside the Earth's atmosphere. Curve (1) is the bolograph itself. Curve (2) is corrected for relative absorption in the spectroscope. This curve is extended at either end to include what may reasonably be estimated as the solar radiation beyond its limits. The shaded area under curve (2) represents an actinometer reading of 1.36 calories per square centimeter per minute.

Curves (3) and (4) give the distribution of the solar radiation in the prismatic spectrum respectively at the Earth's surface and outside the atmosphere. The area under curve (4) is 1.87 times the area under curve (2), and we therefore find from this

[1] In a more exact determination of the solar constant these amounts would require careful measurement, but in this illustration only a rough extrapolation is attempted.

single determination that the solar constant is 2.54 calories per square centimeter per minute.

(The reader will understand that this is but a provisional value, given to illustrate the method, which, it is hoped, may be pursued later under more favorable local conditions. It is, indeed, unlikely that even by much greater labor we can obtain an accurate value of the constant from a station so near sea level as Washington, where the atmospheric absorption is so large and doubtful a factor. It will be remembered also that the writer has elsewhere shown that such values obtained near sea level are always of necessity too small.)

I cannot leave this part of the subject without acknowledging my constant obligation to my assistants, Mr. C. G. Abbot and Mr. F. E. Fowle, for aid in every part of the work.

We now reach the most important part of our study of the so-called solar constant, and of its possible variation.

9. *General absorption of the solar envelope.*—A chief cause of possible variability of the solar radiation is in the solar envelope itself. The absorption of this envelope may well be variable, as I have remarked in earlier articles,[1] in which I have indicated how far the surface temperature of our planet may depend upon it, and Halm has recently[2] put forth an ingenious theory to explain the Sun-spot cycle on this basis. I refer to this now only to point out that bolometric work upon the direct solar image and on the relative amounts of radiation of all wave-lengths, and from its various parts formerly carried on by me elsewhere, has been now taken up at the Smithsonian Observatory, and that a description of it will probably form the subject of a later memoir. Inasmuch as we receive heat and light through different thicknesses of the solar envelope at different parts of the disk, we have here, with the thermal study of faculæ, prominences, and spots, a means of studying solar absorption and radiation which, if continued (as I hope it may be) over a Sun-spot cycle, will determine our knowledge of whether the Sun's radiations to the Earth vary, as I am disposed to believe, from month to month and year to year; and which may, it seems

[1] *Am. Jour. Sci.*, 10, 489, 1875. [2] *Nature*, February 13, 1902.

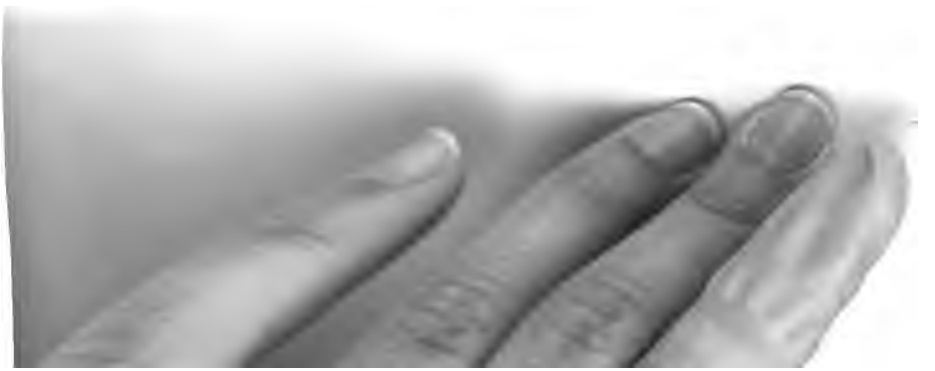
increasingly probable, ultimately bring to light evidence of variation in the heat it sends to the Earth, which may not only interest the astronomer, but be of practical consequence to all men.

SUPPLEMENTARY.

Need of an elevated solar observatory.—The Earth's temperature and the life of its inhabitants, both animal and vegetable, depend on the solar radiation. Yet we confess that even at this late day we do not know, with any certainty, what the total amount of solar radiation is, whether it is constant or variable, or what effect upon terrestrial life and temperature a given change in it would produce. Our ignorance of these fundamental things is largely, though not wholly, due to the variability of our own atmosphere, which prevents us from studying that of the Sun.

The preceding observations are carried on here with extreme difficulty under the almost prohibitive local conditions for such work. We must at least attempt a determination of the solar radiation, and its possible variation under these conditions; although what is more urgently needed, as I believe, than any other desideratum of physical astronomy, is to establish an observatory, placed in some clear and elevated region and charged solely with problems relating to the possible variations of the heat the Earth receives from the Sun. I have referred to the former expedition to Mount Whitney for this purpose, but it is not a temporary expedition, but many years' occupation which is now in question. Now that great undertakings are the order of the day, let us hope that some way may open to reach the solution of a problem which so concerns the whole human race.

SMITHSONIAN INSTITUTION,
Washington, D. C., February 7, 1903.



point O, which marks what may be termed the mean center of illumination of the incident wave-front. Let the angle between this line oO, and the secondary optical axis, oa, *i. e.*, the angle aoO, be τ. Then if the angle oac be κ as before, it is evident that the distance ξ, of an image from the central point c of the plane in which it lies, will be

$$\xi_1 = f \sin \kappa_1 + (z - f) \sin (\kappa_1 - \tau_1) \ . \tag{95}$$

The separation of two images which lie at equal distances on opposite sides of the optical axis and having a total angular separation κ will be

$$\xi = 2\xi_1 = f \sin \kappa + (z - f) \left\{ \sin \left(\frac{\kappa}{2} - \tau_1 \right) + \sin \left(\frac{\kappa}{2} + \tau_2 \right) \right\} ,$$

or since κ and τ_1, τ_2 are all very small angles

$$\xi = z \{ \kappa - (\tau_1 - \tau_2) \} + f (\tau_1 - \tau_2) \tag{96}$$

and

$$\left. \begin{aligned} d\xi &= dz \{ \kappa - (\tau_1 - \tau_2) \} + df (\tau_1 - \tau_2) \\ &= \frac{dz}{f} (\xi - S) + \frac{df}{f} S \end{aligned} \right\} , \tag{97}$$

where S is, as before, the total linear separation of the mean centers of illumination of the incident wave-fronts.

Comparing (97) with (58) we see that in this latter case the change in the separation of the two images involves the consideration of the change in both z and f. It is necessary therefore to know the law of variation of f with t to the same degree of exactness as that with which we know the coefficients of linear expansion a, a', and a'' involved in a change in z.

Expressing the values of dz and df in the same form as before [(63) and (94)], and taking into account the expansions of the micrometer screw and photographic plate, etc., as in (59) and (60), we finally obtain in this case:

First, for direct micrometric or heliometric measurements at the focal plane $f \simeq z + d\Omega$,

$$\begin{aligned} \Delta\xi = {} & \kappa (z + d\Omega) (a - a') \Delta t + d\Omega \left[\kappa - (\tau_1 - \tau_2) \right] \\ & - (\tau_1 - \tau_2) (z + d\Omega) (a - a''') \Delta t \ , \end{aligned} \tag{98}$$

and second, for comparator measurements on photographic plates

$$\sum_{1}^{r}\xi = \kappa(z+d\Omega)\left[(a-a')\,\Delta t+(a'-a'')\,\Delta' t\right] + d\Omega\left[\kappa-(\tau_1-\tau_2)\right]-(\tau_1-\tau_2)(z+d\Omega)(a-a''')\,\Delta t\ . \qquad (99)$$

When $\tau_1=\tau_2$, *i. e.*, when the mean centers of illumination S_1 and S_2 are incident at the same point on the objective, the measured separation between any two images is exactly the same in the case of eccentrical incidence as in the case of centrical incidence (64) and (65), previously considered. From this it follows that any unsymmetrical diaphragming or unsymmetrical absorption (the effect of which is the same) is without effect on $\Delta\xi$, provided only that the wave-fronts in each case fill the whole aperture. The position of each individual image, however, will be shifted by an amount indicated by differentiating (95), and the application of this principle forms the basis of the ingenious methods that have been described by Cornu,[1] Newall,[2] Hartmann,[3] and others for determining the exact focal length of a telescope.

If the focusing were always exact we would have

$$\begin{aligned} d\Omega &= (a-a''')f\Delta t \\ &\cong (a-a''')(z+d\Omega)\,\Delta t\ , \end{aligned} \qquad (100)$$

and under such circumstances (98) and (99) would again reduce to forms identical with (64) and (65). Exact focusing, in the sense in which the term is used in metrological work, is, however, never possible by the ordinary standards of definition; hence it is better to consider these terms separately.

The quantities a, a', a'', and a''', which appear in the first and last terms of (98) and (99) are all small and may be accurately determined once for all. Hence since $d\Omega$ is always small compared to z, and κ is known from the measurements themselves, we can, as we have already shown, determine the value of the first term of these corrections with all requisite accuracy if we know the value of Δt within 3° ($\pm 1^{\circ}6$). In order to do the same for the other two terms we must first of all determine the values of the angles τ_1 and τ_2.

[1] *Ann. de l'École Normale*, (2) 9, 21.

[2] *M. N.*, 57, 572, 1897.

[3] ASTROPHYSICAL JOURNAL, 12, 37, 1900.

The first and most interesting case to be considered is that of the prism spectrograph. Here the separation of the axial pencils of the two extreme wave-fronts is produced by the dispersion of the spectroscope train. The amount of this separation has already been calculated [see equation (34)].

If there were no absorption in the prism train, and the prisms and view telescope had apertures sufficiently large to transmit the entire beam of the two extreme lateral pencils, the separation S given by (34) would be that required in (97). Owing to the unsymmetrical absorption, however, the mean center of illumination O will not be on the axis of the wave-front, but will be displaced toward the refracting edge of the prism by an amount proportional to the coefficient of absorption. With the same notation as that already employed in (8) and (9) we have for the total quantity of light transmitted

$$A = \int_{-\frac{b}{2}}^{+\frac{b}{2}} i_0 e^{-Bx}\, dx = \frac{i_0}{B}\left[e^{\frac{Bb}{2}} - e^{-\frac{Bb}{2}}\right] . \tag{101}$$

Hence the mean center of illumination O will lie at a point S_0 such that

$$i_0 \int_{-S_0}^{\frac{b}{2}} e^{-Bx}\, dx = \frac{1}{2} A , \tag{102}$$

which by comparison with (101) gives at once

$$S_0 = -\frac{1}{B} \operatorname{nap} \log \frac{e^{\frac{Bb}{2}} + e^{-\frac{Bb}{2}}}{2} . \tag{103}$$

For $B = \frac{1.386}{b}$, as previously assumed,

$$S \simeq -\frac{1}{6} b , \tag{104}$$

and therefore

$$\tau_1 = \frac{S}{f} = -\frac{1}{3}\beta ,$$

i. e., the center of illumination is displaced one-sixth the entire diameter of the wave-front toward the refracting edge of the

prism train, and the angle τ_1 for the wave-front passing at minimum deviation is one-third the semi-angular aperture of the view telescope.

In order to avoid displacement of the central image (for which $\kappa = 0$) with change of focus, the center of the objective must coincide with the mean center of illumination O; *i. e.*, the principal axis of the view telescope should not lie on the axial line of the ray transmitted at minimum deviation but should be shifted toward the refracting edge of the prism train by the amount indicated in (103). This is a point generally overlooked in the design of spectrographs.

In the case of lateral pencils the value of the middle term representing the effect of the mechanical change in focus will be less or greater according as the signs of τ_1 and τ_2 are the same or opposite, and according as the sign of their differences is the same or opposite to the sign of κ. If τ_1 has the same sign as κ we may make the term $\kappa - (\tau_1 - \tau_2)$ zero by properly controlling S_1 and S_2, the points of incidence of the centers of illumination of the lateral wave-fronts. In the case of the spectrograph this can be accomplished by so proportioning f, the focal length of the view telescope, and L, the total length of path through the prism train, that the value of S in (97) and (34) is identical with the value of ξ, the separation of the two images at the focal plane. This gives us, from (34) and (35),

$$f(\tau_1 - \tau_2) = f\kappa = \frac{d\theta}{d\lambda}\left[\frac{1}{2}\left\{L_1 + 3L_2 + 5L_3 + \cdots (2N-1)L_N\right\} + n\left\{L_1 + 2L_2 + 3L_3 + \cdots N\frac{L_N}{2}\right\}\right], \quad (105)$$

or since the total angular displacement κ is $2N$ times the displacement $\frac{d\theta}{d\lambda}$ for one refraction, we have for three prisms

$$f = \frac{1}{12}\left[L_1 + 3L_2 + 5L_3\right] + \frac{1}{6}n\left[L_1 + 2L_2 + 3\frac{L_3}{2}\right]. \quad (106)$$

For the Bruce spectrograph already considered

$$f = 10.08 + 16.42 = 26.5 \text{ cm},$$

instead of 44.9 cm and 60.7 cm, as adopted for camera A and camera B.

Since (34) is deduced on the assumption that the prisms and objective of the view telescope are placed as close together as possible, (105) and (106) express the minimum value of f that will satisfy the above condition for the elimination of the term $\kappa-(\tau_1-\tau_2)$. We can, however, satisfy this condition for any *larger* value of f by separating the prisms or by withdrawing the camera objective so that the total path L through the spectroscope train is correspondingly increased. It is so easy to do this in the original design of the instrument, and the advantage resulting from it in the entire elimination of the effect of errors in the scale reading Ω is so great, that it is singular that this particular point of design seems to have been previously overlooked.

We may accomplish the same result in the case of the plane-grating spectrometer by making the distance from the grating to the objective of the view telescope equal to the focal length of the latter. In this case as before the objective of the view telescope must be increased in size sufficiently to receive the entire cross-section of the lateral beams. In this latter case the use for which the instrument is intended, as well as the form of view telescope adopted, must be considered before we can determine in any case whether it is worth while to introduce this modification of the usual design. With reflecting view telescopes the condition is very easily and almost necessarily satisfied, and this is another advantage of this form of grating spectroscope.[1]

In the case of the heliometer we may satisfy the same condition of zero displacement with change in the position of the eye-piece by proper diaphragming. With a full semi-circular aperture the total quantity of light in the transmitted wave-front is proportional to $\frac{1}{2}\pi r^2$. If we diaphragm one edge of the aperture by an amount mR the amount of light transmitted will be

$$\left.\begin{aligned} A &= \int_{-R}^{R(1-m)} \sqrt{R^2-x^2}\,dx \\ &= \frac{R^2}{2}\left[(1-m)\sqrt{2m-m^2}+\sin^{-1}(1-m)+\frac{1}{2}\pi\right] \end{aligned}\right\} . \quad (107)$$

[1] *Phil. Mag.*, 38, 137; ASTROPHYSICAL JOURNAL, 1, 232, etc.

The mean center of illumination of the diaphragmed aperture lies at a point S_0 such that $S_0 = R(1-n)$ and

$$\int_{-R}^{-R(1-n)} \sqrt{R^2 - x^2}\, dx = \frac{1}{2} A \ ,$$

and therefore

$$\left.\begin{aligned} &2\left[(n-1)\sqrt{2n-n^2} + \sin^{-1}(n-1)\right] + \frac{\pi}{2} \\ &= (1-m)\sqrt{2m-m^2} + \sin^{-1}(1-m) = \frac{2}{R^2} A - \frac{\pi}{2} \end{aligned}\right\} = A_m \ . \quad (108)$$

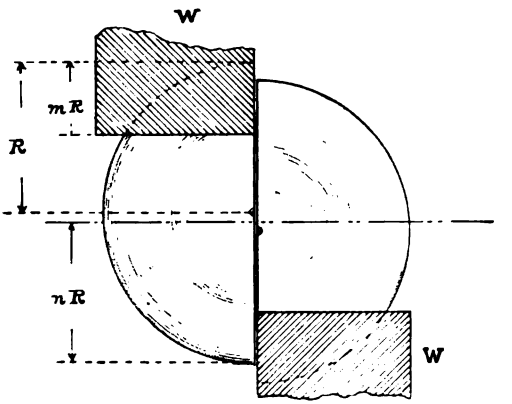


FIG. 12.

Let $A_0 = \frac{1}{2}\pi R^2$ be the amount of light transmitted when there is no diaphragm. Then we have from (107) and (108)

$$\frac{A}{A_0} = \frac{A_m}{\pi} + \frac{1}{2} \ . \quad (109)$$

Equation (108) serves to determine m, the amount of diaphragming required for any given value of n, and (109) gives the ratio between the illumination secured with this diaphragm and the illumination with full (half) aperture. To determine n we have also in this case

$$S_0 = f\tau_1 = f\frac{\kappa}{2} = R(1-n) \ ,$$

whence

$$n = 1 - \frac{\kappa}{2\beta} \ . \quad (110)$$

Assume as before that for a 16 cm heliometer $\beta \cong .03$. Then for two objects separated by an angular interval of 0°.5, $\frac{\kappa}{2} = 0.00436$ and $n \cong 0.855$, *i. e.*, the center of illumination of the incident wave-front should fall upon each half of the divided lens 0.145 of the radius from the center. This is accomplished by giving to m a value obtained by solving (108) for $n = 0.855$. We thus obtain

$$m = 0.475 ,$$

i. e., a little less than one-fourth of each half of the objective should be covered, as in Fig. 12, the screens $W\ W$ being placed so as to shut out in each case the light from the edge which is on the opposite side of the optical axis from the image formed by that half.

By expressing κ as a fractional part of β we can determine the general relation between κ and m for any heliometer. This has been done for values of $\frac{\kappa}{\beta}$ varying by tenths from 0 to 1 and by 0.2 from 1 to 2. The results for A_m, $\frac{A}{A_0}$, and m are given in Table IV and plotted in Fig. 13.

With this relation it is easy to devise a mechanical arrange-

TABLE IV.

$\frac{\kappa}{\beta}$	n	A_m	$\frac{A}{A_0}$	m
0	1.00	1.5708	1.00	0
0.1	0.95	1.3706	0.9363	0.230
0.2	.90	1.1726	.8732	.369
0.3	.85	0.9738	.8099	.490
0.4	.80	.7774	.7473	.600
0.5	.75	.5808	.6849	.705
0.6	.70	.3876	.6233	.805
0.7	.65	.1994	.5634	.900
0.8	.60	.0144	.5046	.988
0.808	.596	.0000	.5000	1.000
0.9	.55	− .1664	.4471	1.085
1.0	.50	− .3422	.3911	1.172
1.2	.40	− .6770	.2845	1.345
1.4	.30	− .9784	.1886	1.512
1.6	.20	−1.2424	.1047	1.678
1.8	.10	−1.4542	.0372	1.840
2.0	.00	−1.5708	.0000	2.000

ment which will automatically move the screens WW of Fig. 12 by the required amount when the two halves of the object-glass are adjusted to bring two images separated by the angular interval κ into coincidence. The great advantage of such an arrangement in the case of the heliometer is that when it is adopted the eyepiece may be moved in or out at will to suit the convenience and personality of the observer without thereby introducing any differences or errors in the settings of the instrument itself. The only drawback is that the light is considerably reduced for objects having considerable separation.

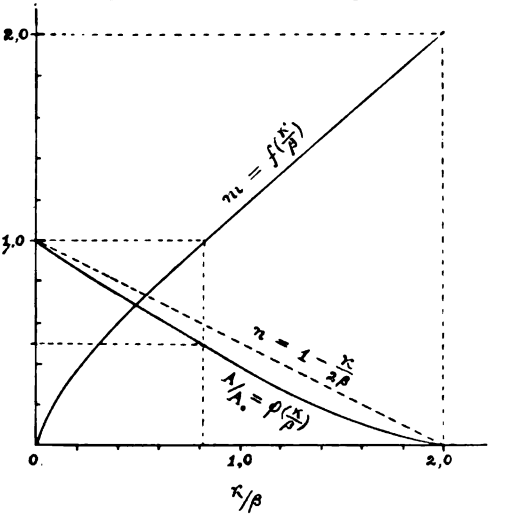


FIG. 13.

In this case, as in the preceding, the values of the angles τ_1 and τ_2, which appear in the third terms of (98) and (99), are determined by the relations

$$(\tau_1 - \tau_2) f = f(\kappa) ,$$

and these terms reduce to the form

$$\kappa f (a - a''') \Delta t , \qquad (111)$$

in which all the quantities are known or may be measured, and which, like the first terms of these equations, may be computed with all necessary accuracy when we know Δt for ordinary object-glasses to within about 3°. Temperatures of the object-glass and telescope-tube may be easily observed with this degree of accuracy, but we may by the special optical construction already mentioned on page 19 eliminate this term entirely by making $a - a''' = 0$.

B (3). It would be impossible and, in fact, quite unnecessary to enter here upon a general discussion of the methods of optical measurement best suited to individual cases. We shall endeavor

to point out only some general principles of working which the previous investigations have shown that it is necessary to regard if we aim at the highest attainable degree of accuracy in our work, and to investigate in this connection one or two individual sources of error in the use of particular instruments which seem either to have escaped previous attention, or at least to have been considered less carefully than their importance demands.

In general, the causes will produce the maximum effects in the optical distortion and displacement of images, and the ones therefore which we must most carefully guard against are those dealt with in sections *A* (2), *A* (3), *B* (1), and *B* (2).

In order to eliminate, or minimize as far as possible, the effects of asymmetrical, spherical, or chromatic aberration, A (2) and A (3), we must employ instruments and methods of work planned with special reference to *avoiding* the following sources of error: (1) the use of too large fields or of too great angles of incidence, particularly when using short focus mirrors or gratings at the principal focus; (2) the use of methods of comparison or of forms of instruments and of optical construction which necessitates that the different images whose relative positions are to be measured are formed by different portions of the surfaces of the optical train; (3) variations in temperature conditions during the interval of measurement or of record; (4) the effect of small chromatic dispersions, particularly when the spectral curve of radiation is different for the different sources; (5) the effect of varying chromatic absorption and unsymmetrical broadening of the spectral lines when the chromatic dispersion is large.

In the case of errors of Class B we must especially guard against (6) the use of too small resolving powers, and (7) the continued effect of temperature variations and asymmetrical illuminations.

Of these various effects those which have heretofore received most attention are (3) and (7). Temperature variations are, indeed, our greatest source of trouble and error in the great majority of physical measurements of all kinds. It appears, however, from the results of the investigations on this point

(pp. 292–4), that these effects are of even larger order of magnitude than is ordinarily assumed, and that in spectroscopic work particularly we must guard against them with the greatest care. As was pointed out on pp. 278, 279 and 292–4, the best method of eliminating residual effects, after reducing the temperature variations to the smallest possible range, is to measure the separation of the images and determine the so-called constants of the instrument, *i. e.*, focal length, pitch of measuring screw or scale, etc., simultaneously with each observation, instead of relying on values determined once for all at considerable intervals, as is usually done.

This method of simultaneous record is carried out most completely at present in determining star places by the chart photographs of the Astrophotographic Survey, in which the constants of reduction for each plate are determined from the photographic record on the plate itself. The next best example is that of the spectrographic method of determining absolute wavelengths and motions in the line of sight by the use of comparison spectra. The only criticism to the present practice of the method in the latter case is that the record of the comparison spectrum and of the spectrum to be measured is not simultaneous, and in the case of star spectra, in which an interval of sometimes two hours intervenes between the records, the residual errors of intermediate temperature variations, small though they may be, are in certain cases quite sensible. It would be much better, as already pointed out, if the exposure on the comparison spectra were made continuously instead of intermittently at the beginning and end of the exposure, and this could be arranged without any great difficulty.

It must be farther noted, however, that this method of simultaneous record and individual reduction of each set of measures, although it may eliminate almost completely the effect of small temperature variations, will not eliminate the effects of (1), of (2), of (4), of (5), nor of the latter part of (7), unless, indeed, the conditions of observation are precisely the same in all of these respects for both the objects upon which the direct measurements are made and the objects upon which the comparison meas-

urements for the "constants" of reduction are executed. It is physically impossible always to fulfill this requirement in the case of (4) and (5), and frequently very difficult in the case of (1), (2), and (6) because of the different parts of the field in which the two sets of images (direct and comparison) are situated, or because of necessary limitations in the size and power of the observing instrument.

For these reasons it is unsafe, if we aim at the highest degree of accuracy in optical measurements, to neglect the causes of optical displacement considered above, or to assume beforehand that their effects are vanishingly small. In this connection it is necessary to insist again on the point already made, that the ordinary tests and standards of optical definition are not sufficiently rigorous to serve as criteria in determining questions relating to the limiting accuracy of measurement. Nor are we safe in considering our results free from constant errors which may arise from instrumental causes, simply because they agree consistently among themselves, or are checked by occasional measurements of known quantities. Our only safe way of procedure would seem to be to investigate in each individual case, as we have already done in the general one, the *maximum individual* effects produced by possible disturbances of theoretical conditions, and either to reduce each of these maximum individual effects (unless there may be two or more which are *always necessarily* compensatory) to a quantity less than ϵ, the limiting metrological power; or to determine the magnitude of each effect to the same degree of exactness, in order that it may be taken care of as a known correction. In such individual cases there may exist possible causes of disturbance of a special character not included in the general classes of errors already discussed. As stated at the beginning of this section, it is unnecessary and undesirable to take up in detail a complete discussion of any number of such cases.

There is, however, one individual case in which an error of measurement due to a cause not yet considered is very likely to be introduced, and which on account of its importance in one of the leading fields of astrophysical research we will consider at

length. This is the case of the slit spectroscope or spectrograph as used for determinations of velocity of motions in the line of sight, or more generally of absolute wave-lengths. Here, in order either to secure greater intensity of spectrum, or to localize determinations with reference to different parts of the sources under examination, images of both the latter and of the comparison sources are first formed on the slit of the instrument, and we measure the relative positions, not of the spectral images of the sources themselves, as they would be formed by an objective prism or grating, but the relative positions of the spectral images of the slit as illuminated by the light concentrated upon it by the image-forming objective. The distinction is an important one, for, as we have seen in the case of A (3), any asymmetrical distribution in intensity in the source of radiation will result in an asymmetrical distribution in intensity in the image, and a consequent displacement of its center of maximum brightness. Hence if the slit is *not* uniformly illuminated across its entire width, its spectral image corresponding to any particular wave-length in the light falling upon it will be displaced from the position it would occupy under conditions of uniform illumination, no matter what the conditions in the source itself may be. The asymmetrical illumination of the slit may arise from (1) asymmetrical distribution of intensity in the image of the source itself on the slit, due either to a real asymmetry of radiation or to asymmetrical distortion or aberration of the image-forming objective; (2) displacements of the center of symmetry of the image with reference to the center of the slit. As the effect on the measurements will be the same in both cases we may consider them jointly.[1]

The general expression for the distribution in intensity in the

[1] This effect is entirely different in nature from that produced by lack of uniform illumination of the collimator objective, which has already been considered in *B* (2). So far as I have been able to ascertain, the effect of asymmetrical illumination of the slit itself it had escaped attention previous to the time when the preliminary results of this paper were informally announced at the Astrophysical Conference at the Yerkes Observatory in 1897. It was then expected that these results would be at once investigated in detail by others more directly interested than the writer in line of sight work, but their bearing on this line of research seems to have been overlooked or not fully realized by those present at that time.

spectral image of a slit *uniformly* illuminated across its entire width by radiation from any individual spectral line has already been given [equation (52)], and this expression has been evaluated for a number of special cases.[1] If the illumination across the width σ is not uniform, we must add another factor, $f(\xi)$, to express this variation in intensity, which gives us in (52)

$$I_{,,,} = A'' \int_{-\frac{\sigma}{2}}^{+\frac{\sigma}{2}} f(\xi)\, \psi_1 \{\xi - \psi,\ a_0 a\}\, d\xi \ . \qquad (112)$$

As in the consideration of a uniformly illuminated slit, we will first examine the case in which the light on the slit is absolutely monochromatic and the spectroscope itself is free from aberration. Then at the focal plane of the instrument the function ψ_1 becomes simply

$$\frac{\sin^2 \frac{\pi a}{a_0}}{\left(\frac{\pi a}{a_0}\right)^2} ,$$

and we have for $I_{,,,}$

$$I_{,,,} = A'' \int_{-\frac{\sigma}{2}}^{+\frac{\sigma}{2}} f(\xi)\, \frac{\sin^2 \frac{\pi}{a_0}(a - \xi)}{\frac{\pi}{a_0}(a - \xi)}\, d\xi \ , \qquad (113)$$

which is the same form as that expressing the distribution in intensity in the image of an individual spectral line [equation (51)], for an infinitely narrow slit.

It is evident that (113) like (51) will be symmetrical or asymmetrical according to the form of the function $f(\xi)$. We will examine a few cases of particular interest in detail.

First, suppose

$$f(\xi) = A + B\xi \ . \qquad (114)$$

This represents a uniformly progressive increase in the intensity of illumination across the entire width of the slit, such as is graphically illustrated in Fig. 14. Taking the origin of co-ordinates at the point corresponding to the geometrical image of

[1] *Loc. cit.*

one edge of the slit (instead of the center as before), the integral (113) becomes

$$I_{\prime\prime\prime} = A'' \int_0^{\sigma} (A + B\xi) \frac{\sin^2 \frac{\pi}{a_0} \left\{ a - \left(\xi + \frac{\sigma}{2} \right) \right\}}{\left[\frac{\pi}{a_0} \left\{ a - \left(\xi + \frac{\sigma}{2} \right) \right\} \right]^2} d\xi \,. \qquad (115)$$

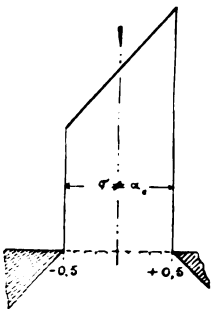


FIG. 14.

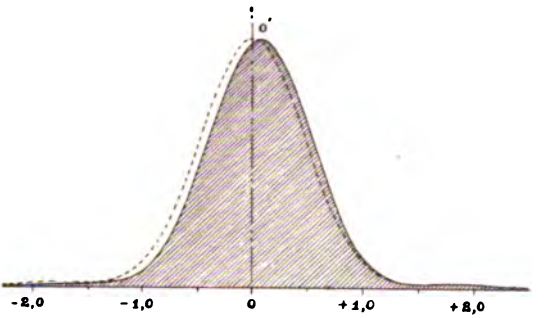


FIG. 15.

I have evaluated this integral by mechanical quadrature for the special case $A = \frac{I}{2}$, $B = \frac{I}{2a_0}$, and $\sigma = a_0$ (*i. e.*, the case in which the angular width of the slit is equal to the resolving power of the view telescope), and the intensity of illumination at one edge, $\xi = 0$, is one-half that at the other, $\xi = a_0 = \sigma$. The results to three places of decimals are tabulated in terms of fractional parts of $\frac{a}{a_0}$ in Table V and plotted in Fig. 15. In both the table and the figure the origin of co-ordinates has been shifted back to the center of the geometrical image.

An examination of these results shows that the position of maximum brightness in the spectral image of the slit is displaced in this case about $0.08\, a_0$ from the center of the geometrical image. This is just a little more than the limiting accuracy ϵ of the instrument, *i. e.*, a uniform variation of 50 per cent. in intensity of illumination from edge to edge of the slit will displace the center of the physical spectral image by an amount somewhat exceeding the limiting accuracy of measurement.

TABLE V.

$\frac{a}{a_0}$	$I_{\cdots}=f(\xi)$	$\frac{a}{a_0}$	$I_{\cdots}=f(\xi)$	$\frac{a}{a_0}$	$I_{\cdots}=f(\xi)$
−2.45	0.011	−0.75	0.235	+0.95	0.148
−2.35	.011	−.65	.317	+1.05	.090
−2.25	.012	−.55	.429	+1.15	.054
−2.15	.014	−.45	.552	+1.25	.036
−2.05	.015	−.35	.672	+1.35	.030
−1.95	.018	−.25	.798	+1.45	.030
−1.85	.020	−.15	.898	+1.55	.032
−1.75	.023	−.05	.968	+1.65	.033
−1.65	.026	+.05	1.000	+1.75	.031
−1.55	.028	+.15	0.987	+1.85	.027
−1.45	.030	+.25	.933	+1.95	.022
−1.35	.032	+.35	.844	+2.05	.017
−1.25	.038	+.45	.727	+2.15	.013
−1.15	.048	+.55	.595	+2.25	.011
−1.05	.068	+.65	.462	+2.35	.010
−0.95	.102	+.75	.337	+2.45	.010
−0.85	.153	+.85	.231	.....	....

Second, suppose

$$f(\xi) = C\sqrt{\xi} \,. \tag{116}$$

This represents a progressively increasing illumination from edge to edge as graphically illustrated in Fig. 16.

With the origin of co-ordinates at one edge of the slit as before, the integral (113) reduces to the form

$$I_{\prime\prime\prime} = A''' \int_0^{\sigma} \sqrt{\xi}\, \frac{\sin^2 \frac{\pi}{a_0}\left[a-\left(\xi+\frac{\sigma}{2}\right)\right]}{\left[\frac{\pi}{a_0}\left\{a-\left(\xi+\frac{\sigma}{2}\right)\right\}\right]^2} d\xi \,. \tag{117}$$

This integral has been evaluated for a slit width $\sigma = a_0$ as before for the special case $C = 1$. The results are tabulated and plotted in terms of $\frac{a}{a_0}$ as in the preceding case in Table VI and Fig. 17.

The position of maximum intensity, o′ is, in this case, displaced about $0.13\, a_0$ from the center of the geometrical image, *i. e.*, about twice the limiting error of measurement.

Cases in which the illumination of the slit is of the nature of that just investigated are frequently met with in spectrometric work on the Sun and planets when the image of these bodies is

placed eccentrically on the slit or when particular portions of the surface, such as the Sun-spots or solar prominences, are under examination. In the latter case the illumination is likely to be of the nature of that assumed in (114) and (115), and the effect of the variation in intensity of the image on the measured

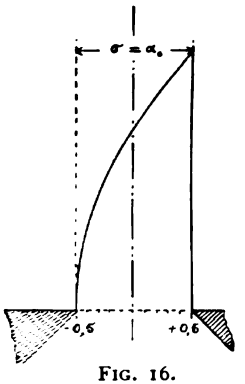


FIG. 16.

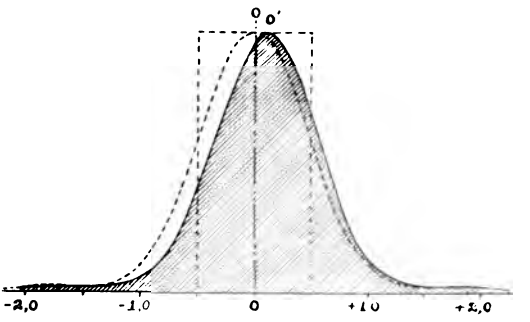


FIG. 17.

wave-lengths of the spectral lines can be disregarded, as we have seen, only when said variation is less than 50 per cent.

In the case of measurements of the velocity of rotation the slit is generally placed tangentially to the limb of the Sun or planet, and the conditions of illumination are then more gener-

TABLE VI.

$\frac{a}{a_0}$	$I_{...}$	$I'=f(\gamma)$[1]	$\frac{a}{a_0}$	$I_{...}$	$I'=f(\gamma)$[1]
−1.0	0.062	0.11	+0.00	0.969	1.00
−0.9	.090		+0.1	1.000	
−0.8	.136	0.24	+0.2	0.985	0.92
−0.7	.204		+0.3	.925	
−0.6	.295	0.45	+0.4	.827	0.71
−0.5	.406		+0.5	.702	
−0.4	.533	0.71	+0.6	.564	0.45
−0.3	.665		+0.7	.426	
−0.2	.784	0.92	+0.8	.301	0.24
−0.1	.896		+0.9	.198	
−0.0	.969	1.00	+1.0	.120	0.11

[1] For the purpose of comparison the values of I', which expresses the distribution in intensity in the image of a slit of the same width uniformly illuminated, are given in columns 3 and 6. See *Phil. Mag.* 43, 323, Table I; ASTROPHYSICAL JOURNAL, 3, 333 *et seq.* The curve $I' = f(\gamma)$ is plotted in dotted lines in both Fig. 15 and Fig. 17.

ally such as are represented by the distribution assumed in (116) and (117). For in the case of the Sun the variation in the intensity of radiation from the center to the edge is of the nature shown by the curves of Fig. 1 of the preceding paper of this series,[1] and may be represented by the general expression

$$i = \phi(\xi^2 + \eta^2)^{\frac{1}{2}} . \tag{118}$$

The effective intensity $f(\xi)$ at any point on the axis $\eta = 0$ will be found for this case by integrating the expression

$$f(\xi) = \int_0^{\sqrt{R^2-\xi^2}} \phi(\xi^2+\eta^2)^{\frac{1}{2}} \frac{\sin^2 \frac{\pi}{a_0}(a-\eta)}{\left[\frac{\pi}{a_0}(a-\eta)\right]^2} d\eta . \tag{119}$$

At the edge of the Sun the value of η increases very rapidly with respect to ξ, and for large values of η the integral

$$\int_0^{\eta} \frac{\sin^2 x}{x^2} dx$$

rapidly approaches a constant value. Hence for the purposes of the present general investigation in which we are concerned more with the form of $f(\xi)$ than with its absolute value, we may neglect the second factor of the term under the integral sign of (119) and simply write

$$f(\xi) \cong \int_0^{\sqrt{R^2-\xi^2}} \phi(\xi^2+\eta^2)\, d\eta . \tag{120}$$

This integral was evaluated by the writer several years ago (in connection with another research by Professor Michelson), for the particular value of $\phi(\xi^2-\eta^2)$ corresponding to wave-length $\lambda = 5790$ tenth-meters, as given in Vogel's tables. The results were published in the form of a curve (Fig. 5) in Michelson's paper in the *Philosophical Magazine*.[2] An examination of this

[1] F. W. VERY, "The Absorptive Power of the Solar Atmosphere." *Misc. Sci. Papers*, Allegheny Observatory, No. 9; ASTROPHYSICAL JOURNAL, **16**, 73. See also VOGEL, "Spectralphotometrische Untersuchungen," *Monat. d. K. Akad. d. Wiss.*, Berlin, March 1877; pp. 23, 24, Plate II.

[2] "Application of Interference Methods to Astronomical Measurements," **30**, 1.

curve will show that for small values of ξ the function $f(\xi)$ can be very closely represented by the parabola

$$f(\xi) = C\sqrt{\xi}$$

which is the form of function used in (116) and (117) already examined.

The effect of this inequality of illumination is, as we have already seen, to displace the spectral image of any line by an amount considerably in excess of the limiting accuracy of measurement. The exact amount of the displacement will depend (*a*) on the width of the slit in comparison with the diameter of the solar image, (*b*) on the position in which the latter is placed with reference to the slit opening. In the case already investigated in (117) we have assumed that the photospheric edge of the solar image is placed tangent to the outer edge of the slit so that when $\xi = o$, $f(\xi)$ is also zero.[1] If the image overlaps the slit, the conditions approximate more nearly those assumed in (114) and (115) and the displacement is less; if the edge of the image is within the slit, the displacement is even greater. Thus when the edge is coincident with the center of the slit, the position in which it is generally placed for velocity of rotation measurements, the displacement of the central point of intensity from the geometrical center will be nearly 0.6 of the half width of the slit, *i. e.*, about $0.3\,a$, or four times the limiting accuracy ϵ.

The direction of the displacement in the spectrum will depend on the position of the solar image with reference to the direction of deviation of light by the spectroscope train. If the edge of the image is on that side of the slit corresponding to the red end of the spectrum, the displacement will be toward the violet, and the wave-length derived from the measurement will be too small; if turned in the opposite direction the wave-length will be correspondingly too large. The sign of the displacement will, therefore, change not only as we shift the solar image so

[1] This of course is only an approximation to the actual conditions at the edge of the Sun. There is, however, such a sharp demarcation in intensity of total radiation between the photosphere and the chromosphere and outlying corona that the approximation is in most cases not far from the truth.

that first one edge and then the other falls on the slit; but also as we reverse the direction of deviation of the spectroscope train, "right" or "left," leaving the position of the solar image unchanged. If we therefore attempt to determine the velocity of rotation of a body by the direct measurement of the separation of the spectra from the opposite edges of the slit image, the total error in any one position of the spectroscope will be twice the error of displacement noted above. If we then take a corresponding series of measurements with the direction of dispersion reversed, the errors of displacement will be of the same magnitude as before, but reversed in sign; hence the total difference in measurement will be four times the individual displacement in any one position of image and spectroscope.

Errors of such magnitude as these cannot, of course, be disregarded, even though they may be eliminated by the exact reversal of the relative positions of image and spectroscope. I have, however, failed to find any reference to this particular source of error in the papers of the investigators who have given us our best spectroscopic determinations of velocity of solar rotation. Crew, indeed, in his second investigation[1] notes that the results of his measurements differ with grating "right" and grating "left," but he ascribes this difference to an entirely different cause, *i. e.*, the local heating of the slit jaws. This will undoubtedly produce an effect unless the greater part of the solar image is cut off by a suitable screen interposed in front of the slit, but even when this is not done the total displacement of the slit jaws due to this cause can hardly, in the opinion of the writer, be as great as it is necessary to assume to explain the observed differences of measurement. To do this it is necessary, as Crew has shown, to assume a change of temperature of about 15° C.[2] of each jaw between each measurement, in an interval of only about one minute, and to assume, further, that the expansion due to this change in temperature is all toward the

[1] "On the Period of Rotation of the Sun," *Am. Jour. Sci.*, 38, 204.

[2] The assumption of a difference of 10 C., made in Crew's paper, p. 210, explains only about two-thirds of the observed differences of equatorial velocities with grating "right" and grating "left."

slit opening. Crew observed actual differences of temperature of 10° between two thermometers placed on the two sides, but the change in the slit jaws themselves would be probably much less on account of their own mass and the masses of surrounding and attached metal. Likewise, the jaws when heated, would, if of the usual double-motion construction, expand in both directions. Lastly, if the change of temperature explanation were the correct one, there would be a continual and progressive shift of the spectral image during the entire interval of measurement, and this effect would not have been likely to escape the attention of so careful and accurate an experimentalist as Professor Crew.

On the other hand, if the width of the slit and the setting of the edge of the solar image corresponded to the conditions assumed in the previous integration of (117) the resulting displacement of the spectral images "right" and "left" would *fully* explain and account for the observed differences of measurement. For, as we have shown, the total difference of measurement to be expected is four times the individual displacement of any one image, and this latter is $0.13a_0$ for the conditions assumed. In this instrument the aperture and the focal length of the view telescope are 4 inches and 94 inches respectively. Hence for the D lines the linear value of a_0 is

$$a_0 f = \frac{0.0005896}{94} \times 4 = 0.014 \text{ mm} .$$

Hence the total difference $= 4 \times 0.13 a_0 = 0.52 a_0$ is in linear measure

$$0.014 \text{ mm} \times 0.52 = 0.0073 \text{ mm} .$$

The total observed difference in measurement amounted to about 0.015 revolutions of the micrometer screw, or, since the latter is of 0.5 mm pitch, to

$$0.0075 \text{ mm} ,$$

or almost exactly the quantity required by theory. When, therefore, the cause of displacement investigated above is taken into account in Crew's measurements, there is no discrepancy remaining to be explained.

The results already obtained indicate the care which it is necessary to take in determining the exact distribution of light in the slit image when the latter is necessarily variable, as in the preceding case. If this cannot be determined, the only way to avoid sensible errors in the wave-length measurements is to secure and maintain an exact centering of the image on the slit, so that the distribution of light over the slit opening is symmetrical about a center line. A case of great importance in this connection is that of determinations of wave-length in stellar spectra for the purpose of measuring motions in the line of sight, etc., with the compound slit spectroscope. The image of the star, as formed on the slit by the main telescope objective, is, if the latter is free from unsymmetrical aberration, a symmetrical diffraction pattern. If the slit is placed in the focal plane of the main objective, the theoretical distribution of light in this image is, for any given wave-length, represented by the usual expression (4), *i. e.*,

$$I_c^2 = A \frac{J_1^2\left(\frac{\pi}{a_0}\xi\right)}{\left(\frac{\pi}{a_0}\xi\right)^2}.$$

In long photographic exposures, the actual effective distribution of light in the slit image is somewhat different from this, owing to atmospheric and instrumental disturbances which broaden the image into what Newall has very aptly termed a "tremor disk." The determination of the exact distribution of light in such a disk is a matter of considerable uncertainty, since the broadening results not only from vibrations of the image, but also from temporary changes of focus and of chromatic dispersion and aberration due to passing air-waves of variable intensity. Under good conditions of "seeing," however, the principal cause of the broadening may be regarded as due to vibrations, and under such conditions we may derive an expression which will represent at least a closely approximate distribution of light in the tremor disk, from the law of probability.

The general expression which represents the probable law of

distribution of errors, or, in this case, of displacements from a central position, is

$$y = e^{-h^2\xi^2} \tag{121}$$

and the most probable error (*i. e.*, displacement) is that for which $y = 0.5$ and is given by the relation

$$\xi_0 = P = \frac{0.4769}{h} \tag{122}$$

In the present case it seems a fair assumption to consider that the most probable displacement of the image will be about $\frac{1}{2}\, a_0$, *i. e.*, half the resolving power of the main telescope objective. This assumption gives us for h

$$h = \frac{0.9538}{a_0} , \tag{123}$$

which substituted in (121) gives us for y

$$y = e^{-0.9097\frac{\xi^2}{a_0^2}} . \tag{124}$$

The resulting distribution in intensity in the tremor disk will then be represented by the integral

$$\int_{-\infty}^{+\infty} e^{-0.9097\frac{\xi^2}{a_0^2}} \frac{J_1\left(\frac{\pi}{a_0}\xi\right)}{\left(\frac{\pi}{a_0}\xi\right)^2} d\xi = f(\xi) , \tag{125}$$

or if we assume a rectangular aperture as before, by

$$I_t^2 = \int_{-\infty}^{+\infty} e^{-0.9097\frac{\xi^2}{a_0^2}} \frac{\sin^2\frac{\pi}{a_0}\xi}{\left(\frac{\pi}{a_0}\xi\right)^2} d\xi = f(\xi) , \tag{126}$$

and the resulting distribution in the spectral image of the slit of width σ_0 will be

$$I_{,,,}^2 = \int_{-(\sigma_2-\sigma_1)}^{+\sigma_1} I_t^2 \frac{\sin^2\frac{\pi}{a_0}(a-\xi)}{\left[\frac{\pi}{a_0}(a-\xi)\right]^2} d\xi . \tag{127}$$

In the integration of (126) and (127) it is convenient to express a_0 in terms of σ_0, the width of the spectroscope slit.

The usual practice is to make this width equal to $2\,a_0$[1] so as to just include the central diffraction disk of the undisturbed star image. Substituting this value of σ_0 in the above equations we obtain

$$I_t^2 = \int_{+\infty}^{-\infty} e^{-3.64\frac{\xi^2}{\sigma_0^2}} \frac{\sin^2 \frac{2\pi}{\sigma}\xi}{\left(\frac{2\pi}{\sigma}\xi\right)^2} d\xi \ . \qquad (128)$$

This has been integrated by mechanical quadrature as before. The results are tabulated in Table VIII and plotted in Fig. 18.

TABLE VIII.

$\frac{a}{\sigma}$	I_t^2	$e^{-A'\xi^2}$	$I_t^2 - e^{-A'\xi^2}$
±0.00	1.000	1.000	0.000
±0.10	0.972	0.973	−.001
±0.20	.888	.892	−.004
±0.30	.771	.773	−.002
±0.40	.632	.633	−.001
±0.50	.490	.490	±.000
±0.60	.361	.358	+.003
±0.70	.254	.247	+.007
±0.80	.171	.161	+.010
±0.90	.111	.099	+.012
±1.00	.069	.057	+.012

The full curve, I_t, closely resembles the exponential curve $e^{-A'\xi^2}$. We have therefore substituted for (128) a second empirical curve of this form having such a value of A' that the two curves coincide at the point $x = a_0 = \frac{1}{2}\sigma_0$. This gives us for A'

$$A' = 2.854 \ ,$$

and for $f'(\xi)$

$$e^{-2.854\frac{\xi^2}{\sigma_0^2}} \cong I_t^2 \ . \qquad (129)$$

The values of $f'(\xi)$ from (129) are tabulated in the third column of Table VIII and plotted as the dotted curve in Fig. 18. Over all that part of the tremor disk which is likely to fall within the slit opening, *i. e.*, from $\xi = 0$ to $\xi = \pm 2\,a_0$, the coin-

[1] This is twice as large as the value of σ assumed in the previous cases. Compare Figs. 14, 16, and 18.

cidence between the two curves, I_t^2 and $f'(\xi)$, as given by (129), is closer than the uncertainty as to the exact form of I_t^2 itself.

Assuming that the effective size of the tremor disk is defined by the limits $m\,m$ such that $I_m^2 = 0.04 I_o^2$, we have under the conditions assumed above

$$m\,m = W \cong 4.5 a_o = 2.25 m_o\,m_o \; ; \qquad (130)$$

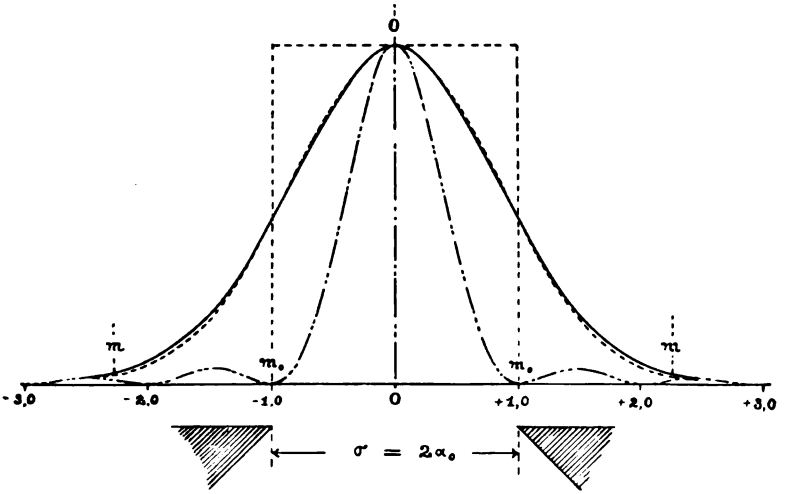


FIG. 18.

i. e., the tremor disk is about two and one-quarter times the size of the true central diffraction disk of the star. When the tremor disk is centered on the slit opening, the percentage of the total quantity of light in the image which enters the slit is of course given by the expression

$$\frac{2}{\sqrt{\pi}} \int_{-\frac{1}{2}\sigma}^{+\frac{1}{2}\sigma} e^{-2.854 \frac{\xi^2}{\sigma_o^2}} d\xi \cong 0.76 \; ; \qquad (131)$$

i. e., the time of exposure is increased about one-third by the effect of atmospheric and instrumental vibrations of the previously assumed probable magnitude of $\frac{1}{2} a_o$.

This result agrees fairly well with the actual exposure times which Campbell finds necessary in obtaining well defined spectra

with the Mills slit spectroscope as compared with those required on the same stars with the Harvard objective prism spectrograph, when we allow for the difference in the resolving power and light efficiency of the two instruments. It is lower than the estimate given by Frost in his description of the Bruce spectrograph, but for some reason the latter is somewhat less efficient in regard to exposure times than the Mills instrument.[1]

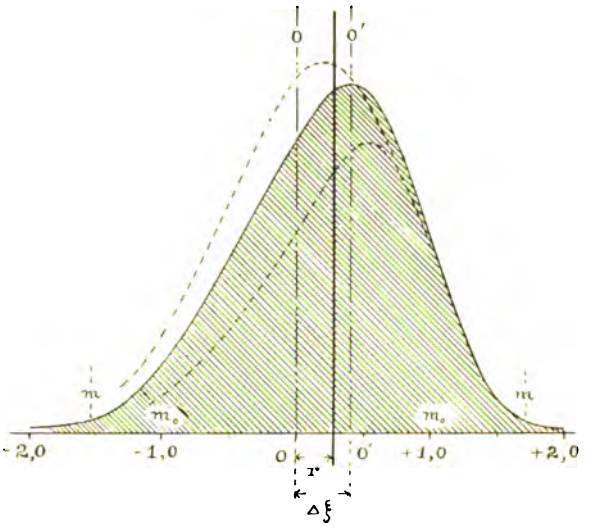


FIG. 19.

Substituting the value of $f'(\xi)$ given by (129) in (127), and expressing a_o in terms of σ_o as before, we obtain

$$I_{,,} = \int e^{-2.854\frac{\xi^2}{\sigma_o^2}} \frac{\sin^2 \frac{2\pi}{\sigma_o}(a-\xi)}{\left[\frac{2\pi}{\sigma}(a-\xi)\right]^2} d\xi \ . \qquad (132)$$

If, as before, we take the origin of co-ordinates at the point corresponding to the geometrical image of one edge of the slit and consider the most general case in which the center of the tremor disk may be located anywhere within the slit opening, we have as before for ξ

$$\xi = \xi' + \frac{\sigma}{2} + \Delta\sigma \ ,$$

[1] Compare result on p. 20 of Frost's paper, ASTROPHYSICAL JOURNAL, **15**, with those given by Campbell, ASTROPHYSICAL JOURNAL, **8**, 141.

where $\Delta\sigma$ is the amount by which the center of the tremor disk is displaced from the center of the slit. Substituting, this gives

$$I''_{,,,} = \int_0^{\sigma_0} e^{-2.854\frac{(\xi+\frac{\sigma}{2}+\Delta\sigma)^2}{\sigma_0^2}} \frac{\sin^2\frac{2\pi}{a_0}\left[a-\left(\xi+\frac{\sigma}{2}+\Delta\sigma\right)\right]}{\left[\frac{2\pi}{a_0}\left\{a-\left(\xi+\frac{\sigma}{2}+\Delta\sigma\right)\right\}\right]^2} d\xi \quad . \quad (133)$$

The only case for which $I''_{,,,}$, (133), is symmetrical is that for which $\Delta\sigma = 0$. The center of intensity is then at the point $\xi = \frac{1}{2}\sigma_0$, and there is no error of displacement.

In order to find the relation between the error of centering, $\Delta\sigma$, and the resulting displacement $\Delta\xi$ of the spectral image, I have integrated (133) for different values of $\Delta\sigma$ from $\Delta\sigma = -0.1\sigma_0$ to $\Delta\sigma = 0.5\sigma_0$. The resulting values for $I''_{,,,}$ are given in Table IX and the curves for $\Delta\sigma = 0.3\ \sigma_0$ (full) and $\Delta\sigma = 0.1\ \sigma_0$ and $0.5\ \sigma_0$ (dotted) are plotted in Fig. 19.

TABLE IX.

$\frac{a}{\sigma}$	$I_{,,,}$ FOR DIFFERENT VALUES OF $\frac{\Delta\sigma}{\sigma}$				
	$\Delta\sigma = 0.1\sigma$	$\Delta\sigma = 0.2\sigma$	$\Delta\sigma = 0.3\sigma$	$\Delta\sigma = 0.4\sigma$	$\Delta\sigma = 0.5\sigma$
−1.00			0.018		
−0.90			.021		
− .80			.034		
− .70			.058		
− .60			.107		
− .50			.191		
− .40			.296		
− .30			.422		
− .20			.550		
− .10			.678		
− .05	0.948		.742		
± .00	.977	0.909	.803	0.677	0.545
+ .05	**.995**	.948	.858	.741	.611
+ .10	**1.000**	.975	.903	.799	.672
+ .15	0.988	**.986**	.935	.844	.727
+ .20	.959	**.978**	[**.946**]	**.872**	.765
+ .25	.912	.948	.934	**.877**	**.783**
+ .30		.893	.896	.855	**.776**
+ .35		.817	.832	.807	.742
+ .40		.724	.746	.732	.682
+ .50			.532	.534	.519
+ .60			.315		
+ .70			.150		
+ .80			.061		
+ .90			.024		
+1.00			.016		

The positions ξ_0 of the points of maximum intensity in the curves $I''_{,,,}$ are found to be as follows:[1]

$$\begin{aligned}
\text{For } \Delta\sigma &= 0.1\sigma_0, \ \xi_0 \cong 0.595\sigma_0 \quad \Delta\xi \cong 0.095\sigma_0 = 0.19a_0 \\
\Delta\sigma &= 0.2\sigma_0, \ \xi_0 \cong 0.655\sigma_0 \quad \Delta\xi \cong 0.155\sigma_0 = 0.31a_0 \\
\Delta\sigma &= 0.3\sigma_0, \ \xi_0 \cong 0.700\sigma_0 \quad \Delta\xi \cong 0.200\sigma_0 = 0.40a_0 \\
\Delta\sigma &= 0.4\sigma_0, \ \xi_0 \cong 0.735\sigma_0 \quad \Delta\xi \cong 0.235\sigma_0 = 0.47a_0 \\
\Delta\sigma &= 0.5\sigma_0, \ \xi_0 \cong 0.765\sigma_0 \quad \Delta\xi \cong 0.265\sigma_0 = 0.53a_0 .
\end{aligned}$$

The relation between $\Delta\sigma$ and $\Delta\xi$ is plotted in Fig. 20 (full curve). As will be seen by the dotted curve it can be represented very closely by the empirical equation

$$\Delta\xi = \frac{7}{8}a_0 \sqrt[3]{\left(\frac{\Delta\sigma}{\sigma}\right)^2} . \tag{134}$$

In measuring the apparent position of the displaced diffraction image the tendency will be, as before, to compromise between the point of maximum intensity, o', and the point midway between the apparent edges, $m\ m$, (defined as before), of the line. The resultant setting, r, for the cross-wire will be at a point not far from $\frac{2}{3}\Delta\xi$. We may therefore write

$$\left.\begin{aligned}
\Delta_m\xi &= \text{measured displacement} \cong \frac{2}{3}\Delta\xi \\
&\cong 0.58a_0 \sqrt[3]{\left(\frac{\Delta\sigma}{\sigma}\right)^2}
\end{aligned}\right\} . \tag{135}$$

In order that the displacement shall not be great enough to affect the accuracy of measurement we must have as before

$$\Delta_m\xi \leqq \epsilon ,$$

or in this case from (14) and (135)

$$0.07 \geqq 0.58 \sqrt[3]{\left(\frac{\Delta\sigma}{\sigma}\right)^2} , \tag{136}$$

or

$$\frac{\Delta\sigma}{\sigma_0} \leqq .04 .$$

i. e., the star image must be centered on the slit with an error not exceeding 4 per cent. of the slit width.

This is a condition which has apparently never before been

[1] The points of maximum lie between the numbers in heavy type in the table.

recognized as necessary to accuracy in the use of the slit spectroscope. Care has of course always been taken to keep the star image as nearly central as possible in the slit opening, but the object aimed at has been simply to utilize the greatest amount of light and shorten the time of exposure as much as possible. But the above investigation shows that the requirements of accuracy and the avoidance of constant errors require a far greater degree of care in centering and following than is usually deemed necessary, greater in fact than many of the devices now used for following make possible.

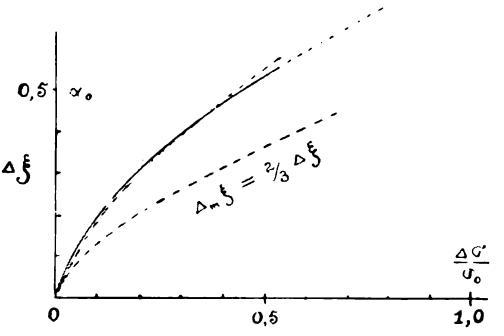


FIG. 20.

This question is one of such great importance in view of the minuteness of the displacements upon which determinations of motions in the line of sight depend, that it may be interesting to calculate the error in kilometers per second produced by an error of say 10 per cent. in the centering of the star image on the slit. From the previous results we find, for $\frac{\Delta\sigma}{\sigma_0} = 0.10$,

$$\Delta_m\xi = 0.13\alpha_0 \,. \qquad (137)$$

Since the angular resolving power α_0 is the same for both collimator and view telescope, we can readily establish a relation between α_0, $\frac{d\theta}{d\lambda}$ = the dispersion of the spectroscope train, and v_s the motion in the line of sight of the body under examination. From the well-known relations

$$v_s = \frac{\Delta\lambda}{\lambda} V$$

$$f\alpha_0 = \frac{\lambda}{b} f = \frac{\lambda}{\beta}$$

and

$$\Delta\lambda = \frac{\Delta\xi}{\frac{d\theta}{d\lambda}} \,,$$

we get at once

$$v_s = \frac{\Delta\xi}{\frac{d\theta}{d\lambda}} \cdot \frac{V}{\lambda},$$

and for the value of $\Delta_m\xi$ assumed in (137)

$$\delta v_s = \frac{0.13V}{b\frac{d\theta}{d\lambda}} = \frac{0.13V}{R} \qquad (138)$$

$$= \frac{38000}{r} \text{ km per sec.},$$

where r is the spectroscopic resolving power of the prism train.

The following table contains the approximate resolving powers of the more important instruments that have been used in line of sight work and the corresponding error δv_s produced by a 10 per cent. error of centering of the star disk from (138):

TABLE X.

Name of Instrument	Spectroscope Train	Approximate r for $H\gamma$	δv_s Kilometers per Second
		about	about
Potsdam II (Vogel)	2 Compound Prisms	40,000	0.9
Pulkowa (Bélopolsky)			
Bruce I (Newall)[1]	1 Simple Prism	22,000	1.7
Mills (Campbell)[2]	3 " Prisms	74,000	0.5
McMillin (Lord)[3]	2 " "	42,000	0.9
Potsdam III (Vogel)[4]	3 " "	63,000	0.6
Bruce II (Frost)[5]	3 " "	110,000	0.2

An examination of this table shows that even for this small error in centering the errors in the resulting determinations of v_s are sufficiently large to account for a large part of the differences of measured velocities between some of the different observers. When we remember the difficulty of keeping a faint star *accurately* centered on the slit, and remember, also, that any constant asymmetry of the tremor disk itself due to peculiar instrumental conditions will notably increase the effect of a given displacement from the center, we can readily see that we have here what may be a constant source of error of the most serious kind, and in some cases of even greater magnitude than indicated in the above table. If, for example, the error of centering were

[1] ASTROPHYSICAL JOURNAL, 3, 266. [3] *Ibid.*, 4, 50.

[2] *Ibid.*, 8, 123. [4] *Ibid.*, 11, 393. [5] *Ibid.*, 15, 1.

for any reason constant in direction with reference to the large telescope, any reversal of the spectroscope with reference to the latter would introduce differences in observed positions of the lines, and hence of v_s of twice the amount indicated in Table X.

As this error is purely one of manipulation, we cannot in general make any correction or allowance for it *after* the photographic record has been taken. The utmost care, therefore, must be exercised in order to attain the accuracy of centering and following requisite to eliminate it completely, as indicated in (136). To do this we should, as far as possible, fulfill the following general conditions:

1. Have a sharp, well-defined, and symmetrical image of the star formed on the slit of the spectrograph. If the main image-forming objective is a visual refractor, this will necessitate in general the use of a correcting lens, and this should be so designed as to be free from spherical and chromatic aberration for the region of the spectrum under examination, and so mounted that its principal optical axis *coincides* with that of the main objective and the axis of collimation of the spectroscope itself. The light from the other portions of the spectrum should be cut out from the following eyepiece by use of suitable screens or screening devices[1] placed between the slit and eyepiece. On account of the perfect achromatism of the reflecting telescope this form of objective has great advantages over the refractor in this connection; another reason why such instruments should be given the preference for spectroscopic work.[2]

2. The guiding device should use as a reference mark for centering the star image, some point on the slit itself, in order

[1] Such, for example, as the optical color screen described by the writer in this JOURNAL, 3, 169.

[2] The importance and value of the reflecting mirror as compared with the refractor in spectroscopic work have been urged upon the attention of astrophysicists by the writer for many years. See *Phil. Mag.*, July and October, 1894; *A. and A.*, December, 1894; ASTROPHYSICAL JOURNAL, January, 1895; *ibid.*, March, 1895; *ibid.*, March, 1896; *ibid.*, May, 1896; *ibid.*, October, 1896; *ibid.*, February, 1897; *ibid.*, February, 1898; *Pop. Astron.*, February, 1898, etc. The advantages of the reflector in this line of work are now being more generally recognized, and a number of large instruments of this type are planned or in actual course of construction by our large astrophysical observatories.

to avoid any errors of parallax or relative displacement of the slit and guiding cross-wires. For this reason I believe a modification of the Huggins reflecting slit and following device is better adapted to this purpose than any other form yet invented. The use of an auxiliary independent guiding telescope such as is proposed with the new Potsdam instrument would seem to be especially dangerous.

3. If the required accuracy of following cannot be attained with the usual slit width $\sigma_0 = 2\,a_0$, then the width should be reduced and the time of exposure correspondingly increased, or else greater spectroscopic resolving power should be used. The effect of decreased slit width will be to reduce the limits of integration in (127) and (133), and correspondingly reduce the resultant asymmetry and displacement of the center of intensity of the spectral image. The result of increasing R will be to decrease the value of δv_s in (138) for a given value of $\Delta\sigma$.

In this connection it may also be noted that "bad seeing," which results in an increase in size of the tremor disk, will reduce the effect of a given percentage error of centering $\frac{\Delta\sigma}{\sigma_0}$ by flattening the curve I_i^2 of Fig. 18, and thus reducing the difference between the intensity of illumination at different parts of the slit image. When the "seeing" is "bad," therefore, the slit may be opened wider without increasing the effect of errors of following.

B (4). It was the intention of the writer to take up also, in connection with the present paper, the discussion of effects of errors of mechanical design and construction, with reference particularly to the avoidance of flexure and strain resulting from changes in position or changes of temperature of the instrument during use. The present discussion has, however, so far exceeded the limits originally set that it seems better to reserve this part of the subject for a future paper in which the description of the spectrographs and some other astrophysical instruments of the new Allegheny Observatory will also be taken up.

ALLEGHENY OBSERVATORY, 1901–1902.

ON SCREENS TRANSPARENT ONLY TO ULTRA-VIOLET LIGHT AND THEIR USE IN SPECTRUM PHOTOGRAPHY.

By R. W. Wood.

Anyone who has repeated Tyndall's beautiful lecture experiment of kindling a pine stick in the dark heat focus of a burning-glass, concentrating light from which the visible radiations have been removed by means of a solution of iodine in bisulphide of carbon, must have wished that we possessed a screen opaque to visible light and transparent to the ultra-violet.

I have recently succeeded in making a screen quite transparent to these radiations, though a gas flame cannot be seen through it. By combining it with a large condensing lens and an arc-lamp, it is possible to form a dark focus of ultra-violet light in which a lump of uranium nitrate glows with a vivid green phosphorescence like a great emerald.

Aside from giving us the means of performing a most beautiful lecture experiment, these screens make it possible to photograph the ultra-violet lines in grating spectra of higher orders than the first, entirely uncontaminated by the visible radiations which overlie them. Other applications at once suggest themselves, such as the complete removal of the highly actinic blue and violet rays, in certain investigations of the ultra-violet region where the long exposures necessary are apt to produce fogging of the plates. It seems quite possible, too, that photographs of the Moon, planets, and nebulæ, taken by means of ultra-violet light, may furnish valuable data, as I shall attempt to show at the end of this paper.

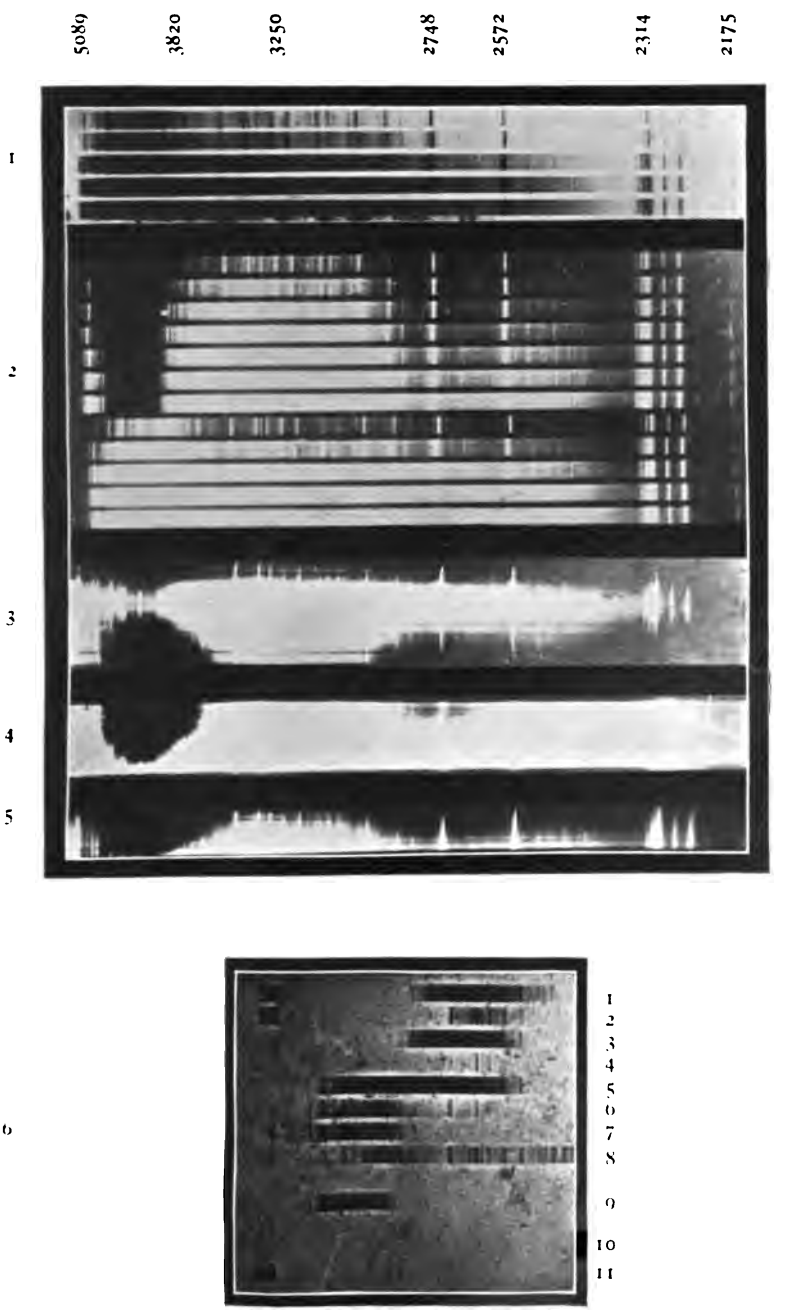
The substance which has made possible the production of such a screen is nitroso-dimethyl-aniline, the remarkable optical properties of which I have already alluded to in a previous paper. As I have already said, a prism formed of this substance yields a spectrum about thirty times as long as a quartz prism of the same angle, the dispersion somewhat resembling that of selenium. I

was of the opinion that the absorption, which commences at about wave-length 0.0005 mm, would increase continuously from this point down to the end of the spectrum, as was found to be the case with selenium. On commencing a study of the absorption, however, I was astonished to find that it ended abruptly a little beyond the H and K lines, and that from this point on the substance was transparent even down to the last cadmium line, of wave-length 0.0002 mm. It at once occurred to me that, if some substance or substances could be found, absorbing the red, yellow, and green, and transparent to the ultra-violet, we could, by combining them with the nitroso compound, produce the long-sought screen.

Very dense cobalt glass, coated with a thin film of gelatine lightly stained with the nitroso, was found to be transparent only to the extreme red and the ultra-violet, and the red was eventually removed by means of a thin sheet of Chance's "signal-green" glass, such as is used for one of the reflectors in the Ives Kromskop. This combination is wholly opaque to visible light, while freely transmitting everything between wave-lengths 34 and 38. Of course, the employment of glass screens limits the ultra-violet transmission, and a screen of this description is useful chiefly for lecture demonstrations. Considerable care must be used in the adjustment of the strength of the solution of the nitroso in gelatine, otherwise the intensity of the ultra-violet light is considerably weakened. The best strength is such as will be just sufficient to remove the blue and violet light transmitted by dense cobalt glass. Quite a number of trials will be found necessary in adjusting the densities of the three components of the screen to secure the maximum effect, but when the balance is just right, it is possible to form a focus in which a piece of paper is quite invisible, while a mass of crystals of the nitrate of uranium (which I have found superior to anything else) glows with sufficient intensity to be seen from the back of the largest lecture-room. It is best to exclude carefully all light which does not pass through the screen.

With the assistance of one of our students, I am at the present time investigating the absorption of a large number of sub-

PLATE XII.



EFFECTS OF ABSORBING SCREENS.

stances which, so far as I know, have not been previously studied, and I hope in time to dispense with glass entirely, and produce an opaque screen which transmits ultra-violet light down to the end of the spectrum. A tube filled with iodine vapor and furnished with quartz ends, on one of which is a thin film stained with nitroso, transmits all the ultra-violet, and only the extreme red, but it is very inconvenient to work with. For use as a screen in spectrum photography there is no especial object in removing the red, yellow, and green, the nitroso alone blocking out completely the actinic portions of the visible spectrum, which overlie the ultra-violet in the second, and third-order spectra, and I shall next consider solutions of the substance in various fluids in connection with spectrum photography.

I have found that the best method of quickly securing a record of the absorption of a solution is to bring a prismatic layer of the liquid, contained in a quartz cell, before the slit of a quartz spectrograph, and photograph the spectrum of the cadmium spark. We secure in this way a record of the absorption of the liquid in various thicknesses, in the form of a curve, quite similar to the curves laboriously constructed from the readings obtained with the spectro-photometer.[1]

The curve obtained with a solution of the nitroso in glycerine is shown in Plate XII, Figs. 4 and 5. It will be noticed that after a certain thickness has been passed we begin to get a noticeable absorption in the ultra-violet, the form of the curve in this region being well shown in Figs. 3 and 4. The band in the blue and violet is, however, so much heavier that, by employing a film of suitable thickness, we can get complete opacity in this region, combined with almost perfect transparency in the ultra-violet.

[1] It is my intention to prepare a monograph on the absorption of a large number of the aniline dies, and other organic compounds such as the nitroso-dimethyl-aniline' which have not been previously investigated. The spectra will be approximately normal, all on the same scale, and will extend from the C line down to the end of the spectrum. They will be photographed in the manner which I have described, and will, I hope, make it possible for the spectroscopist or physicist to pick out at once the combination necessary to produce any desired result. Preliminary experiments are now in progress to determine the best form to give the apparatus, and the most suitable source of light, and I shall be very glad of any suggestions pertaining either to the apparatus or to particular substances worthy of investigation.

The nitroso is soluble in water, glycerine, ether, alcohol, bisulphide of carbon, and many other fluids, and the region of heaviest absorption varies somewhat with the nature of the solvent, the shift of the band not, however, following Kundt's rule in every case. A stained gelatine film on a quartz plate forms a fairly suitable screen, if we do not wish to photograph below the group of cadmium lines at wave-length 2314. It is opaque, however, to waves much shorter than this. The glycerine solution transmits down to the last cadmium line, $\lambda = 2147$, and some other solvents appear to work equally well.

In photographing the spectrum of the cadmium spark in the ultra-violet of the third order, with the fourteen-foot concave grating, I found that the prolonged exposure of the solution in glycerine to the light of the spark resulted in its decomposition. Gas bubbles formed in the thin quartz cell, and, by bridging across the space between the two plates, allowed the passage of blue and violet light. The same thing occurred with pure glycerine under a quartz plate, while glycerine under glass was unaffected, showing that the decomposition was caused by the extreme ultra-violet. In addition to the formation of bubbles a gradual bleaching of the solution occurred. To obviate this difficulty I constructed a small cell of quartz, by cementing two plates together, with a space of about 0.5 mm between them, the cell thus formed being cemented to the bottom of a small thistle tube with a very small bore. By filling the thistle tube with the glycerine solution, a flow took place through the cell at the rate of about a drop every two minutes. This device worked admirably and gave no trouble at all, the cell being placed close to the slit of the grating camera in the path of the convergent beam from the quartz lenz. Another very satisfactory screen can be made by dissolving celluloid (previously boiled for some time in water) in amyl acetate, adding a little nitroso, and flowing the solution on a quartz plate. It is, however, opaque to the last two cadmium lines. The use of the screen necessitates considerable increase in the time of the exposure, the amount varying from two to ten, or even twenty, times, according to the density of the screen. The strength of the glycerine solution must be

PLATE XIII.

A B

EFFECT OF SCREENS TRANSPARENT ONLY TO ULTRA-VIOLET LIGHT.

adjusted according to the work required of it; a strong solution gives a wider band in the blue and violet, but diminishes the intensity of the ultra-violet as well. In general, the best results are obtained when the blue line of wave-length 4799 in the spark spectrum of cadmium can be just barely discerned.

In Plate XII, Fig. 1, the wave-lengths of the principal lines in the spark spectrum of cadmium are given for reference. The action of the nitroso screen is well shown in Fig. 2, the spectra being photographed with a small quartz spectrograph made by Fuess. The first seven spectra were taken through the glycerine nitroso cell which I have just described, with the following times of exposure: 5, 10, 15, 20, 30, 45, and 60 seconds. The cell was then removed and the following six spectra taken with exposures 2.5, 5, 10, 15, 20, and 30 seconds. A study of these spectra enables us to calculate just what can be done with this screen, and the necessary increase in the time of exposure resulting from its use.

In Fig. 6, which is a negative, we have the absorption spectra of the various components of the screen which I mentioned in the first part of this paper, taken with exposures of 20 seconds each. The spectra were taken through screens as follows:

1. Nitroso in gelatine on glass (thin film).
2. Nitroso in gelatine on glass (thick film).
3. Nitroso in gelatine on cobalt glass (thin film).
4. Nitroso in gelatine on cobalt glass (thick film), strong ultra-violet absorption.
5. Dense cobalt glass.
6. Turnbull's blue in gelatine.
7. Chance's "signal green" glass (two thicknesses).
8. No screen, 3 seconds' exposure.
9. Cyanine in Canada balsam.
11. Aurantia in collodion.

These photographs were taken on an orthochromatic plate, the yellow and yellow-green being compressed into the small strip which appears alone in No. 11.

The utility of the nitroso screen in photography with the concave grating is very clearly brought out in the photographs of the iron spectrum shown in Plate XIII. These were made with

a fourteen-foot grating, with a glycerine nitroso cell before the slit during one of the exposures. Figs. 1 and 2 are from the same plate. Strip B in each was made through the screen, and shows the ultra-violet of the third order, uncontaminated by the blue of the second. In strips A, which were made without the screen, the two orders are mixed. Strips C were made through a glass screen, which cut off the third order ultra-violet, leaving the blue of the second. I have marked a few of the wave-lengths to aid in the identification of the lines. The times of exposure were for strips A and C 10 minutes, for B 50 minutes.

The group of cadmium lines in the neighborhood of wave-length 2314 is, in the second-order spectrum, mixed up with a lot of blue air lines of the first-order spectrum. The separation of the two by the nitroso screen is well shown in Fig. 3, in which the two orders are shown superposed in strips A, and the ultra-violet of the second order in strip B. The exposures in this case were 15 minutes and 2 hours respectively.

Another screen which I believe may prove useful in astrophysical work is made by combining nitroso-dimethyl-aniline with a small amount of the dye uranine, the latter removing the bluish-green portion of the spectrum which affects the photographic plate. By a proper adjustment of the two in gelatine on glass, a screen can be formed which, when used with an ordinary (*i. e.*, not orthochromatic) plate, gives us a photograph made exclusively by ultra-violet light, comprised between wave-length 345 and 365—a rather narrow range. I have made a few photographs with a screen of this description which have brought out some interesting points. In a photograph of the full Moon, taken by ultra-violet light, the contrast between the bright and dark areas is very strongly accentuated, while in photographs of landscapes made in the same way there is almost no contrast at all, except between white objects and objects not white. I have also photographed a collection of rocks and minerals with ultra-violet light and with yellow light. In the negative taken by yellow light there is a great deal of contrast and detail, especially in the marbles and conglomerates, while

in the negative taken by ultra-violet light all this is absent, the white specimens coming out very black, with everything else of a thin and almost uniform gray. I hope in the near future to have an opportunity of making some lunar photographs on a large scale, the only instrument at my disposal at the present time being the nine-inch equatorial of the University. Photographing by ultra-violet light appears to diminish the contrast between all objects not white, and to increase the contrast between white objects and those not white. I do not wish to be hasty in drawing conclusions, but it appears to me to be probable that the more luminous portions of the lunar surface, if not as white as plaster of Paris, must at least be much whiter than gray sandstone.

In Plate XIII, Fig. 4, are reproduced two photographs of the same landscape, taken at the same time and under similar conditions of illumination, the one (A) taken on an orthochromatic plate by yellow light through a screen of dense aurantia, the other (B) taken on an ordinary plate by ultra-violet light. The absence of contrast between the chimneys and walls in B is especially noticeable in the right-hand part of the picture. I tried various times of exposure, and the picture reproduced is the best of the lot. Another curious effect is the almost complete absence of shadows in the ultra-violet picture (it was taken in full sunlight like the other), showing that most of the ultra-violet light comes from the sky, which is what we should expect, though we should hardly anticipate that the effect would be so pronounced. This is best seen on the monument and on the snow in the middle distance. The increase of "atmosphere" in the ultra-violet picture is very marked. It is so strong that under-exposed plates fog in the shadows of objects not over one hundred yards from the camera, a circumstance which shows the great scattering power of the air for these short waves. The two pictures are also interesting as showing that our eyes have developed a maximum sensibility for that region of the spectrum which shows terrestrial objects in strongest contrast. Nitroso-dimethyl-aniline is the only substance, other than the ordinary aniline dyes, that I have examined thus far, and I feel

very hopeful of finding among the large number of allied substances, absorbing media even more transparent to the ultra-violet radiations than the one which I have described in this paper.

PHYSICAL LABORATORY,
JOHNS HOPKINS UNIVERSITY,
Baltimore, January 1903.

SYSTEMATIC ERRORS IN THE WAVE-LENGTHS OF THE LINES OF ROWLAND'S SOLAR SPECTRUM.

By G. EBERHARD.

In their investigations with their interference apparatus Messrs. Fabry and Perot[1] have measured the wave-lengths of 33 lines of the solar spectrum, and have compared their values with those given by Rowland in his preliminary table of solar spectrum wave-lengths. In this way they have shown that Rowland's system contains systematic errors of considerable amount in comparison to the accuracy of the relative measurements for small regions of spectrum. They give for a number of lines the ratio $\frac{\lambda\,(\text{Rowland})}{\lambda\,(\text{Fabry and Perot})}$, which in the absence of systematic errors should be constant, and for the sake of the subsequent comparison I have also plotted the numbers of their table on a curve given below.

Although any doubt as to the reality of these systematic errors in Rowland's table is wholly dispelled by the great accuracy attained by Messrs. Fabry and Perot in their investigations, it nevertheless seemed to me to be of interest to examine another independent series of observations—the wave-lengths of 300 solar lines measured by Müller and Kempf—and see whether a comparison of their values with Rowland's would exhibit the same thing. In a similar comparison Müller had already found that "in the central portions of the spectrum between wave-lengths about 550 and 610, negative signs prevail, which leads to the inference of systematic inequalities in one of the two series of measures," etc.[2] I had to take into account the fact that, in consequence of the nature of the grating then employed, which was decidedly inferior in quality to modern gratings, accidental errors of measurement would occur of much larger amount than in the case of Rowland, and that these could only be in

[1] ASTROPHYSICAL JOURNAL, 15, 270–273, 1902.

[2] *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, 8, 51.

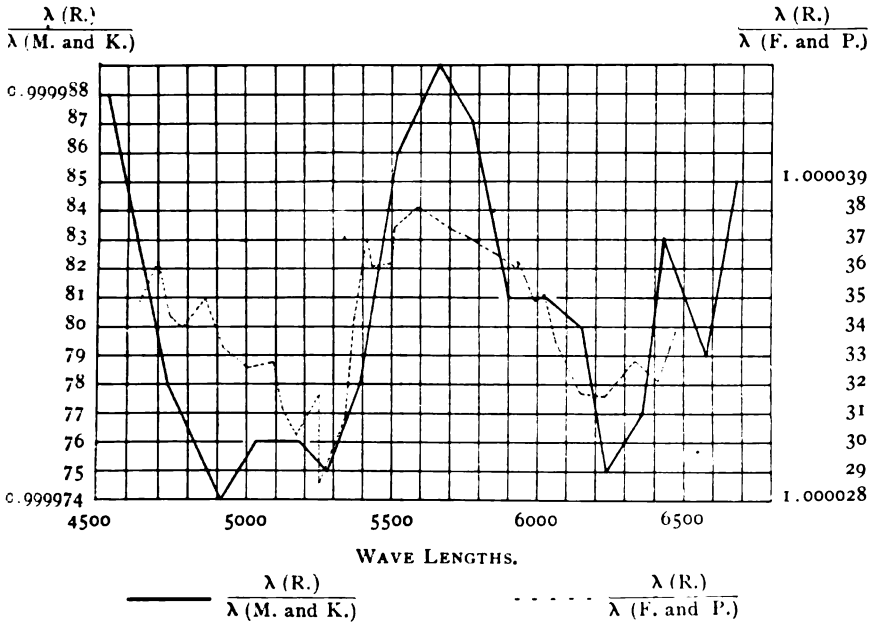
some measure compensated for by taking the means from a large number of lines.[1] But, on the other hand, Müller and Kempf proceeded with the greatest possible care in building up their system, so that the occurrence of systematic errors, which were dependent upon the wave-length, did not seem to me probable, aside from those errors which might arise from the determination of the constant of the grating.

On carrying out the computations, my suspicions were fully confirmed, as I found that the quotient $\frac{\lambda\ (\text{Rowland})}{\lambda\ (\text{Müller and Kempf})}$ exhibited very nearly the same course as the similar quotient between Rowland and Fabry and Perot; but the values of the first quotient disclose very much greater differences for neighboring lines than in the case of Fabry and Perot.

λ (Rowland)	$\frac{\lambda\ (\text{Rowland})}{\lambda\ (\text{M. and K.})}$	λ (Rowland)	$\frac{\lambda\ (\text{Rowland})}{\lambda\ (\text{M. and K.})}$	λ (Rowland)	$\frac{\lambda\ (\text{Rowland})}{\lambda\ (\text{M. and K.})}$	λ (Rowland)	$\frac{\lambda\ (\text{Rowland})}{\lambda\ (\text{M. and K.})}$
4494.738	1.000001	5281.971	0.999967	5775.304	0.999990	6246.535	0.999970
4501.448	0.999986	5283.802	0.999976	5806.950	0.999983	6252.773	0.999964
4508.455	1.000000	5288.705	0.999973	5816.601	0.999988	6256.572	0.999986
4563.939	0.999971	5307.541	0.999978	5831.821	0.999976	6265.348	0.999979
4572.156	0.999984	5324.373	0.999980	5862.582	0.999987	6318.239	0.999971
4703.177	0.999993	5353.571	0.999974	5896.155	0.999984	6322.907	0.999976
4754.225	0.999963	5367.669	0.999977	5934.881	0.999982	6335.554	0.999974
4861.527	0.999977	5383.578	0.999981	5948.765	0.999972	6344.371	0.999980
4890.948	0.999969	5389.683	0.999973	5977.007	0.999983	6355.246	0.999966
4903.502	0.999974	5393.375	0.999964	5987.290	0.999982	6358.898	0.999986
4920.685	0.999979	5405.989	0.999987	6003.239	0.999985	6380.958	0.999973
4924.107	0.999971	5415.416	0.999981	6013.715	0.999981	6393.820	0.999984
4973.281	0.999976	5434.740	0.999987	6024.281	0.999983	6408.233	0.999982
4980.352	0.999966	5487.959	0.999980	6042.315	0.999976	6411.865	0.999982
4999.689	0.999978	5497.735	0.999983	6056.227	0.999977	6421.570	0.999977
5050.008	0.999992	5501.683	0.999975	6065.709	0.999983	6431.066	0.999992
5090.954	0.999968	5528.041	0.999993	6078.710	0.999980	6430.293	0.999987
5133.870	0.999975	5543.414	0.999995	6102.937	0.999980	6450.033	0.999977
5159.231	0.999967	5555.122	0.999991	6122.434	0.999994	6534.172	0.999980
5162.440	0.999971	5624.760	1.000001	6141.938	0.999983	6546.479	0.999972
5172.856	1.000001	5634.171	0.999995	6162.390	0.999977	6563.045	0.999986
5183.791	0.999973	5675.647	0.999980	6180.420	0.999977	6609.360	0.999979
5215.353	0.999960	5688.436	0.999987	6200.527	0.999970	6633.995	0.999979
5233.122	0.999983	5731.084	0.999985	6213.644	0.999978	6643.876	0.999988
5242.658	0.999983	5754.881	0.999996	6219.494	0.999981	6678.235	0.999982
5269.723	0.999967	5763.218	0.999997	6230.943	0.999968	6717.940	0.999984

[1] Müller and Kempf give as the probable error of a line on the average ±0.03 tenth-meters (*Publicationen des Astrophys. Obs.*, 8, 145), but it may be seen from the table, pp. 147 ff., that the probable error of a line depends very much upon its character.

I employed as the basis of my computation the list of 127 lines of the Potsdam list of wave-lengths which Müller[1] had already selected in making his comparison of the Potsdam values with Rowland's standards of 1889. Of these 127 lines I omitted, however, 23 more, as the Potsdam values of the line might have been affected by close and often fainter lines, in conse-



quence of the small dispersion and resolution employed in the measurements. Although it would have been desirable to have as great a number of common lines as possible for my comparison of the two systems, I nevertheless refrained from identifying a larger number of the 300 lines of the Potsdam list[2] with those of Rowland, as I have never made extensive observations with the old Potsdam glass grating, and therefore could not have made an advantageous selection of the lines. Rowland's values are of course taken from his "Preliminary Table."

The individual values of the above table exhibit a somewhat irregular behavior, but on plotting on section paper a progres-

[1] *Ibid.*, 49, 50.

[2] *Ibid.*, 5, 147 ff.

sion is also clearly visible. This becomes more evident on combining into a mean the values of the ratio $\frac{\lambda\,(\text{Rowland})}{\lambda\,(\text{Müller and Kempf})}$ for lines 20 tenth-meters apart. But also in taking this mean still far too few single values are combined in each mean, and I therefore also took means for every 100 tenth-meters, which are represented in the accompanying curve. The close agreement with the curve of Fabry and Perot cannot fail to be recognized, and hence the occurrence of systematic errors in Rowland's table is also evident from the Potsdam determinations of wavelengths by Müller and Kempf, in the manner first established by Fabry and Perot.

ASTROPHYSIKALISCHES OBSERVATORIUM,
Potsdam, January 3, 1903.

PRELIMINARY NOTE ON SOME MODIFICATIONS OF THE MAGNESIUM LINE AT λ4481 UNDER DIFFERENT LABORATORY CONDITIONS OF THE SPARK DISCHARGE.

By SIR WILLIAM HUGGINS and LADY HUGGINS.

IN his "Note on the Wave-Length of the Magnesium Line at λ4481"[1] Professor Crew points out that an interesting problem still remains, namely to discover the laboratory conditions under which the line becomes sharp, as in some stellar spectra.

For some years, at intervals, experiments have been made here in the laboratory on the spectrum of magnesium, with the hope of throwing light on the physical conditions of the stellar atmospheres which we may assume to be indicated by the character of this line when present; a line which in the laboratory is subject to a very wide range of modifications, both of character and of intensity.

As it may be some time before these experiments are sufficiently complete for publication, it seems desirable to reproduce at once with this preliminary note, out of the very large number of spectra which have been taken, a few representing the most typical forms of the modifications of this line.

The teaching of these experiments suggests that the condition of the spark-discharge which is most potent in bringing about modifications of this line both in intensity and in character is the greater or less suddenness of the blow of the discharge. To a small extent only does the character of the line appear to be affected by the quantity and the electro-motive force of the electricity which is in action; indeed, such changes as may appear are probably brought about indirectly by the larger mass of material acted upon as the discharge is made more powerful.

The appearance of the line at λ4481 in the spectrum at the top of the plate may be taken as representing its normal condi-

[1] ASTROPHYSICAL JOURNAL, 16, 246, 1902.

tion with capacity in the secondary of the coil. When the jar is taken out of circuit, and the discharge of the secondary takes place directly between the magnesium electrodes, the line becomes thin, defined, and of small intensity, as in spectrum No. 2. In this case the electric blows are less sudden through the incoming of the full self-induction of the coil itself.

The researches of Schuster, Hemsalech, Schenck, Huff, and others have shown that a similar effect follows when the jar-discharge is slowed down by the introduction into the circuit of an independent self-induction. The condition of the line in spectrum No. 3 shows the effect of the introduction of a self-induction, the conditions of the discharge remaining otherwise the same as in photograph No. 1.

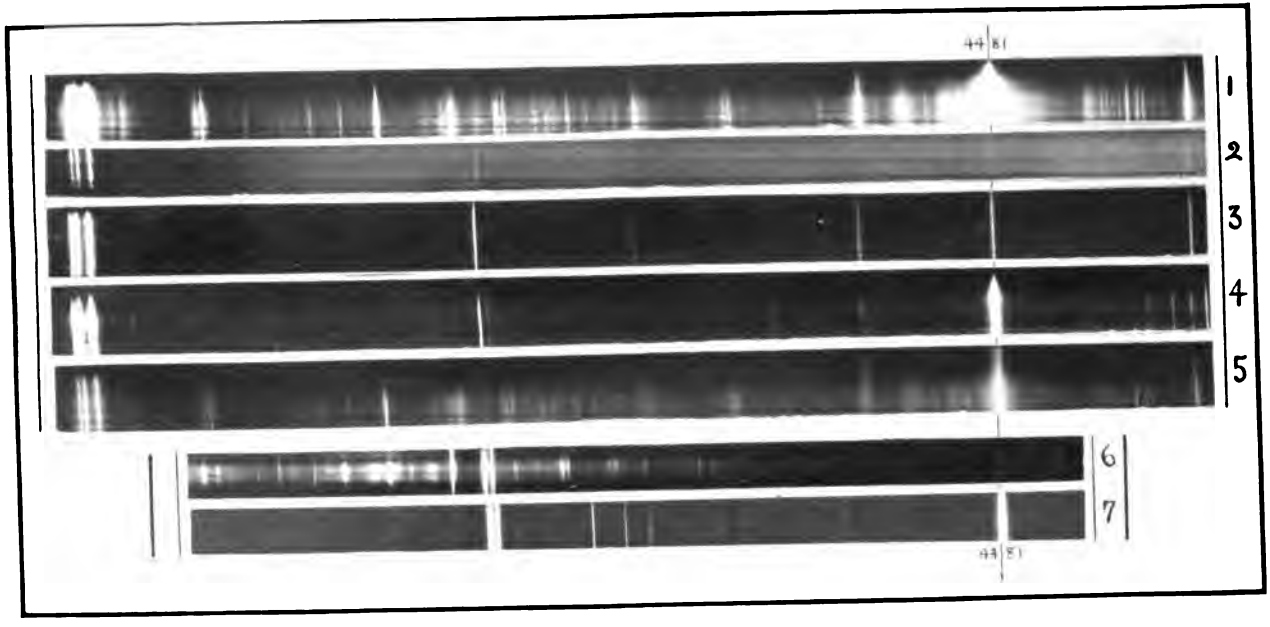
In spectrum No. 5 a stronger alternating current and a capacity four times as great as in No. 1 were employed; but the photograph is feeble from over-exposure. On the contrary, in No. 4 the coil was excited by a feeble continuous current and the capacity in the circuit was reduced to a small jar.

The two spectra placed below were taken some years ago with another spectroscope. They are of interest in showing the great variation of intensity which the line at λ 4481 may undergo without assuming its normal diffused character, as in No. 1. The line in spectrum No. 4 appears to be intermediate in character between that of the line in No. 7 and that in No. 1.

We have still before us the task of the interpretation of the differences in the mode of production of the electric spark, especially as to greater or less suddenness, so as to enable us to connect them with definite conditions of temperature and density of those stellar atmospheres in which the magnesium present absorbs radiations similar in character to those photographed in the laboratory.

LONDON,
January 1, 1903.

PLATE XIV.



SPARK SPECTRA OF MAGNESIUM.

ON THE FLAME SPECTRUM OF RADIUM.

By C. RUNGE and J. PRECHT.

DR. GIESEL has recently found that very small quantities of sufficiently pure radium bromide give a characteristic color to the flame of a Bunsen burner.[1] He describes the spectrum on the whole as consisting of two red bands and a bright blue line. In the violet he saw some other less prominent lines.

At the request of Dr. Giesel we have investigated the spectrum more closely, for which purpose he was kind enough to supply us with 13 milligrams of radium bromide of his own making.

To fix the substance on the platinum wire we proceeded in the same way as described by one of us in his paper on the spark spectrum of radium.[2] The platinum wire was doubled up, twisted into a loop, and heated by an electric current. If a fragment of the salt is touched with the glowing wire, it sticks to it, melts, and forms a small drop. When the current is cut off, the drop hardens and begins to give out a bright phosphorescent light. If the wire is again heated, the light ceases at once, but is renewed as soon as the heating is stopped.[3]

The wire carrying a small fragment of the substance was brought into the hottest part of the Bunsen flame, which was placed before the slit of a spectroscope. The spectroscope consisted of a Rowland concave grating of one meter radius. It was mounted in the way described by one of us.[4] If the wire is electrically heated while the substance is in the flame, the intensity of the spectrum is increased, but the substance is consumed more rapidly.

The wave-lengths were read off on a paper scale. The light

[1] F. GIESEL, *Physikal. Zeitschrift*, 3, 578, 1901–2 and *Ber. d. deutsch. chem. Gesellschaft*, 35, 3608, 1902.

[2] C. RUNGE, "On the Spectrum of Radium," ASTROPHYSICAL JOURNAL, 12, 1.

[3] This phenomenon has already been described by Giesel.

[4] *Wied. Ann.*, 61, 644, 1897.

is not sufficient for a photograph if only small quantities of the salt are at the disposal of the observer.

When the substance is first brought into the flame, there are some lines to be seen, which rapidly lose their intensity, so much so that one must be quick to determine their wave-lengths before they disappear. The blue line at λ4826 is the last to lose its intensity. It seems to possess the same character as the barium line λ5536, the strontium line λ4607, and the calcium line λ4226, which are also most prominent when these substances are brought into the Bunsen flame.

In the following table we have noted all we have seen. We believe all the observed lines to be due to radium except the barium line λ5536 and the two sodium lines. The wave-lengths

Wave-Lengths	Remarks
4405	Weak, diffuse
4500	Weak, diffuse, observed only once
4592	Diffuse
4680	Weak, observed only once, perhaps identical with the strong line of the spark spectrum at λ4682.3
4718	
4750	
4826	Strong, sharp line, very likely identical with the line at λ4826.1 in the spark spectrum, as Giesel presumes
509-513	Weak band, observed only once
5210	Observed only once
5360	Observed only once
5535	Impurity, barium λ 5535.7
5655	Observed only once
5685	Observed only once
5890	Impurity, sodium
5896	Impurity, sodium
590-605	Weak band
613-633	Strong band
6210	These lines are seen in the band, but apparently do not form the band
6216	Observed only once
6228	Observed only once
6247	
6250	Observed only once
6260	Observed only once
6269	
6285	
6329	Strong
6349	Strong
653-670	Strong band
6653	Strong line; there is a minimum of intensity of the band, in the neighborhood of the line
6861	Uncertain, observed only once

of the lines that have been observed clearly and repeatedly we believe to be correct to one or two Ångström units. With those lines, however, which have been observed only once, the error may amount to several Ångström units. The bands have no definite limits. When an ample amount of the salt is brought into the flame, they may appear much broader than we have seen them. It may be that the bands are due to the spectrum of the compound. This might be tested by observing the flame spectrum of the chloride.

HANNOVER,
January 1903.

FIVE STARS WHOSE RADIAL VELOCITIES VARY.

By EDWIN B. FROST and WALTER S. ADAMS.

IN the course of the observations we have been making with the Bruce spectrograph during the past year on stars having spectra of the *Orion* type, the following four stars of this class have been found to vary in respect to their motions in the line of sight, in addition to the three previously announced in this journal (*η Orionis*, *o Persei*, *β Cephei*).[1]

The determinations of velocity so far made are given below. The G. M. T. of the observations is not now communicated, as we do not deem the data at present available to be sufficient for the accurate determination of the orbits of these stars. The number of star lines measured is given for each plate in the fifth column.

δ CETI ($\alpha = 4^h\ 31^m$; $\delta = -0°\ 6'$; Mag. = 4.1)

Plate	Date	Taken by	Velocity	No. of Lines	Measured by
A 291	1901, Nov. 1	F.	+ 8 km	15	A.
B 227	Nov. 13	F.	+12	10	F.
A 295	Dec. 19	A.	+16	7	A.
A 298	1902, Jan. 4	A.	+ 6	7	A.
B 383	Aug. 7	F.	+13	8	A.
B 388	Aug. 11	A.	+ 6	9	A.
B 400	Aug. 27	A.	+ 9	9	A.
B 404	Sept. 3	A.	+12	8	A.
A 376	Sept. 6	A.	+12	10	A.
A 381	Sept. 7	F.	+ 6	10	A.
B 432	Oct. 29	A.	+ 9	8	A.

The spectrum is of Miss Maury's Class IV*a* and has numerous oxygen lines and the usual ones of helium, silicon, hydrogen, and magnesium. The lines are fairly sharp, so that the spectrum is relatively well measurable. The above range of variation in velocity is not large, from +6 to +16 km per sec., but we cannot question its reality.

[1] ASTROPHYSICAL JOURNAL, **15**, 214, 340, 1902; **17**, 68, 1903.

ν ERIDANI ($\alpha = 4^h\ 31^m$; $\delta = -3°\ 33'$; Mag.=4.1)

Plate	Date	Taken by	Velocity	No. of Lines	Measured by
B 242	1901, Nov. 20	A.	+12 km	5	A.
B 438	1902, Oct. 30	F.	+28	10	F.
			+26	12	A.
B 443	Oct. 31	A.	+18	7	A.
B 447	Nov. 6	A.	+ 3	8	A.
B 472	Dec. 18	F.	+12	8	F.
B 494	1903, Feb. 5	A.	+25	7	F.

The spectrum resembles that of δ *Ceti*, but the lines are more difficult to measure. The range so far observed is 24 km.

π^5 *ORIONIS* ($\alpha = 4^h\ 49^m$; $\delta = +2°\ 17'$; Mag. = 3.9)

Plate	Date	Taken by	Velocity	Number of Lines	Measured by
A 332	1902, March 4	A.	+ 1	3	A.
B 469	Dec. 17	A.	+ 58	5	A.
B 475	Dec. 31	A.	+70	5	A.
			+72	6	F.
B 480	1903, Jan. 1	Ellerman	+ 32	4	A.
A 384	Jan. 16	A.	+ 7	6	A.
B 488	Jan. 21	F.	− 35	4	A.
			− 32	4	F.
A 390	Jan. 22	A.	+ 73	4	A.

The spectrum of this star is not well adapted for accurate measurement, the lines being broad and very diffuse. A few oxygen lines are present, but they are faint and difficult to set upon. The period is evidently short.

ζ *TAURI.*

The spectrum of the star is peculiar, as has been noted by Lockyer and probably by others. The conspicuous feature in the region of spectrum covered by our plates is the remarkable sharpness and intensity of $H\gamma$. Settings can be made upon this line with such accordance that the radial velocity determined from this line alone is probably not much less reliable than that based on several lines in the case of most other stars of the *Orion* type. Other lines are faintly discernible in the spectrum, chiefly the enhanced lines of titanium and iron. These lines are in general so faint and broad, however, that they have commonly not been employed in the determination of the star's motion.

ζ *TAURI* ($\alpha = 5^h\ 32^m$; $\delta = +\ 21°\ 5'$; Mag. = 3.0).

Plate	Date	Taken by	Velocity	Number of Lines	Measured by
B 219	1901, Nov. 8	A.	+ 17	1	A.
			+ 16	1	F.
A 317	1902, Feb. 12	A.	+ 23	1	A.
			+ 24	1	F.
B 332	April 23	F.	+ 15	1	A.
			+ 11	1	F.
B 410	Sept. 13	A.	+ 18	6	A.
B 425	Oct. 15	A.	+ 34	1	A.
B 440	Oct. 30	F.	+ 31	1	A.
			+ 28	2	F.
B 452	Nov. 6	A.	+ 19	1	A.
			+ 21	1	F.
B 462	Nov. 19	A.	+ 14	1	A.
B 470	Dec. 17	A.	+ 7	1	A.
B 473	Dec 18	F.	+ 9	1	F.
B 476	Dec. 31	A.	+ 4	1	A.
B 482	1903, Jan. 8	A.	+ 2	1	A.
B 485	Jan. 9	A.	+ 5	1	A.
A 386	Jan. 16	A.	+ 4	1	A.

The period is probably rather long. Plate B 452 of the above series is underexposed, and the value derived from it is somewhat uncertain.

η *VIRGINIS* ($\alpha = 12^h\ 15^m$; $\delta = -\ 0°\ 7'$; Mag. = 4.1).

This star has a composite spectrum, both components belonging to Vogel's type I*a* 2, or Miss Maury's VIII*a*. On account of the weakness of the lines of the fainter component the discussion of the star's motion will be based mainly upon the absolute velocity of the brighter component. The binary character of the star was established by the first plate, taken and measured by A.

Plate	Date	Taken by	Brighter Component		Fainter Component		Measured by
			Velocity	No. of Lines	Velocity	No. of Lines	
A 388	1903, Jan. 16	A.	− 31.5	14	+ 42	4	A.
B 493	Feb. 4	F.	+ 0.7	16	+ 60	4	F.
			+ 0.2	15	+ 63	6	A.
A 399	Feb. 5	A.	+ 3.4	16	..	..	A.

NOTE ADDED FEBRUARY 18, 1903.

When it was decided to publish at this time the observations of the above stars, we expected also to include the *Orion* type star π^4 *Orionis*, the first two plates of which indicated a range of variation of about 15 km. It was, however, withheld until we should obtain additional plates, as the third and fourth plates gave results nearly like that of the second. We see today in the *Publications of the Astronomical Society of the Pacific* (**15**, 20, 1903) a notice by Dr. H. M. Reese, stating that spectrograms of this star, taken with the Mills spectrograph of the Lick Observatory, on October 6, 1902, and January 4 and 12, 1903, yielded values respectively of +43, ±0, and +6 km. per second.

There is accordingly no reason for longer withholding our observations, which are as follows:

π^4 *ORIONIS* ($\alpha = 4^h\ 46^m$; $\delta = +5°\ 26'$; Mag. 4.0)

Plate	Date	Taken by	Velocity	Number of Lines	Measured by
A 332	1902, March 4	A.	+13 km	8	A.
B 466	Nov. 27	F.	−2	5	F.
			−1	7	A.
B 468	Dec. 17	A.	+1	6	F.
A 389	1903, Jan. 22	F.	+3	7	F.
			+0	6	A.

YERKES OBSERVATORY,
February 8, 1903.

SECOND NOTE ON THE SPARK SPECTRUM OF IRON IN LIQUIDS AND COMPRESSED GASES.

By GEORGE E. HALE and NORTON A. KENT.

IN a note published a year ago[1] it was shown that the position and the reversal phenomena of the lines in the spectrum of a spark between iron poles in liquids depend upon a variety of circumstances, chief among which are the electrical constants of the circuit and the nature of the liquid. It was also stated that certain lines in the blue part of the spectrum of iron had been photographed in air at pressures ranging from 1 to 20 atmospheres, and that the shifts of the lines were (approximately) directly proportional to the pressure, thus confirming and extending the results previously found by Humphreys and Mohler for the low potential discharge in air at pressures up to 14½ atmospheres. It was at the same time remarked that the lines thus investigated in air showed none of the peculiar phenomena of reversal which had been observed in the case of the high potential discharge in liquids.

Since this first note was written, the investigation of the spectrum of iron in liquids has been completed. The reversal phenomena previously studied with low dispersion have been photographed with a concave grating of ten feet (3.05 m) radius, on a scale sufficiently great to permit the displacements of the best lines to be measured with errors not often exceeding 0.01 tenth-meter. In the note referred to above it was stated that the preliminary experiments on the effect of self-induction in the discharge circuit were inconclusive. With a coil afterwards constructed, in which the self-induction can be varied from 0.000042 to 0.000426 henry in five steps, change of self-induction has been found to produce more marked effects than change of capacity, length of spark, diameter of terminals, physical or chemical properties of the liquid, or any other variable previously shown to influence the reversal phenomena. Accordingly,

[1] GEORGE E. HALE, "Note on the Spark Spectrum of Iron in Liquids and in Air at High Pressures," ASTROPHYSICAL JOURNAL, 15, 132, 1902.

changes of self-induction in the discharge circuit have been employed to produce a series of spectra, which pass by gradual degrees from a spectrum consisting almost wholly of bright lines (highest self-induction), resembling that of the iron spark in air at atmospheric pressure, to a spectrum consisting for the most part of dark lines (no self-induction) in the region $\lambda 3550$–$\lambda 4500$. Such a series of spectra, if photographed with low dispersion, would closely resemble the spectra reproduced in Plate XI of the note referred to above. A careful series of measures of the lines has shown that as the self-induction is decreased the bright lines move gradually toward the red, while the absorption lines, which begin to appear in the earliest stages, may at first have some apparent shift[1] toward the violet, though later they may be displaced one or two hundredths of a tenth-meter toward the red, rarely more. Full details of the measurements will be given in a paper which will appear soon in the *Publications of the Yerkes Observatory*.

In his paper "On the Interpretation of the Typical Spectrum of the New Stars,"[2] Professor Wilsing describes his experiments on the spectrum of high potential discharges between metallic poles in water, and bases an explanation of the characteristic pairs of bright and dark lines in the spectra of temporary stars upon the phenomena he observed. Wilsing concludes that the pressure resulting from the spark discharge in liquids amounts to several hundred atmospheres. As our own investigations led us to the belief that the pressure, at least in the case of our iron spark, was in reality much lower, we thought it desirable to extend our earlier work on spark spectra in air at high pressures to the more refrangible region studied in the case of liquids. As the apparatus used for such work a year ago proved inadequate, new apparatus has been constructed in the Observatory instrument shop, and with this we have had no difficulty in photographing the spectrum of the iron spark in gases at pressures ranging from 1 to 53 atmospheres. The present note contains a preliminary account of the results hitherto obtained, but many

[1] Doubtless due in part to the displacement of the lines of the comparison spectrum, caused by pressure in the condensed spark.

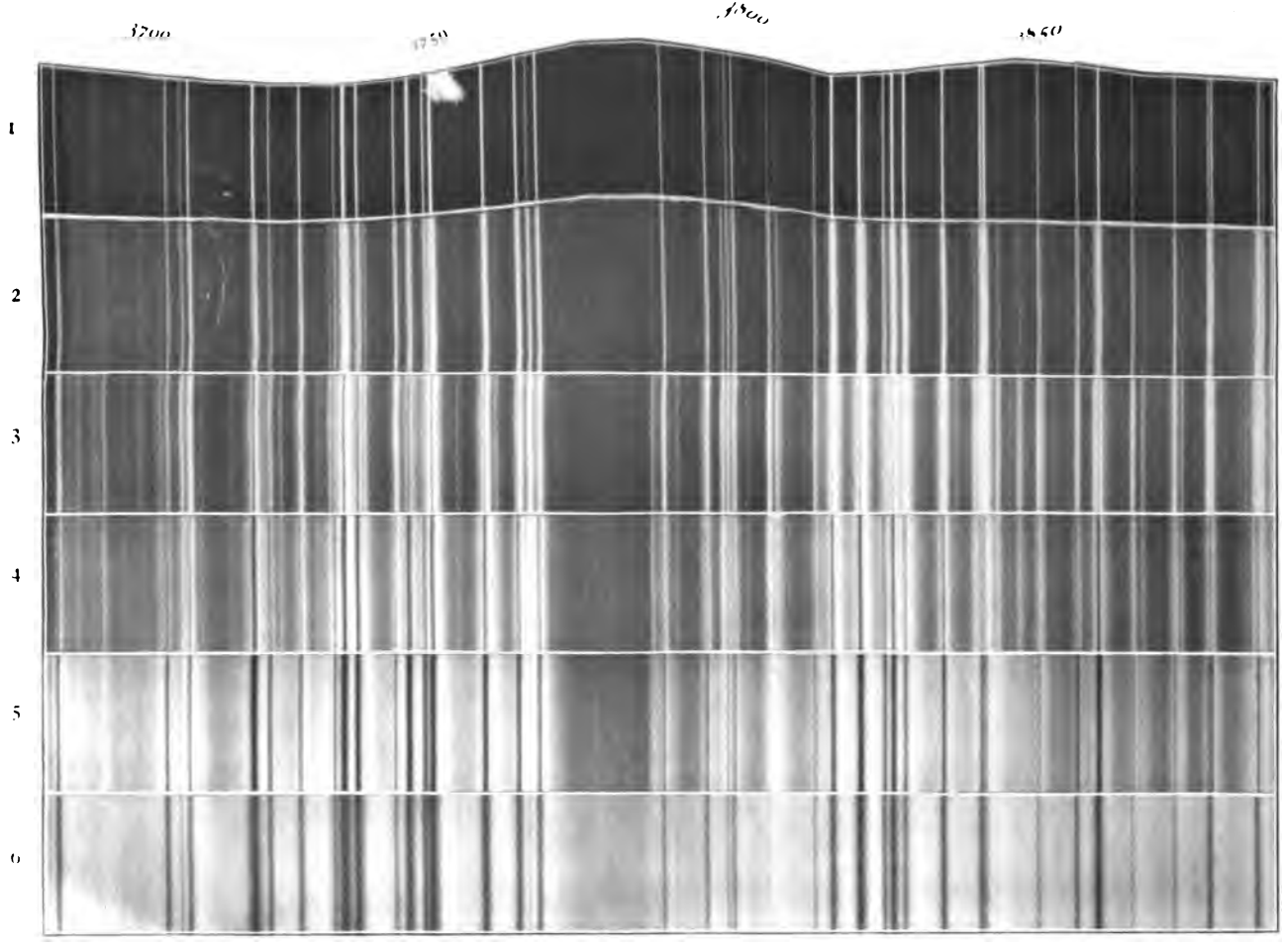
[2] ASTROPHYSICAL JOURNAL, 10, 113, 1899.

details must be reserved for the more complete paper which is to appear in the Observatory *Publications*.

In the recent experiments the spark between iron terminals has been observed through a glass window, in the side of a steel chamber provided with a pressure gauge, and connected by means of a heavy copper tube with a steel reservoir containing compressed air or liquid carbon dioxide. The terminals enter the steel pressure chamber through long cylinders of ebonite, and special precautions have been taken to insure the most perfect insulation. The spark is produced by means of a transformer giving about 15,000 volts on open circuit, ordinarily used with a condenser of 0.0066 microfarad capacity, connected with the secondary terminals. In order to insure successful operation of the spark at high pressures, an air break is essential in the discharge circuit. The photographs of spectra reproduced in Plate XV were made in the first order of a concave grating of ten feet radius, ruled with 14,438 lines to the inch (5684 to the cm).

As stated above, the spectra photographed a year ago at high pressures did not extend into the ultra-violet. In a previous note[4] it has been shown that the reversals of the lines in the case of spark spectra in liquids first appear in the ultra-violet, and advance gradually toward the less refrangible region as the conditions for reversal become more favorable. A similar result was shown many years ago by Liveing and Dewar to obtain in the case of arc reversals. It is for this reason that the phenomena described in the present note were not encountered by us in our earlier work at lower pressures on the less refrangible region of the spectrum of the iron spark in air. Nevertheless, we were somewhat surprised to find, in our first test of the new apparatus, that at an air pressure of 14 atmospheres the reversal phenomena in the region $\lambda 3550$–$\lambda 4500$ closely resembled those we were accustomed to photograph in the case of the iron spark in water with but little self-induction in the discharge circuit. It was at once evident that a series of photographs made at different air

[4] GEORGE E. HALE, "Selective Absorption as a Function of Wave-Length," ASTROPHYSICAL JOURNAL, 15, 227, 1902.



PHOTOGRAPHS OF THE SPARK SPECTRUM OF IRON IN COMPRESSED GASES.

1. In air at atmospheric pressure.
2. In air at 3 atmospheres.
3. In air at 7 atmospheres.
4. In air at 14 atmospheres.
5. In CO_2 at 27 atmospheres.
6. In CO_2 at 53 atmospheres.

pressures would resemble the series previously obtained with the spark in water by varying the self-induction. Photographs have now been made at pressures ranging from 1 to 53 atmospheres, portions of which are reproduced in Plate XV. In the temporary absence of compressed air at pressures greater than 14 atmospheres, liquid carbon dioxide was used to give higher pressures. It has since been found, however, that at a given pressure the reversal phenomena are more marked in an atmosphere of air than in one of carbon dioxide. This result will render necessary an investigation of the effect of other gases. It has also been found that changes of self-induction in the condenser circuit affect the reversal phenomena in the same sense as in the case of the spark in water, but over a much smaller range.

An examination of Plate XV will show that the phenomena closely resemble those previously described in the case of liquids. At an air pressure of three atmospheres the lines are for the most part bright, though a few cases of reversal appear. It will be noticed that a few of the lines have at this stage increased very markedly in relative intensity. Such lines, as the subsequent photographs show, pass through a maximum of intensity and afterwards reverse. At a pressure of 7 atmospheres the bright lines are broader and more diffuse, and the absorption lines are becoming prominent. It is evident from inspection that many of the reversals are not symmetrical, the bright line being relatively displaced toward the red. At a pressure of 14 atmospheres the dark lines have become very conspicuous, and many of the bright lines have almost disappeared. Some evidences of a continuous spectrum begin to appear at this pressure. At 27 atmospheres (in carbon dioxide) the continuous spectrum is quite conspicuous. The absorption lines are strong and the bright lines are much fainter than before. At 53 atmospheres (also in carbon dioxide) the bright lines have practically disappeared and the dark lines remain, broad and diffuse, on a background of bright continuous spectrum. At this pressure most of those lines whose intensity is increased at moderate pressures are reversed, and the relative intensities of all the dark lines closely resemble those of the bright lines in the spectrum

of the iron spark at atmospheric pressure. Even at the highest pressures very few lines less refrangible than λ4415 are reversed. In the region λ4800–λ4900 there are several strong lines unsymmetrically broadened in carbon dioxide at 53 atmospheres, and similarly affected in water. It seems to be true that the order in which various lines reverse is not precisely the same in gases as in liquids.

A careful study of the shifts of the bright and dark lines at these various pressures has brought out certain facts, which are of interest in connection with the results obtained by Humphreys and Mohler on the low potential discharge in air at high pressures, and our own results on the spectra of high potential discharges in liquids. The following tables contain measures of a few lines which may be regarded as fairly typical. Table I gives the shifts of certain lines in the spectrum of a spark between iron poles in water, resulting from changes of self-induction in the discharge circuit. In this series the length of the water spark was 0.3mm; air spark 10mm; diameter of terminals (flat ends) in water 2.3mm; diameter of terminals (rounded ends) in air 4mm; metal in both cases Bessemer steel; electric fan used to prevent arcing at air gap; distance of terminals below surface of water 56mm; 15,000 volt transformer; capacity in discharge circuit 0.0066 microfarad. The shifts of the lines toward the red are expressed in tenth-meters, and are positive unless otherwise indicated. The types of the lines, as indicated below, are given in parentheses.

(1) = narrow emission line.
(2) = broad symmetrical emission line.
(3) = broad emission line, diffuse toward red.
(4) = symmetrical absorption line.
(5) = absorption line superposed symmetrically on broad emission line.
(6) = absorption line superposed on broad emission line, which is strongest on red side.
(7) = similar to (6), but with violet component of bright line lacking.

E signifies that the measured shift refers to an emission line, *A* that it refers to an absorption line. In lines of types (6) and (7) both the emission and absorption components may be measured. In such a case the measurement of the red component (the violet component is not measured) of the emission line does

not give the true shift of the line, since the setting is made at the center of that part of the line which lies on the red side of the absorption line. Such measures are given here incidentally only because of their previous use by Wilsing.

TABLE I.

IRON SPARK IN WATER.

Shifts corresponding to changes of self-induction in discharge circuit.

Plate Number		659	651	650	658	666	665
Self-Induction Henries		0.000426	0.000310	0.000161	0.000080	0.000042	0.0
Wave-Length	Intensity						
		t. m.	*t. m.*	*t. m.*	*t. m.*	*t. m.*	*t. m.*
3606.85	6	(2) 0.01 *E*	(2) 0.02 *E*	(2) 0.04 *E*	(2) 0.08 *E*	(2) 0.14 *E*	(7)–0.01 *A* .28 *E*
3765.70	5	(2) .02 *E*	(2) .03 *E*	(2) .01 *E*	(2) .09 *E*	(2) .10 *E*	(3) .21 *E*
3815.99	9	(3) .05 *E*	(3) .07 *E*	(3) .12 *E*	(6) .00 *A*	(6) .01 *A*	(6) .03 *A*
3827.98	9	(3) .02 *E*	(3) .05 *E*	(2) .09 *E*	(7)– .01 *A* .35 *E*	(6) .00 *A*	(4) .02 *A*
4063.75	10	(2) .03 *E*	(2) .02 *E*	(2) .04 *E*	(6) .06 *A*	(6) .02 *A*	(4) .02 *A*
4294.32	6	(2) .02 *E*	(2) .02 *E*	.....	(2) .02 *E*	(2) .06 *E*	.....
4308.06	10	(2) .02 *E*	(3) .05 *E*	(2) .06 *E*	(6)– .05 *A*	(6)– .01 *A*	(6) .02 *A*
4325.94	10	(3) .02 *E*	(3) .04 *E*	(2) .03 *E*	(6)– .01 *A*	(6)– .03 *A*	(6) .01 *A*

TABLE II.

IRON SPARK IN AIR AND CO_2.

Shifts corresponding to pressures ranging from 3 to 53 atmospheres.

Plate Number		842	839	833	853	852
Pressure in Atmospheres		Air 3	Air 7	Air 14	CO_2 27	CO_2 53
Wave-Length	Intensity					
		t. m.	*t. m.*	*t. m.*	*t. m.*	*t. m.*
3606.85	6	(2) 0.06 *E*	(2) 0.14 *E*	(7) 0.05 *A* .39 *E*	(4) 0.05 *A*	(4) 0.12 *A*
3765.70	5	(2) .09 *E*	(2) .15 *E*	(7) .05 *A* .33 *E*	(4) .07 *A*	(4) .12 *A*
3815.99	9	(6) .04 *A*	(6) .05 *A*	(6) .08 *A*	(4) .08 *A*	(4) .14 *A*
3827.98	9	(2) .11 *E*	(6) .06 *A*	(6) .10 *A*	(6) .07 *A*	(4) .20 *A*
4063.75	10	(2) .07 *E*	(5) .06 *A*	(5) .07 *A*	(5) .10 *A*	(5) .18 *A*
4294.32	6	(2) .04 *E*	(2) .09 *E*	(2) .16 *E*	(2) .21 *E*	.....
4308.06	10	(2) .09 *E*	(5) .08 *A*	(5) .09 *A*	(5) .12 *A*	(5) .18 *A*
4325.94	10	(2) .07 *E*	(5) .04 *A*	(5) .10 *A*	(5) .09 *A*	(5) .22 *A*

Table II contains the shifts of the same lines as observed at pressures of 3, 7, and 14 atmospheres in air, and at 27 and 53

atmospheres in carbon dioxide. The constants of the circuit were the same as in the case of water, except that the external air gap was 5 mm long.

As Humphreys and Mohler have shown, the reversals in the case of the arc in air at high pressures are generally symmetrical, the absorption lines being displaced toward the red equally with the emission lines. In the case of the spark in water we have found, as the table shows, that the absorption lines are but little displaced toward the red, though they sometimes have a small apparent displacement toward the violet. The emission lines, however, are considerably displaced toward the red, and this displacement increases as the self-induction in the discharge circuit decreases. A knowledge of the true displacement of the bright lines is obtained from a study of those lines which do not reverse before the last stages of the process are reached. Thus the errors resulting from the presence of the absorption lines are avoided. At moderate pressures the absorption lines are displaced less than the bright lines, though their displacements are considerably greater than in the spectrum of the spark in liquids.

These are only a few of the lines which we have already measured, but much additional material will be required before the results can be discussed satisfactorily. It may be noted, however, that the relative shifts of several lines in air and water indicate that the pressure produced by our iron spark in water (no self-induction in the discharge circuit) is much less than Wilsing's value of several hundred atmospheres. An extension to high pressures of the law connecting pressure and displacement will be possible as soon as a complete series of spectra has been obtained in a single gas. But on account of the lack of agreement among the shifts obtained for different lines, and the possibility that in the high potential discharge the pressure of the gas is not the sole controlling factor, it is hoped that the present work may be supplemented by an investigation of the discharge at potentials ranging from 110 volts to 15,000 volts.

YERKES OBSERVATORY,
February 11, 1903.

Minor Contributions and Notes.

REPLY TO E. VON OPPOLZER'S REMARKS ON BIGELOW'S "ECLIPSE METEOROLOGY."[1]

On p. 738 of *Astronomy and Astro-Physics*, **12**, 1893, E. von Oppolzer writes: "Assuming that the solar atmosphere is made up of hydrogen, the diminution of temperature per second [$1'' = 720860$ m] is $\theta = 13740°$. At a distance of one second above the photosphere, we have, therefore, a prevailing temperature 14000° lower than on the surface of the photosphere." Inasmuch as the prevailing temperature of the photosphere is by general consent in the neighborhood of 7000°C., this statement is so far in error as to discredit the theory upon which the result is based. His description in the note mentioned of the work summarized in my *Eclipse Meteorology*, pp. 77–83, is hopelessly in error, and the reader is referred to my original text. There should be no difference of opinion about the deduction of the formulæ employed, as they follow the lines laid down in my reports, *International Cloud Observations*, 1898, pp. 487–490, or *Barometry of the United States*, 1902, chap. 2, which is common meteorology. Since the height of a homogeneous atmosphere on the Sun has a fundamental bearing in the studies of the pressures and temperatures of the solar atmosphere, it may be proper to deduce this constant again, and to explain the terms that enter into it. A homogeneous atmosphere is one which has throughout its length a constant mass per unit volume, this mass being that of the lowest cubic volume of the gas composing the atmosphere itself, and the total pressure being that of the actual atmosphere. In the case of the Earth this pressure is, in units of weight,

$$p = \sigma_m B_u = 13595.8 \times 0.760 \quad \text{in kilograms/meter}^2,$$
$$= \sigma_o l \quad = 1.29305 \times 7991.04,$$

where σ_m, σ_o are the weights of the unit volume of mercury and air, and B_u, l the heights of the homogeneous columns, respectively.

Similarly the terms may be computed for a hydrogen atmosphere on the Sun, as follows: To deduce the mass of the unit volume at the

[1] Astrophysical Journal, **16**, 334, 1902.

Sun from the terrestrial constants, we must take the pressure and temperature of the Sun's photosphere. Gravity is $g = 27.6\, g_o$, and the temperature I have assumed to be $7535^\circ = 27.6 \times 273$, or $T = 27.6\, T_o$. There are two reasons for using $T = 7535^\circ$C.: (1) The computed temperature of the solar surface hovers around 7000°C., while the figure 7535 "serves to systematize the solar and terrestrial constants, for hydrogen and air respectively, in a single numerical scheme," as given on p. 77. (2) The observed ordinates of the normal solar spectrum, from Professor Langley's results, in the region of the wave-lengths 1.5μ to 1.7μ are longer than correspond to radiation temperatures of less than $T = 7535^\circ$. There seem to be certain waves which pass through both the solar and terrestrial envelopes without depletion, and which will not match any lower temperature curves. This is shown in the *Monthly Weather Reviews* December 1902. Taking the usual notation, Earth (P_o, T_o, σ_o, g_o), Sun (P, T, σ, g), we have

$$P_o = g_o \sigma_m B_n \qquad \text{for the Earth,}$$

and

$$P = 27.6\, g_o \sigma_m B_n \quad \text{for the Sun.}$$

Since the density varies by the formula,

$$\sigma = \sigma_o \frac{P}{P_o} \frac{T_o}{T} = \sigma_o \frac{27.6\, P_o}{P_o} \cdot \frac{T_o}{27.6\, T_o} = \sigma_o ,$$

it is inferred that we can use the terrestrial densities on the Sun for the temperature $T = 7535^\circ$, and thus unify the two systems in one numerical scheme. It is evident that I have not omitted the gravity factor in $g = n \cdot g_o$, as von Oppolzer stated in his notes.

For the homogeneous atmosphere of hydrogen we have, by the thermodynamic and the dynamic formulæ, respectively,

$$l = RT = 420.552 \times 7535 = 3168860 \text{ m} ,$$

and

$$l = \frac{\sigma_m B_n \times 27.6}{\sigma_n} = \frac{13595.8 \times .760 \times 27.6}{0.089996} = 3168860 \text{ m} .$$

Hence the barometric constant for common logarithms is $K = \frac{l}{M} = 7296570$, as used in my formulæ.

On p. 77 there are numerous checks by cross-computations among the formulæ, and the numbers are consistent with one another. From this point onward the discussion follows well-known lines, and they need not be indicated more fully. My results vary somewhat in detail from Fényi's, but generally agree closely enough to say that we have reached similar conclusions by different methods.

It should be noted that, if a pressure of five atmospheres be assumed

for the upper strata of the photosphere, the computed diminution of pressure outward seems to be in harmony with the observed extent of the inner corona. Also, from these data one may compute the pressures on the inner strata below the surface of the photosphere, assuming the structure to be arranged practically in adiabatic layers, and thus construct an approximate solar meteorological system. It is very desirable to determine whether this use of the temperature $T = 7535^\circ$ is not permissible from the observational data. It would make the solar constant at the Earth about 4.0 gram-calories per minute, and there are reasons for thinking it is higher than 3.0 gram-calories. Indeed, there are several directions in which these constants can be utilized, unless my suggestion meets with well considered objections.

FRANK H. BIGELOW.

WEATHER BUREAU,
January 5, 1903.

CORRECTION.

IN the December number, p. 337, of the ASTROPHYSICAL JOURNAL, the visual rediscovery of *Eros* last August at the Chamberlin Observatory of the University of Denver was attributed to me. The rediscovery was really made by Dr. Charles J. Ling, to whom the credit is due.

HERBERT A. HOWE.

ERRATA.

ASTROPHYSICAL JOURNAL, Vol. **16**, November 1902:

Page 189, third line from foot, *for* 63300, *read* $63300\,\pi$.
190, fourth line from foot, *for* r, *read* z.
190, fourth line from foot, *for* γ, *read* γ^3.
190, last line, *for* (m^2-n^2), *read* $(m^2-n^2)^3$.
191, sixth line from top, *for* zcn, *read* $2\,cn$.

Vol. **16**, December 1902:

In equation (23) the first term within the bracket $= D$, should be written $\frac{3}{2}\rho$ instead of ρ and the second term should be $\frac{\rho^2}{u}$ instead of ρ.

In the three equations following (31) *for* $\theta_{max} = 2^\circ\ 20'$ and $\theta_{max} = 6^\circ$, *read* $\theta_{max} = 0^\circ.22$ and $\theta_{max} = 0^\circ.6$ respectively.

Vol. **17**, No. 1, January 1903:

Plate I, exchange $-15'$ and $+15'$.
III, *for* Light Curve, *read* Velocity Curve.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

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THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

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THE EVOLUTION OF SOLAR STARS.[1]

By ARTHUR SCHUSTER.

THE contemplation of our solar system, in which a number of planets move around the Sun in the same direction, and nearly the same plane, gave rise to the idea that our Sun has gradually condensed from a nebulous mass. This idea was confirmed by the discoveries of the telescope, which enabled men to see nebulous masses suspended in the sky, some spread out irregularly, some in spiral forms with denser portions, and some of globular shape, with a starlike nucleus in the center. It was clearly a tempting subject to astronomers and others to speculate on the gradual formation of the stars out of the original nebulous chaos. No wonder, then, that the interest which had been aroused by Kant and Laplace, when they formed their celebrated Nebular Hypothesis, has been growing steadily, especially since the spectroscope gave us the means of studying the material out of which the stars are formed.

The readers of this JOURNAL must all be familiar with the main facts of stellar spectroscopy and the general idea of stellar

[1] Read before the *Royal Philosophical Society of Glasgow*, November 6, 1901. Revised for publication in the ASTROPHYSICAL JOURNAL, January 1903.

evolution which these facts have suggested. Everyone recognizes that some kind of evolution is clearly indicated by the manner in which star spectra classify themselves into groups which, though distinct, are yet connected with each other by intermediate types. But while agreeing on a general process of evolution, there is still a good deal of room for differences of opinion on the life-histories of particular stars. One of the important questions which may be raised is this: Does each star or, at any rate, the great majority of them, pass through each of the stages of a uniform evolution? Has, for instance, our Sun at one time given a spectrum identical with that of *α Leonis?* Further, are we justified in concluding that all stars are made up of the same chemical elements in the same proportion? And lastly, admitting a uniform evolution, what is the meaning, as the star grows older, of the gradual displacement of the hydrogen in its atmosphere, first by calcium, and ultimately by iron and other metals?

Before we enter into a fuller discussion of these points, I shall briefly review the methods of experimental investigation which are at our disposal. The simple spectroscopic analysis, which only tells us of the presence of an element, is now complicated, but improved, by the observed fact that spectra are found to vary according to the experimental conditions. If we volatilize a metal by a powerful spark, it sends out waves which are different from those which are seen with a weak spark; and if we replace the spark by the voltaic arc, which volatilizes more material, but is probably at a lower temperature, or by the oxy-hydrogen flame, distinct differences due to the different condition of the luminous vapor generally appear in the spectra. Opinions are not quite concordant as to the cause of these differences, how far they depend on temperature simply, and how far pressure and density may affect them; or whether, finally, the dissociating power of high electric tension may alone be sufficient to explain the observed facts. The metallic absorption in the solar spectrum seems roughly to correspond to that of our electric arc, while, according to Sir Norman Lockyer, the metallic lines seen in the groups of stars intermediate between

the helium and solar spectrum, correspond more closely to the spectra observed in our strongest spark discharges.

In addition to the changes which are produced by temperature and density, we have an effect due to pressure, which consists in a slight lengthening of the waves sent out by the molecules. This effect, which was discovered by Messrs. Mohler and Humphreys, allows us to determine that the pressure to which the vapors in the Sun are subjected is somewhere between two and seven atmospheres.

The investigation of thermal radiation forms another avenue through which we may approach the all-important question of the surface temperature of the stars. It is only quite recently that Mr. E. F. Nichols has succeeded in comparing the total heat radiation of some of the brightest stars. *Vega* and *Arcturus* judged by the eye have the same magnitude, which means that the same amount of that radiation which affects our eyes reaches us from each of these two stars. But when measured by an instrument which is sensitive to all radiation, *Arcturus* is found to have more than double the intensity. As the proportion of total radiation to luminous radiation diminishes with rising temperature, this would indicate a lower temperature for *Arcturus*, and confirm the conclusion, arrived at on other grounds, that the hydrogen stars have a higher surface temperature than the solar stars. Without actual measurements, we may derive the same result from an inspection of the ultra-violet region of the spectrum. This region is made up of rays which are too short to affect our retina, but which produce a photographic effect, and ought to be stronger and more extended, the higher the temperature of the radiating body. We find that in general the hydrogen stars are those which are richest in this ultra-violet light.

These different lines of argument, all leading to the same result, justify us in saying that the surface temperature of the hydrogen stars is higher than that of the solar stars. An extension of the same reasoning leads to the belief that the helium stars have a temperature which is higher still.

The outward appearance and internal constitution of the

stars is not solely defined by the temperature of their surface, and we are in some cases in possession of important information concerning their size, mass, and density. Doppler's principle has received important applications, and will probably yield further results. It might, *e. g.*, give us some indications whether any star is near the point at which instability sets in owing to centrifugal force.

Were a star to revolve around an axis which does not point toward the Earth, and with sufficient velocity to be near the point at which it could throw off a planet or break up into two stars, we could not fail to notice it by the broadening of its lines; but at present there seems little hope that we shall ever witness so interesting an event as the formation of a double star out of one rotating body.[1] Yet there are several cases of double stars where the two bodies must be nearly in contact with each other, and some of these must have been formed, not so very long ago, by the splitting up of a single rotating mass.

Passing on to the light thrown by theoretical investigations on the constitution of stars and other systems, I must notice in the first place those researches which refer to the internal state of gaseous masses condensing under the action of gravitational forces. A mass of gas sufficiently great to collect into a globular body will be denser at the center than near the surface, because the whole weight of the outer layer will cause pressure, and therefore increased density of the central portions. As regards the temperature, we might suppose, in the first place, that there is no difference throughout the mass; but even if this were the case at any one time, it could not long remain so. If the gas be warmer than the surrounding space, radiation will take place, accompanied necessarily by a lowering of temperature in some parts, and consequently the setting up of ascending or descending convection currents. These convection currents will ultimately establish a distribution of temperature which is

[1] (Note added February 1903.) Is it possible that the phenomena of so-called new stars may be due to the sudden violent disturbance produced by the formation of a double star? In such a splitting up, gases which are enormously hotter than the surface gases must suddenly be brought to the surface.

not uniform, and has been named by Lord Kelvin the "convective equilibrium."

If a star is a mass of gas in a state of convective equilibrium, its temperature must increase downward. The rate of increase depends to a great extent on the internal constitution of the molecules. At places where the gravitational attraction is the same, the increase of temperature with depth is proportional to the molecular weight multiplied by a number which is 0.4 in the case of gases like mercury vapor, containing one atom in each molecule, and about 0.3 in the case of gases which contain two atoms in a molecule. But as it is exceedingly likely that at the temperature of the stars all molecules are monatomic like mercury vapor, we may base our calculations on that assumption. The investigations of Homer Lane, Ritter, and Lord Kelvin allow us to solve the problem of the distribution of temperature and density within a star, assuming the interior to behave like a perfect gas in a state of convective equilibrium. Lane was the first to see the importance of a conclusion, which may appear paradoxical at first sight, but which is based on strict mathematical reasoning. According to him, a star, while it radiates heat into space, does not cool, but actually becomes hotter, and this is due to the fact that the contraction, which accompanies the loss of heat, is accompanied by an evolution of heat which more than compensates for the loss. We may imagine radiation to take place chiefly from the outside, and there would no doubt be a lowering of the temperature in these outer layers, if all convection currents were artificially stopped. But at the same time the contraction of the outer shell, inclosing the deeper layers, would cause a rise in temperature in the inside, and consequently a disturbance of the thermal equilibrium. This could be re-established only by convection currents, which would supply the lost heat to the outer layers. Calculation shows that the temperature of the center of a star increases in the same proportion as the diameter of a star diminishes; but it must be clearly understood that all these conclusions are based on the supposition that the whole mass of the star behaves like a perfect gas, a supposition which fails to be true at the surface of a star, and

also near its center, but probably holds very nearly in the intermediate layers.

If a star contained only perfect gases, its surface would be formed where the absolute zero of temperature is reached. But the metallic vapors which to a great extent compose the stars condense into a liquid at a temperature of more than 1,000° above the absolute zero, and we must therefore imagine the boundary of the stars to be formed, not at the zero of temperature, but at the place where a cloud-like condensation of vapors takes place, the temperature of these clouds being probably somewhere between 4,000° and 20,000° C. This formation of clouds, though it precludes us from applying Lane's results to the outer layers of stars, does not affect his calculations as to their internal constitution, which probably give us a good representation of the state of a star from the photosphere down to considerable depths. Ultimately, and especially in the case of stars which are already advanced in their condensation, the equations will fail, because when the molecules of a gas become as near to each other as they are in liquids, molecular forces come into play, which prevent the gases from behaving in the ideal manner of a perfect gas. The molecular forces diminish the compressibility, and ultimately the heat which is generated by compression will fail to compensate for the heat lost by radiation. When that period has been reached the star will begin to cool, pass into the liquid state, and soon cease to be luminous.

The questions which meet us when we try to interpret stellar spectra will be more easily understood after we have examined more closely what happens on the surface of the Sun. We may, in the first place, inquire with advantage whether our knowledge of the constitution of that body supplies us any arguments for or against the theory of convective equilibrium, as presented to us by Lane and Ritter. I have calculated, chiefly from the data supplied by Ritter, the density, the pressure, and the temperature of the inside of a star having the same mass and size as the Sun, and behaving like a perfect gas in a state of convective equilibrium. The numbers are given in the accompanying table.

A simple calculation allows us to extend the results to a period of time when the Sun had a larger diameter. The first column of the table defines the position of the different layers in terms of the fraction obtained by dividing the distance of any layer from the center, by the distance of the uppermost layer. The second column gives the density, and the third the pressure in megadynes per square centimeter, that unit being very nearly equal to our atmosphere. The last column shows the temperature, which depends, however, on the nature of the gas; the numbers given apply to hydrogen, assuming it to retain its biatomic constitution, and should in other cases be multiplied by the molecular weight. Thus assuming the whole of the inside of the Sun to be made up of hydrogen split up into monatomic elements, the temperature at the center would be 12,000,000 degrees. If made up entirely of monatomic iron, the temperature would have fifty-six times that value. To apply the table to a previous period in which the diameter of the Sun was, *e.g.*, twice as great, we should have to divide the second column by 8, which is the cube of 2, the third column by 16, which is the fourth power of 2, and our fourth column by 2. It is a curious fact, which is not perhaps without significance, that the central density of the Sun, as calculated on the assumption of its being a perfect gas, is only very little in excess of the density which, according to the most careful recent estimate, is to be ascribed to the central portion of the Earth, and again that that estimate is very little in excess of the density of solid iron.

GASEOUS SPHERE IN CONVECTIVE EQUILIBRIUM.

Total mass = mass of Sun = 2×10^{33} grams.
a = molecular weight compared to hydrogen.
R = radius of sphere.
= 700,000/ β kilometers,
where β is a factor which is equal to unity in the case of the Sun.
r = distance from center.
$x = r/R$.
Mean density = $1.406/\beta^3$.

x	$\beta^3 \times$ Density	$\beta^4 \times$ Pressure in Dynes per Square Centimeter	Temperature
0	8.44	8.65×10^{15}	$24.60 \times 10^6 \times \alpha/\beta$
0.1	8.17	8.19	24.06
0.2	7.39	6.93	22.51
0.3	6.23	5.21	20.07
0.4	4.88	3.47	17.07
0.5	3.54	2.03	13.77
0.6	2.33	1.01	10.43
0.7	1.36	0.41	7.28
0.8	0.65	0.12	4.45
0.9	0.20	0.017	2.02
1.0	0.00	0.00	0.00

If it is allowable to imagine the Sun to consist chiefly of iron vapor, that vapor, when above the so-called critical temperature, might be expected to follow the laws of a gaseous compressibility until the density is nearly equal to that of liquid iron, and, whatever the temperature, we shall not be able to compress gaseous iron to a density greater than that of solid iron. It is also allowable to conclude from this reasoning that the distribution of density in the interior of the Sun is not very much different from that indicated in the table, being probably rather less in the central portions, and hence rather greater in the outer portions of the solar mass.

Though the distribution of density and pressure in the interior of a star is probably fairly well represented by the table, the temperature almost certainly is considerably less through the greater portion of the mass. The failure of our equations in this respect is due to diminished compressibility, and also to our having left all effects of radiation and conduction of heat out of consideration. The effects of conduction will be most marked where, owing to increased density and diminished gravitational attraction, the convection current becomes less effective, as for instance near the center of the star. The effects of radiation will be most marked near the surface, and especially in those portions of the star which lie above the cloudy condensations which we have considered to make up its surface. Gases like hydrogen will reach to some height above the surface, and form an atmosphere around the main body of the star. Were it

allowable to neglect radiation altogether, these gases would be in convective equilibrium and rapidly diminish in temperature as they rise above the surface; they could not in fact rise much above the surface before they would be reduced to the zero of temperature.[1] The fall in temperature with altitude above the surface of the Sun is, according to calculation, $26 \times a$ degrees centigrade per kilometer, where a is, as before, the molecular weight. Thus for monatomic iron vapor the diminution in temperature would be 73,000° C. for each 100 kilometers.

It may not be unnecessary to say a few words as to the evidence we possess that the convection currents which play so important a part in the theoretical investigation actually exist in the Sun. The surface radiates an amount of heat into space of which we can form a very fair estimate by measuring the quantity which reaches the Earth. The number so obtained is 1,340,000,000 calories per square meter of the solar surface, the unit of heat being the amount necessary to raise one gram of water through one degree. We obtain an idea of what that number means if we imagine the Sun to be surrounded by a shell of ice; the heat supplied by radiation could melt in each minute a layer of ice fifty-eight feet thick. Or, expressing it with Lord Kelvin in terms of power, we may say that the solar surface does work by radiation equivalent to 131,000 horse-power for each square meter of his surface. The heat thus lost by radiation must be supplied from the inside of the Sun, otherwise the solar surface would cool down in a fraction of a second to a temperature at which it would cease to be luminous. If the heat is carried from the inside to the outside by convection alone, the velocity of the currents of vapor must be very great. Taking the pressure of the vapor near the surface to be one atmosphere, we may say that all the heat contained in a layer having a thick-

[1] (Note added February 1903.) Unless radiation can by itself alone establish a state of equilibrium, convection currents must still take place, and be the predominant factor in the distribution of temperature. Professor Sampson has tried to establish a state of temperature for the case of radiation alone, but his distribution is unstable. As far as my present results go, radiation cannot seriously affect the temperature distribution in the inside of the Sun, unless the material composing it is very much more transparent than we have a right to expect.

ness of 370 meters is lost by radiation in each second of time, and this number does not depend on the nature of the vapor or on its temperature. A layer of that thickness would have to be replaced by convection in every second if the temperature of the surface is to be maintained. From this I calculate that if the difference in pressure between the descending and ascending currents is one atmosphere, the velocity of the convection currents must be 616 meters per second, or about 1,000 miles per hour. These up-and-down draughts of vapor must take place with the calculated velocity, unless an appreciable portion of the heat is supplied from the inside in some other way, as for instance, by radiation. It is difficult to form an estimate as to how far radiation can help to keep up the temperature of the surface. I have made some calculations on that point which, though they have yielded interesting results, cannot at present be expressed in definite numbers. It is sufficient for the present argument to maintain that, even if radiation takes a prominent part in the determination of the distribution of temperature, we cannot escape the conclusion that convection currents must bring about a continuous interchange of matter between the inside and outside of the Sun. This theoretical conclusion is amply confirmed by observation, as is shown by the violent motion observed in the chromosphere and prominences.

In the solar eclipse which was observed in the West Indies in the year 1896 one of these prominences reached to a distance of 140,000 miles. This is by no means the greatest height that has been observed, a prominence being photographed in 1895 by means of an ingenious method due to Professor Hale, which reached to a distance of 281,000 miles from the Sun's limb. The rapidity with which these prominences appear to rise and change their appearance is not perhaps a conclusive proof that the gases which they contain move with great velocity, for these gases are quite possibly always present, and the prominence may only be a sudden lighting up of the gas, or the rapid transmission of an effect, which does not actually require the transmission of a material body. But the tangential velocities observed at the limb of the Sun within the chromosphere have not at present been

explained in a satisfactory way except by assuming that there is an actual motion of hydrogen and of the other gases which form the chromosphere. This tangential motion is far more violent than anything which is required for the convection currents necessary to maintain the temperature of the solar surface. Professor C. A. Young, of Princeton University, states that a velocity of a hundred miles a second is often exceeded, and that twice this velocity is occasionally reached. There can hardly be a doubt as to the facts, but their explanation seems to me more difficult than has been generally recognized. Professor Young says on this point:[1]

> It would seem that thus we might explain how the upper surface of the hydrogen atmosphere is tormented by the uprush from below, and how gaseous masses thrown up from beneath should, in the prominences, present the appearances which have been described. Nor would it be strange if veritable explosions should occur in the quasi pipes or channels through which the vapors rise when, under the varying circumstances of pressure and temperature, the mingled gases reach their point of combination; explosions which should fairly account for such phenomena as those represented in Figs. 69 and 70, where clouds of hydrogen when thrown to an elevation of more than 200,000 miles with a velocity which must have exceeded, at first, 200 miles per second, and very probably taking into account the resistance of the solar atmosphere, may, as Mr. Proctor has shown, have exceeded 500—a velocity sufficient to hurl a dense material entirely clear of the Sun's attraction, and send it out into space, never to return.

My doubt as to the correctness of the above explanation is based on the fact that the highest velocity that a gas can reach when forced to move by differences of pressure is equal to the velocity of sound in the gas, where, of course, the temperature of the gas has to be taken into account in calculating the speed of sound waves. In order that a velocity of 100 miles a second may be possible in monatomic hydrogen, it is necessary that the temperature of the gas should be more than two million degrees, for in no other way can the velocity of sound in hydrogen reach so high a value.[2] On the other hand, if at a temperature of

[1] *The Sun*, p. 207.

[2] The velocity of sound may for violent disturbances be greater than that calculated by the ordinary formula, but there is reason to believe that the calculated velocity cannot be exceeded many times.

(Note added February 1903.) Since the above was written, Professor Julius, of

10,000 C., at which we may imagine the solar surface to be, a gas can rush out under the action of a pressure, however great, with a speed of 100 miles a second, the mass of its molecules must be over 200 times less than the mass of the hydrogen atom such as we know it. Sir Norman Lockyer has recently expressed the opinion[1] that the true spectrum of hydrogen such as is seen in the prominences is due, not to the hydrogen atom as we know it, but to a much smaller one, derived from it by dissociation, and he has estimated this smaller atom to have a mass about sixty times smaller than the hydrogen atom. If this view were accepted the difficulty would disappear, and the observed high velocities might be explained by internal pressure. But there is a difficulty in believing such small masses to be capable of emitting visible radiations.

Leaving out of account for the present the possibility of such a great subdivision of atoms, there are, to my mind, only three courses open to us. We may, in the first place, deny the necessity of admitting the existence of velocities as great as those I have named. As regards radial motion outward from the Sun, the evidence in favor of the reality of these velocities is not perhaps conclusive.[2] But the tangential velocities at the limb of the

Utrecht, has made the ingenious suggestion that many of the appearances we observe on the Sun's limb are not real, but are due to an optical illusion, produced by anomalous dispersion. I do not think that, so far, the efforts to account for prominences in this way have been successful, but at present I only wish to point out that the explanation does not get rid of the difficulty which has been pointed out in the text, for the rapid change in appearance, which indicates apparently a large radial velocity, would, according to Professor Julius, still be accounted for by the propagation of some disturbance, though not by a projection of matter. But the velocity of sound is the limiting velocity for the propagation of any disturbance, whether of matter itself or of any arrangement of matter, and, therefore, a large observed velocity is equally fatal to the theory of anomalous dispersion and to the older theory.

[1] *Inorganic Evolution*, p. 182.

[2] (Note added February 1903.) Professor Hale (*Astronomy and Astro-Physics*, **2**, 611 and 917) has described a remarkable outburst on the Sun's surface, the vapors produced by the outburst quickly covering a small portion of the Sun's disk, and obscuring two Sun-spots and a number of faculæ.

After the outburst, however, it was found that no permanent change in the appearance of Sun-spots and faculæ had taken place, and Professor Hale concludes from this that the obscuring vapors must therefore have been well above the layer of spots and faculæ. The velocities of those vapors during the outburst must have been very large indeed.

Sun, which are velocities parallel to the solar surface, have been observed to be as great as the radial velocities, and these must be real unless some other cause may produce displacements of spectroscopic lines. One such cause recently discovered, and already mentioned, is pressure, which increases the wave-length. Other effects might be thought of, which may act in the same direction, but it is much more difficult to imagine any cause which can produce a *shortening* of wave-length. As far as I can judge from the published accounts and drawings, a great tangential velocity at the Sun's limb is as often observed to take place *toward* us as *away* from us; hence, even if two causes may act, one shortening and the other lengthening the waves, it does not seem probable that the two causes would act with equal frequency and to an equal extent in both directions. For the present we are forced to admit only known effects, and hence we are forced to recognize the reality of the great velocities which have been deduced from the observations.

Various attempts have been made to account for a number of phenomena which are observed on the solar surface by electrical actions, such attempts being based on the supposition that the Sun as a whole is a highly charged electrified body. But we know that no body at the temperature of the solar surface could permanently retain an electric charge. If there is therefore a permanent electric force, there must also be a permanent electromotive force tending to drive negative electricity from the inside to the outside, or *vice versa.* There is nothing improbable in such a supposition, as the phenomena of atmospheric electricity show. The surface of the Earth is charged with negative electricity, and though we know that every burning fire, and every wave of the sea breaking into spray, tends to dissipate that charge, it yet is permanent, and remains without apparent diminution. We conclude that some cause, upon which meteorologists and physicists are not yet agreed, exists, which tends to bring the dissipated electricity back to the Earth, and we are confirmed in this conclusion by the fact that falling drops of rain are more frequently charged negatively than positively.

We are therefore quite at liberty to admit a high charge of

electricity at the surface of the Sun, which probably does not, however, exert any electric force, except near its surface; the outside being screened by opposite electrification. The only way which occurs to me as possibly causing an appreciable electric force at a distance greater than the solar diameter would be to suppose the existence of highly eccentric meteoric swarms, which, passing near the Sun, would carry the neutralizing charge out into space. The streamers of the solar corona might be due to electric discharges between such retreating swarms and the Sun.

If the Sun is a highly charged electrified body, velocities like those observed in the atmosphere are possible, if these velocities are radial, *i. e.*, from the center of the Sun or toward it, but I am unable to satisfy myself that the observed tangential velocities can be explained by electric action. There remains only one way of accounting for these velocities, and that is to conclude that they are either directly due to meteoric matter circulating around the Sun, or indirectly to meteoric matter falling into the Sun, and locally generating a temperature sufficiently high to allow of molecular velocities of 200 miles per second. The velocity of a piece of matter circulating around the Sun and close to its surface in a circular orbit is about 270 miles per second. Such a piece of matter would tend to carry with it the very tenuous gases which are floating above the solar surface, and may well impart to them a velocity ranging from 100 to 200 miles. If actually falling into the Sun, the conversion of their motion into heat, or the kinetic energy supplied by the splash, would be sufficient to account for the observed velocities.[1]

The explanation of the violent disturbances which are observed to take place on the solar surface does not come within the range of my main subject, but it was necessary to point out

[1] (Note added February 1903.) Matter falling into the Sun and producing what has been called a "splash" could account for velocities very much larger than that of the velocity of sound; in fact, there is no limit to the velocity that could be generated in this fashion, as the energy of impact is, at the first instant, concentrated into a very small amount of matter. If the splash takes place near the Sun's limb, but on the visible portion, great receding tangential velocities may be observed. If the splash takes place behind the Sun's limb, the tangential velocities would be approaching the Earth.

that the convection currents, which are necessary to the temperature distribution which is now generally admitted by astronomers, are actually observed to take place on the surface of the Sun with greater violence than we might *a priori* have expected. A detailed study of solar phenomena is in my opinion the only sure guide in our investigations on stellar constitution, and it is probable that many of the unsolved problems, which still meet us in the interpretation of stellar spectra, will be cleared up in the solar and terrestrial laboratories rather than by mere statistical comparisons and classifications. There are, in fact, many similarities between solar and stellar phenomena. One of the most curious facts revealed by the photographs of star spectra is, as has already been pointed out, the peculiar behavior of calcium, which forms the main connecting link between hydrogen and metallic stars; and this behavior of calcium shows itself with equal persistency in the spectra of solar prominences, which, as regards the hydrogen lines, do not present a spectrum far different from that of *α Aquilæ* or *Procyon;* while the similarity between the spectrum of the chromosphere and that of some of the stars has been pointed out by Sir Norman Lockyer.

Before leaving the subject of the Sun, I may refer briefly to an argument which seems to me to be fatal to any theory which involves the decomposition of the elements right through its mass. We may say with certainty that no amount of pressure can increase the density of any substance much beyond what it is in its liquid state. Liquid hydrogen has a density of about 0.09, or less than the fifteenth part of the average density of the Sun. I conclude that the interior of the Sun cannot be made up of hydrogen or of any substance which might be formed by the breaking up of hydrogen, for such decompositions are, as far as we can judge, never accompanied by an increase of density. On the other hand, the mean density of the Sun is quite consistent with the supposition that its interior is mainly composed of the same substances as are known to us on the Earth.

We may now return to the main subject of our inquiry, which is the critical discussion of the arguments that have convinced the great majority of astronomers of a process of evolution

which in the course of time makes each star pass successively through a number of stages, in which the spectrum changes from that of the helium stars to that of the hydrogen stars, and hence to that of stars with prominent calcium lines and of the solar stars.

The first fact which requires explanation is the one which is generally, though not universally admitted, that the temperature of the photosphere of the stars diminishes in the order in which they have just been named. If the hydrogen star is always hotter than the solar star, this would suggest that the chemical composition stands in some direct causal relationship to the temperature, but it is open to discussion which is the cause and which the effect. Is the photosphere of *α Leonis* hotter because it is surrounded by an atmosphere chiefly containing hydrogen, or does *α Leonis* only show us the hydrogen spectrum because its atmosphere is too hot to show anything else? The discussion of this point is altogether independent of our ideas regarding evolution. Even if we do not wish to enter at all into the previous history of a star or its future development, we are bound to search for an explanation of the constitution of the present universe, which shows us hydrogen stars and solar stars, the former being apparently hotter than the latter.

But if we take the theory of evolution into account, we have further to explain the fact that, if the above be true, a star cools as it grows older, while the theory of Homer Lane, of which an outline has been given, states that the star should get hotter. The apparent disagreement between theory and observation has been a stumbling-block to astronomers, but it is due in great measure to the want of definiteness in our meaning, when we speak of the "temperature" of a star. We do not observe the temperature of the center, or the temperature of the gaseous mass below the photosphere, and it is with this temperature that the theoretical analysis deals. What we can observe is the photosphere and the absorbing layer above it, and the temperature of these portions of the Sun are not touched by Lane's theory. If our ideas of the photosphere are correct, and it consists of condensed clouds of metallic or carbon

vapor, the temperature of these clouds will be quite independent of the temperature inside the star; and, for all we know, might under certain circumstances remain the same for a long period of a star's life, during which time the star may condense to a fraction of its original volume, and its interior become hotter and hotter, until the condensation has reached the point at which the laws of gaseous compression no longer hold. The surface of a star is pouring out energy in the form of radiation, and the temperature of the surface will depend on the balance of a number of delicately poised conditions. Equilibrium is reached when the loss of heat by radiation is balanced by an equal gain of the heat from the inside or from the outside. The gain in the case of our Sun is in great part due to the convection currents from beneath, which keep the photosphere at the temperature at which, under the existing pressure, the metallic vapors condense. So far we should expect the photosphere to become more luminous as the star contracts, because the greater the intensity of gravitational attraction, the more active the convection currents may be expected to be. But two factors may operate in the opposite direction. In the first place, we must not assume that the inflow of outside meteoric matter, which not so long ago was considered to be the chief cause of the maintenance of solar heat, is altogether inactive. Even in the case of the Sun, it has already been pointed out that several phenomena point directly to the generation of heat at the surface of the Sun by the impact of falling masses. What in the Sun is a subordinate cause may in some of the stars become predominant, and the photosphere may recuperate itself for the loss of energy which it radiates into space, not only from the inside, but also from the outside. If this is the case, we need not be surprised that the temperature of the photosphere is apparently higher in the younger stars, or that the spectrum of these younger stars resembles that of the solar prominences.

But even without having recourse to outside influence, it is possible to account for the higher temperature of the hydrogen stars in a more direct way, by making the hydrogen atmos-

phere itself responsible for it. The temperature of the photosphere must be largely affected by the absorbing properties of the gases surrounding it. If, for instance, these gases were largely to absorb the infra-red radiation, the effect of such absorption would be observed in a rise of temperature. We need only point, in illustration of this, to the way in which the glass roof and sides of a hothouse protect the plants inside against loss of heat by radiation into space. If it were possible to imagine hydrogen to absorb infra-red rays, this gas would, by stopping the loss of these rays, increase the visible, and especially the blue and violet, radiations, and the fact that the metallic lines shown by hydrogen stars are principally the high temperature lines would thus be accounted for. Such an explanation will only remain a mere surmise, unless it is confirmed by independent evidence, but perhaps the phenomena accompanying the formation of faculæ on the Sun may be found to furnish such evidence, although only of an indirect character. The faculæ are bright streaks on the solar surface specially seen in the neighborhood of spots, and, according to recent observations, they are closely connected with the prominences which seem chiefly to lie above them. If the suggested explanation for the high temperature of hydrogen stars be correct, the same explanation would apply to any portion of the solar surface which has masses of hydrogen hanging over it, and the increased luminosity of those portions of the Sun which lie underneath the prominences, would be a necessary consequence. To prevent misunderstanding, it is well to point out that the infra-red absorption need not be due to the same molecules of hydrogen which give the well-known hydrogen spectrum, and which are almost certainly not identical with the diatomic hydrogen molecule which we prepare in the laboratory. The luminous hydrogen in the stars must be surrounded by cooler hydrogen, and the space surrounding the prominences will similarly contain masses of cool hydrogen, having radiating and absorbing properties differing from those of the luminous substance. If I have dwelt on an explanation which at present is little more than a guess, it is only to emphasize that we need

not hesitate on theoretical grounds to accept the evidence of observation, that the photosphere of the hydrogen stars is hotter than the photosphere of a star giving a solar spectrum. The suggestions I have put forward are sufficient to show that the temperature of the photosphere is regulated by considerations which lie altogether outside the calculations of Lane.

We are now prepared to admit that the hydrogen star is hotter than the solar star, and that, if there has been a process of evolution, this hydrogen period of a star is earlier than the solar period. The reason for this second statement will appear more clearly farther on. This brings us to the next stage of our problem. Why does the hydrogen disappear in the process of cooling, and why is its spectrum replaced by that of the metallic vapors? The simplest explanation — simple because it cuts the Gordian knot — is that offered by Sir Norman Lockyer. If the hottest star shows no metallic lines, it is because the temperature is too high for the existence of the molecule which alone can emit the radiation corresponding to these lines. The atoms are decomposed or dissociated, or whatever name we may attach to what must practically be a splitting of what is generally considered an "atom," or, in other words, unsplitable by chemical agencies. When a star cools, and its photosphere has reached the temperature at which the more complex molecule can exist, the corpuscles, according to Lockyer's theory, will recombine and ultimately form metallic vapors such as we know on the Earth. This explanation involves a hypothesis that is possible and consistent, but which, before it can be generally accepted, must either be shown to be the only hypothesis consistent with the facts, or to be supported by strong outside evidence. A great difficulty of the dissociation hypothesis lies in the fact that, as in the case of the Sun, so also in the case of the *Algol* variables, of which the density is approximately known, that density is greater than the density of solid hydrogen, although these stars have spectra which are generally of the hydrogen or calcium type. In order to maintain the theory, it would be necessary to imagine that dissociation only takes place on the surface of the star, and that the pressure inside is sufficient to produce recombination, in spite of the higher temperature which reigns there.

What are the alternative suggestions which have been made? Sir William Huggins, who touched on this in his presidential address delivered to the British Association at Cardiff, draws attention to the effect of convection currents in mixing up different layers of the gaseous matter forming the star. If convection currents could be completely stopped, the heavier gases would sink to lower levels, and the outer layer of a star would be made up of hydrogen and the lighter metallic vapors. It is owing to convection that a mixing takes place, and the stronger the convection the more complete is this mixing. The following quotation will show the position Sir William Huggins takes up in this matter:

Now, the conditions of the radiating photosphere and those of the gases above it, on which the character of the spectrum of a star depends, will be determined, not alone by temperature, but also by the force of gravity in these regions; this force will be fixed by the star's mass and its stage of condensation, and will become greater as the star continues to condense.

In the case of the Sun the force of gravity has already become so great at the surface that the decrease of the density of the gases must be extremely rapid, passing in the space of a few miles from the atmospheric pressure to a density infinitesimally small; consequently the temperature-gradient at the surface, if determined solely by expansion, must be extremely rapid. The gases here, however, are exposed to the fierce radiation of the Sun, and, unless wholly transparent, would take up heat, especially if any solid or liquid particles were present from condensation or convection currents.

From these causes, within a very small extent of space at the surface of the Sun, all bodies with which we are acquainted should fall to a condition in which the extremely tenuous gas could no longer give a visible spectrum.

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Passing backward in the star's life, we should find a gradual weakening of gravity at the surface, a reduction of the temperature-gradient as far as it was determined by expansion, and convection currents of less violence producing less interference with the proportional quantities of gases due to their vapor densities, while the effects of eruptions would be more extensive.

At last we might come to a state of things in which, if the star were hot enough, only hydrogen might be sufficiently cool relatively to the radiation behind to produce a strong absorption. The lower vapors would be protected, and might continue to be relatively too hot for their lines to appear very dark upon the continuous spectrum; besides, their lines might be possibly to some extent effaced by the coming in under such conditions in the vapors themselves of a continuous spectrum.

In such a star the light radiated toward the upper part of the atmosphere may have come from portions lower down of the atmosphere itself, or at least from parts not greatly hotter. There may be no such great difference of temperature of the low and less low portions of the star's atmosphere as to make the darkening effect of absorption of the protected metallic vapors to prevail over the illuminating effect of their emission.

In a discussion before the Royal Society in 1897 I took up an essentially similar position.[1] "The chief difference (I then wrote) between a hydrogen and a solar star lies in the more or less effectual mixing up of the constituents. If we could introduce a stirrer into *α Lyræ* there can be no doubt whatever that the low-temperature lines of iron would make their appearance, while, on the other hand, if we could stop all convection currents on the surface of the Sun, the hydrogen which now lies under the photosphere would gradually diffuse out and give greater prominence to its characteristic absorption."

I still believe this statement to be true, but I have modified my opinion in so far as I do not now believe the difference in the condition of the surface of stars like *Sirius* or our Sun to be sufficient to eliminate convection almost entirely in one case and make it the predominant factor in the other. The mean density of *Sirius* is probably not greater than that which our Sun had when its diameter was about five times as great as it is now. But even then, with a gravitational force still greater than that on the surface of the Earth, convection currents must have been active in stirring up and mixing the strata down to a considerable depth below the surface. The conditions which I imagined, in 1897, to hold in the hydrogen stars are not, according to my present opinion, consistent with the formation of a photosphere, which, I now believe, necessarily involves effective convection currents. Hence, there must be some other cause for the elimination of hydrogen out of the atmosphere of stars, when they have reached the solar stage. We are reasoning, of course, on the supposition that the difference in the type of spectra is not due to any inherent chemical difference in the composition of the stars. The evidence for this assumption will have to be further examined, especially in view of the fact that, if my suggestion

[1] *Proc. R. S.*, 61, 209.

should prove to have a solid foundation, and if the presence of masses of hydrogen is the cause and not the result of the higher temperature of the photosphere, the difficulty we are now trying to overcome disappears. Here, as in the previous discussion, it is not so much my intention to argue in favor of one hypothesis or another, but rather to show the different possibilities which may, in a natural way, account for much that is obscure at present.

If we had only to account for those stars which chiefly show hydrogen and calcium, we might attribute their spectra to the action, in an exaggerated form, of the same cause which produces the prominence spectrum of our Sun. And if we believe that the influx of meteoric matter is directly or indirectly responsible for the prominences, we should be led to suppose that the stars of the *Procyon* type are bodies in which the aggregation of meteoric masses from outside still plays an important part in regulating the temperature and spectrum of the superficial layers. This, though in many ways a satisfactory explanation, does not account for the spectra in which hydrogen is seen without the calcium, nor for other types of spectra which are generally considered to precede the *Procyon* type.

But the disappearance of hydrogen as the star condenses is not perhaps a phenomenon which should surprise us so much. Hydrogen is readily absorbed by many metals, even at a high temperature, and it is highly probable that gases show phenomena of molecular absorption like solids or liquids, when they are subjected to a pressure so high that their density approaches the density of the liquid state. I can see nothing improbable in the supposition that when a star condenses, and its pressure reaches a high value in the interior, it should begin to absorb hydrogen, helium, and possibly oxygen, nitrogen, and the other constituents which have either not been observed in the Sun, or only give faint evidence of their presence. If this opinion is correct, a quantity of matter, suddenly transported from the interior of the Sun to the outside, would violently give up the hydrogen which it was able to contain under its original pressure. The phenomena observed on the surface of the Sun would seem

to lend countenance to such a view; at any rate they do not contradict it. Another and perhaps simpler explanation is suggested by considering the process of the formation of a star from its first beginnings. Its consideration may therefore be deferred until we are prepared to look at the question of evolution as a whole. I repeat that it is only my intention to put forward suggestions, which may be found to have some truth in them, and which should, therefore, be taken into consideration.

But from such speculative inquiries we may once more turn to the more solid search for further facts. Results which have an important bearing on our subject have been obtained from the observation of double stars, for they allow us to obtain a value for the state of condensation of the matter composing some of these stars. Professor E. C. Pickering and, later, Mr. Monck have deduced a remarkable equation, which connects together the intrinsic brightness of a star's surface, its mean density, and other quantities which may be obtained by observation. We may thus calculate the density on the supposition that all stars emit an equal amount of light per unit surface. As regards stars showing a similar type of spectrum, this supposition is probably nearly correct, but the results have to be used with caution when comparing a hydrogen and a solar star, for the mere fact that the spectrum of the latter is filled with absorption lines would induce us to believe—quite apart from any question as to the temperature of the surface—that the amount of light leaving the star is less per unit surface for stars giving a solar spectrum, than for stars which only show hydrogen lines. The result of calculation showed that on the average the density of the solar stars was fifteen times greater than the density of the hydrogen stars;[1] but this number was founded on information which, as regards the nature of the spectrum emitted, was, in many cases, deficient. Pending a renewed inquiry into this important subject, I am struck by the slight systematic difference shown between the stars of different spectroscopic types. A density fifteen times as great means, for the same mass, a diameter reduced in the value of $2\frac{1}{2}$ to 1, and even this difference would

[1] PROCTOR-RANYARD, *Old and New Astronomy*.

disappear if the hydrogen stars had an emissive power six times as great as that of the Sun.

The differences of density shown by individual stars of the same spectral type are considerably greater. Thus γ *Leonis* gives a spectrum almost identical with that of *Arcturus*, which is generally considered to belong to a later period than the solar stage; its density on the assumption of equal emissive powers is 0.0002 as compared with that of the Sun. It is not possible to admit an emissive power 300 times as great as that of the Sun, and hence the density of γ *Leonis* must be very considerably less than that of the Sun. As far as the observations go, the stars which are purely hydrogen stars show smaller variations in density than the solar stars, and have a density which is unmistakably smaller, but great variations are found in the intermediate stages in which the calcium lines are prominent. η *Cassiopeiae*, for instance, has a density almost equal to that of the Sun, being the third in order of density of all known stars, while γ *Virginis*, giving a similar spectrum, has a density sixty times smaller. The difficulty which, in the case of the binaries we have just discussed, arises from our ignorance of the intrinsic brightness of their surface, is overcome in the case of another set of close double stars, the so-called *Algol* variables. These binaries are characterized by the fact that they consist of two stars of unequal brightness, one of which passes periodically in front of the other so as to produce a variation in the combined brightness of the stars. But I must resist the temptation of entering into a detailed description of the peculiarities of these interesting stars, and content myself by referring to the conclusions of A. W. Roberts and H. N. Russell, which have given an average density considerably less than that of the Sun. Thus Roberts finds for the mean of four of these variables 0.18 for the greatest density consistent with the observations, while the corresponding density found by Russell for the average of seventeen variables is 0.20, closely agreeing with the former result. This means a density equal to about one-eighth of the solar density. For the fainter component of *S Velorum*, the greatest possible density is only 0.03 as compared with the Sun. The spectra of these stars all seem to

belong to the pre-solar type, and their low density therefore confirms the results arrived at from the consideration of other binaries, though we should not lose sight of the fact that the average density of the stars of the solar type seems to be considerably less than that of the Sun itself, so that the average density of the *Algol* variables is not much less than half the average density of the solar stars.

Interesting facts are brought to light when we investigate the distribution of types of spectra in different parts of the heavens. Such investigations are subject to the dangers accompanying all statistical inquiries, especially when the discussion must, to a great extent, turn on the differentiation between accidental and systematic deviations from the average. We owe a spectroscopic survey of the sky to Professor E. C. Pickering, who, with instruments belonging to the Henry Draper Memorial equipment, classified the spectra of 10,345 stars north of 25° of southern declination, and has since completed the investigation for stars which lie farther south. He has very fully discussed the results of the first of these surveys. Almost exactly half belong to the hydrogen type, 10 per cent. to the intermediate or calcium type, 12 per cent. to the solar type, and 25 per cent. to the *Arcturus* type. If we only take the stars which are brighter than magnitude 6.25, the hydrogen type includes relatively more, viz., 61 per cent., while only 5 per cent. belong to the solar type. The percentage of the Arcturian type is reduced to 18. If the region considered is divided into two equal portions, one lying as much as possible along the Milky Way, and the other away from it, it is found that, of all the stars considered, the Milky Way shows a preference for the stars of the hydrogen type, 3,560 of these stars being mapped in the region of the Milky Way, and 1,658 away from it; the corresponding numbers for the calcium type are 650 and 430. Neither the solar stars nor those having a spectrum similar to that of *Arcturus* show, when their total number is considered, a preference for any particular part of the sky. Out of the total number of stars, 6,252 belonged to the portion of the sky which included, and 4,095 to that portion which did not include, the Milky Way.

When we consider the distribution of stars of different magnitudes, we find that the hydrogen stars, which are of the fourth magnitude and brighter, seem to be distributed pretty evenly all over the heavens, and that it is only the weaker stars which show this effect of clustering in the regions of the Milky Way. The brighter solar stars, on the other hand, seem to be relatively more frequent in the Milky Way than away from it, and the more even distribution shown by the solar and Arcturian stars seems rather due to the fact that the stars of smaller magnitudes belonging to these types are chiefly found in the regions which lie away from the Milky Way. Thus, taking the stars down to the sixth magnitude, we find the numbers in the district of the Milky Way, and away from it, for the hydrogen stars to be 152 and 84, while for the Arcturian stars it is 453 and 341, in both cases an increased number in the Milky Way.

The results obtained by Dr. Frank McClean, who confined himself to stars above the 3.5 magnitude in both hemispheres, are not altogether in accordance with the above. Out of a total number of 276 stars, 30 per cent. only were found to belong to the hydrogen type, 17 per cent. to the *Procyon* or calcium type, while 31 per cent. were Arcturian or solar. An unusually large number, viz., 32 per cent., were classed as helium stars, and this leads to the supposition that a number of stars which figure in Pickering's list as hydrogen stars belong really to the helium subdivision. These helium stars show a very decided tendency to cluster in the Milky Way, but the distribution of the other types is, according to McClean, remarkably uniform. Thus, dividing the sky into two equal portions, the first of which includes the Milky Way, the numbers for the hydrogen type are 20 and 16, a very slight excess in favor of the Milky Way. For the calcium type the numbers are 21 and 27, and for the solar and Arcturian types combined, 45 and 40. The total number of stars of the hydrogen and calcium types is thus remarkably nearly equal to that of the solar and Arcturian types.

A promising line of inquiry has been entered upon by Mr. W. H. S. Monck, who finds that the apparent proper motion of solar and Arcturian stars is considerably greater than that of the Sirian

stars. Out of over 5,000 of the latter stars found in the *Draper Catalogue*, 225 are known to have a proper motion of not less than one tenth of a second per annum in one or other of the elements; this gives a percentage of less than 4.5. On the other hand, the percentage of solar and Arcturian stars having the same proper motion is 20 and 15 respectively. This points to the fact that the hydrogen stars are farther away than the solar and Arcturian stars of equal magnitude; or that, on the supposition of an equal distribution in space, and equal average real motion, the hydrogen stars are more luminous, a fact which is quite in agreement with our previous conclusions.

But a novel and unexpected result is the position of the Capellan or solar stars as being the nearest to us, while the Arcturian stars are intermediate between them and the hydrogen stars. Mr. Monck concludes that the Arcturian stars are not cooled-down Capellans, but we shall find that if the explanation I have suggested as to the higher temperature and greater luminosity of the hydrogen stars is correct, the apparently anomalous position of the Arcturian stars is readily explained.

I have intentionally confined myself to a detailed discussion of only two of Secchi's types of stars, but must now briefly refer to other celestial bodies, so that we may be able to obtain a general view of the evidence on which the theory of stellar evolution rests.

There are two kinds of nebulous bodies which may be distinguished by their spectra. One of them, of which the nebula in *Orion* may be taken as a specimen, shows us bright lines of hydrogen, of helium, and of some unknown substance. The second kind, of which the nebula in *Andromeda* is a conspicuous example, apparently give a continuous spectrum, which is weakened so much by spectroscopic dispersion that it is extremely difficult to form a definite judgment as to the nature of the light emitted. While Professor Scheiner believes that he has obtained by photography the absorption lines corresponding to the darkest groups of solar lines, the observations of Sir William and Lady Huggins seem to give very strong evidence of the presence of bright lines in the spectra of these bodies. It is to be hoped that

the matter may soon be cleared up, as these nebulæ include all those of spiral form, in which local condensations occur, suggesting a similarity to what we may suppose to be the origin of the formation of more compact celestial bodies. The gaseous nebulæ of the *Orion* kind are nearly all situated close to the Milky Way, while the nebulæ having the *Andromeda* character show the opposite behavior, and obviously avoid the plane of the Galaxy. Passing on to bodies which appear to be intermediate in character between stars and nebulæ, we also find them confined to the Milky Way. These bodies, as seen through a telescope, appear to be stars, but their light, when resolved by the spectroscope, shows bright lines, either alone or in conjunction with dark lines. Their distribution along the Milky Way is irregular, and they tend to cluster together in certain parts of it. These bright line stars vary to some extent in composition; they show as a rule the lines of hydrogen and some unknown lines. Helium appears in some, but not in all of them. There appears to be a gradual transition from these bright line stars to the helium stars, which also are chiefly found in the neighborhood of the Milky Way, and many of which are grouped together; thus all the bright stars of *Orion*, except *Betelgeuze*, and most of the weaker stars are helium stars. The high temperature of these helium stars is testified by the presence of oxygen lines, first identified by Dr. F. McClean, the particular spectrum of oxygen which appears in them being only obtainable in terrestrial oxygen by very intense sparks. The oxygen lines which, for instance, are found in the solar spectrum undoubtedly belong to a more complex molecule, and a lower temperature. There is again, apparently, a continuous transition from the helium stars to the hydrogen, calcium, solar, and Arcturian stars which I have described. The remaining types of spectra belong to lower temperatures still, as in place of the metallic lines, or in addition to them, certain bands appear, which experiments show us invariably belong to lower temperatures than the lines of the same element. Secchi's Type III possesses bands, as to the identity of which there is no consensus of opinion. These stars also cluster in the Milky Way, and a majority of them have the peculiarity of being variable in

intensity. The variations are of longer period than that of the *Algol* variables, and at the maximum the bright lines of hydrogen are seen in many cases. Lockyer holds that the carbon bands appear as bright lines, and this view, as far as I am able to judge of the evidence, is probably correct.

Nothing is known as to the reason of the light-variation, but among the causes which can produce the same effect at regularly recurring intervals there is only one which we can at present apply to celestial bodies with any show of reason, and that is the orbital revolution of the bodies around each other; but this question lies beyond the range of our present discussion.

The bands shown by Secchi's fourth type of spectra have been identified, and belong to carbon. The stars showing these spectra are all weak, but over 200 are known. Dunér, to whom we owe the first systematic investigation of these stars, has shown that they also congregate in the Milky Way, not only absolutely, but also relatively to the other stars. Mr. T. E. Espin has further investigated and confirmed this point. Out of a total of 224, the region within ten degrees of the Milky Way includes 123, or more than half, while 74 per cent. lie within twenty degrees of it.

We can only form vague guesses as to the manner in which matter was originally spread through space and has gradually condensed, probably, though not necessarily, through an intermediate nebular stage into the numerous luminous spherical bodies which we observe at night. If an evolutionary process has been going on, which is similar for all stars, there is little doubt that from the bright line stars down to the solar stars, the order has been: (1) helium or *Orion* stars, (2) hydrogen or Sirian stars, (3) calcium or *Procyon* stars, (4) solar or Capellan stars. The Arcturian stars are placed by most observers after the solar stars, but as mentioned above, the researches of Monck seem to bring them to an earlier stage of evolution. Opinions are divided as to the proper place to assign to the stars of the third type. There is, on the one hand, no definite boundary line between them and the Arcturian stars, the transition being gradual; the evidence of an evolution from the second or solar

to the third type is as strong as that which is generally recognized to indicate an evolutionary process from the Sirian to the solar type. On the other hand, the facts that these stars are nearly all variable, that they probably contain bright lines, and that they aggregate in the Milky Way, lend force to Lockyer's contention that the stars are in an early state of formation, and that the low temperature of their absorbing layer indicates that it is still rising in temperature. The carbon stars of the fourth type have been uniformly placed at the end of the succession of spectroscopic changes, though, according to many astronomers, this last stage does not succeed the third type stage, but follows the solar stage. These authorities would either, like Lockyer, remove the third type into the early features of a star's history, or derive it independently from the solar stage. According to this latter view, stars having reached the solar stage would bifurcate, and, according to their chemical composition, develop either the spectrum of the third or that of the fourth type.

The views I have expressed, and suggestions I have made in the previous pages, have led me to the following succession of events, as being in harmony with observed facts.

We may start from matter distributed with approximate uniformity through space, and leave out of account the question whether that original matter was in the form of our present elements, or in some primordial state, out of which our elements have been formed. If the latter case, the conditions must have been such that in different portions of space this primordial matter would condense into our elements, nearly, though probably not absolutely, in the same relative quantities. The formation of the elements I assume to take place simultaneously with the condensation into larger conglomerations.

The first point I want to draw attention to is that the effect of the first condensation must have been accompanied by a rejection of helium, hydrogen, and other light gases, because the development of heat which accompanies the early agglomeration, and raises the temperature of the gaseous bodies, must through the known laws of expansion increase their volume. The gravitation toward the condensed portions of matter not

being sufficient to retain the hydrogen, it will diffuse and tend to spread through the adjoining portions of space. In regions where there is no violent motion, it appears to me that matter will tend to concentrate itself around certain nuclei, which will begin to attract each other and clash together. A number of stars will then form, and these stars will at first not contain hydrogen or helium in appreciable quantities. These lighter gases will be left behind as nebulous masses, because it has been shown that unless the gravitation toward the center of a celestial body exceeds a certain value, light gases cannot form a permanent constituent of the atmosphere. This seems to be a not unlikely explanation of the gaseous nebulæ. These bodies are by observation found to be connected with stellar clusters, as has been shown by the remarkable photographs of the nebulous regions surrounding the *Pleiades*, and the spectroscopic investigation of the great nebula of *Orion*.

As soon as a star has grown sufficiently to be capable of retaining the hydrogen and helium, atmospheres of these gases will form around the stars, and the temperature of the photosphere will rise to its maximum. A process of diffusion of hydrogen and helium into the star will at once begin, and may be helped by the process of absorption which has already been alluded to. The helium will be retained first, as it is denser than hydrogen, and we may therefore expect to find helium stars showing the hydrogen lines either weakly or not at all. As a star grows in size the hydrogen will be more and more condensed on its surface from the outside, and we may get a considerable atmosphere of that gas forming around the star. The helium which has first condensed will also first diffuse toward the inside, and we then get the typical hydrogen star. The process of diffusion of the hydrogen, helped quite probably by an absorption due to molecular action, will continue to go on until a stage of equilibrium is reached, in which there is some, but possibly very little, hydrogen near the surface of the star.

It may be objected that the above explanation leaves out of account the peculiar behavior of calcium in the intermediate stage between the hydrogen and solar star. My answer is that

the same objection applies to all other explanations. We must at present accept it as a fact that there is a peculiar connection between some of the lines of calcium and the hydrogen line. This is shown by the phenomena which take place on the Sun, as strongly as by those which take place in the stars. The connection may be a chemical one, or may have a hitherto unsuspected origin. I have spoken of these lines which Fraunhofer designated by H and K as "calcium" lines, as so far we have only been able to obtain these lines when there was ground for the supposition that calcium was present. At the same time I do not think many spectroscopists would be surprised if it were found that these lines did not belong to the metal calcium at all. That was the opinion arrived at by Stas for reasons which to my mind are insufficient, but the opinion may ultimately turn out to be correct, even though for the moment experiment does not lend much countenance to it.[1] At present, therefore, we must content ourselves with accepting the peculiar behavior of these lines and their connection with hydrogen as an observed fact, but the acknowledged want of a sufficient explanation of the fact cannot be quoted as an objection to any particular view on the connection between different types of spectra.

A word may also be said as to the peculiarity of helium, which does not show its presence among the Fraunhofer lines, although it is known to be present in the Sun, because its lines appear bright in the chromosphere. There are two causes which may prevent a line from being visible as an absorption line: the vibration which gives rise to the line may be too nearly homogeneous, or it may be too intense.

It has never to my knowledge been pointed out, though it is obviously true, that an absolutely homogeneous vibration can never give rise to an absorption line in instruments of finite resolving powers. That is a possible, though perhaps not a very probable, explanation of our failure to detect helium among the

[1] (Note added February 1903.) The fact that the difference in the wave-numbers of H and K is nearly double that between the first and second line of the characteristic calcium triplets weighs strongly in favor of the lines being due to calcium. While writing the above I had for the moment forgotten this argument.

absorption lines. The second explanation is contrary to what is generally accepted as true, but only because writers are accustomed to consider the radiation from the solar photosphere to be the radiation of a perfectly black body. If the drops or liquid masses which form the photosphere have any power to reflect or scatter light, such as for instance we know rain drops to possess, the solar radiation need not, as regards intensity, exceed, say, half that due to a black body of the same temperature. It would then become quite possible for a *cooler* body in front either not to show any absorption lines, or even to show them as *bright* lines.[1] The absence among the Fraunhofer lines of the high temperature radiations observed in the chromosphere is probably due to this cause.

Returning now to the secular changes in the stars, I may point out the distinctive features of the views which I have suggested. These views, in the first instance, open out the possibility of much greater variations in their life-history than has generally been admitted. The amount of hydrogen, according to my present view, which a star is able to condense depends on its mass, and on the amount of hydrogen which happens to be present in the neighborhood. Whatever there was originally may already have been drawn toward a previously formed and bigger star. Hence the possibility that a star may form and never pass through the hydrogen stage. Even when the star has as small a density as γ *Leonis* probably has, it may give the spectrum of a solar star, simply from the want of a hydrogen atmosphere. On the other hand, there may be stars which, having attracted a large quantity of hydrogen, but being of comparatively small total mass and small size, are not able to absorb the gas completely, and may remain hydrogen stars without passing through the solar stage at all. Finally, the difference between the Arcturian and solar star may not be one of age at all, but of mass. If the Arcturian star is one which is bigger, it will be able to absorb the hydrogen more completely, and the final

[1] (Note added February 1903.) I have obtained interesting results by a mathematical discussion of the phenomena of radiation through a foggy atmosphere. These I hope soon to be able to publish in this JOURNAL.

state of equilibrium will be such that the hydrogen lines will be thinner than in the Capellan or solar star.

This seems a plausible explanation of the results of Mr. Monck, who finds the Arcturian star to have a smaller proper motion than the solar stars. They are, if my views are true, brighter though cooler, because their surface is larger, and hence, on the average, they are farther away. The theory, if I may call it so, also gives an explanation of a very curious fact, which I venture to think has not so far been satisfactorily accounted for. In the case of double stars, it is often found that the brighter one is yellow and gives a solar spectrum, while the smaller one is blue and gives a hydrogen spectrum. The larger one, though it may originally have attracted more hydrogen to itself, will be able to absorb it more rapidly, and thus pass through the stages of spectroscopic evolution more quickly.

There is a marked difference between this explanation and that advocated by Sir William Huggins, according to whom a star of small mass would run rapidly through the various stages of evolution. We both agree, of course, in the main fact, that a small mass would lose heat more rapidly, but, according to the views here put forward, there is a counterbalancing tendency in the fact that a large mass would absorb the hydrogen more quickly, and therefore show a more rapid tendency to pass from the state in which it gives a hydrogen spectrum to the state in which the metallic lines become prominent.

I will not discuss the question of the spectra of the third and fourth types, for the reason that we have not sufficient data to form any decided opinion about them. As regards the spectra of the third type, it has already been mentioned that opinions differ as to whether their position is anterior to the hydrogen or posterior to the solar star, and there are valid arguments on both sides. The carbon stars are really disconnected from the others, and though they may ultimately be found to have a place in a general system of classification, there is at present nothing to show that they are not *sui generis* and the result of condensation in a space where carbonaceous matter happened to

be abundant. It has already been mentioned that they seem to show bright lines, and therefore probably belong to an early, rather than a late, stage of condensation. I know that the great principle of uniformity will be quoted against any supposition that a particular class of stars is essentially different in its composition from others, but I believe, on the other hand, that the skies bear ample evidence of real differences in composition. There is only need to mention, for instance, the dark bodies which by their passage in front of companion stars, produce the variation of light of the *Algol* variables. The obscuring stars have a density considerably less than that of water, and, as their temperature is low, they must be composed of elements differing widely from those which make up the Earth or the Sun. I cannot therefore admit the validity of an argument based on the so-called law of uniformity, which has always proved a fallacious guide.

Examples are plentiful in the history of science where the law of uniformity might have been quoted, and has been quoted, in support of obsolete moribund theories. Thus the savage, knowing that fire can be made by intelligent hands, unconsciously applied the law of uniformity to conclude that the lightning which set fire to the forests was caused by intelligent beings, surpassing him in grandeur as much as a lightning flash surpassed the feeble fire he could strike himself. When he saw the Sun apparently moving in his orbit, he was compelled by the law of uniformity to conclude that an intelligent being must carry that body in a chariot. When the mediæval magician felt that everything in this world seemed created and centered around man; when, moreover, he saw the Moon obviously describe an orbit around the Earth, he could quote the law of uniformity against the Copernican doctrine, which needlessly removed the center of the universe from what to him seemed its evident position. Even in more recent times false analogies, and conscious or unconscious appeals to the law of uniformity, have been constant sources of deception.

We are led by pure reasoning, and without any consideration of imaginary laws, to consider the universe to be in the state of

a clockwork which is running down. We can form some idea guided by our experience and observation, how a star may have formed and may pass through its various stages to extinction; but to say that all stars must necessarily pass through the same stages, to conclude that *Sirius* will ever look like *Arcturus*, is to put ourselves in the position of one, who having discovered that there is a certain law which apparently connects the ages of children with their height, calls the law of uniformity to witness that a man who is five feet high is necessarily younger than one who measures six. If the law of uniformity had reigned at creation, there could have been no life, for there can only be uniformity in death; but if there were sufficient diversity of position, of mixture, and of composition, to allow of aggregations of matter culminating in the formation of worlds, we may be sure that we shall be able to trace that diversity in the present composition of the stellar system. The universe shows law, order, and regularity, but it refuses to be forced from birth to death through a single channel. There is uniformity no doubt, but it is a uniformity which at all times, and in all places, is relieved by endless variety.

A NEW VARIABLE STAR OF UNUSUALLY SHORT PERIOD.[1]

By G. MÜLLER and P. KEMPF.

IN the course of the zone observations for Part III of the Potsdam *Photometric Durchmusterung* it appeared that the two regular measures of the brightness of the star of the seventh magnitude *B. D.* $+56°$ 1400 ($\alpha = 9^h\ 36^m\ 44^s$; $\delta = 56°\ 24'.6$ [1900]) in 1899 and 1901 differed from each other by an amount greater than that considered permissible for this *Durchmusterung*. Although the revision observations in the period from April 19 to June 4, 1902, left no doubt as to the variability of the star, they nevertheless gave no indication as to the character of the variation. The measures were continued until the end of July, 1902, and later resumed after the appearance of the star in the eastern heaven, without our succeeding in detecting the character of the variation. It was not until the 13th of January of this year, when the star was several times observed during a period of three hours in the course of the evening, that a decline and rise of the light could be established and the time of minimum approximately derived as about $9^h\ 20^m$ Potsdam M. T. This showed that the light changes occurred in a comparatively short time, and the star was therefore observed on the same night at intervals of ten minutes until shortly before sunrise. A definitive conclusion as to the still somewhat doubtful character of the light-variation was reached through the observations of January 14, which were carried on without interruption from $4^h\ 48^m$ until $9^h\ 19^m$ Potsdam M. T. They furnished a complete view of the whole light-curve, and thus led to the discovery of a variable star with the extraordinarily short period of only four hours, the shortest so far known.

All of our measures of the new variable are summarized in tabular form below. The first six values are taken from the zone

[1] Translated from advance proofs, furnished by the authors, of a paper to appear in the *Sitzungsberichte der K. Akad. der Wiss. zu Berlin.*

observations for Part III of the *Potsdam Durchmusterung* in which the variable is compared with fundamental stars. Fundamental stars were also used for comparison at the next three observations on June 10 and 25, 1902. But from June 28, 1902, the near-by star *B. D.* + 54°1329 ($\alpha = 9^h\ 41^m\ 44^s$; $\delta = 54°\ 43'.7$ [1900]) served exclusively as the comparison star. For its magnitude we obtain from comparisons with fundamental stars the following ten values: 7.65, 7.73, 7.81, 7.77, 7.68, 7.82, 7.64, 7.63, 7.75, and 7.85, the mean of which is 7.73.

The first five columns of the following table contain successively the date of observation, the local sidereal time, the Greenwich Mean Time, the designation of the observer, and the magnitude of the variable as derived from the measures. The last three columns of the table will be explained later.

Date	Sid. Time	G.M.T.	Obs.	Mag.	C.	O.-C.	Epoch
1899, May 29	$15^h\ 16^m$	$9^h\ 55^m$	K	7.76	7.94	−18	−7942
1901, January 17	4 50	8 12	M	8.33	8.04	+29	−4358
1902, April 19	14 13	11 32	M	8.12	8.18	− 6	−1617
April 22	14 2	11 9	K	8.58	8.40	+18	−1599
June 2	15 29	9 55	M	7.89	7.90	− 1	−1354
June 4	15 22	9 40	K	7.97	7.90	+ 7	−1342
June 10	15 15	9 10	M	7.84	8.01	−17	−1306
June 25	15 47	8 43	M	8.18	8.29	−11	−1216
	16 3	8 59	K	8.19	8.17	+ 2	
June 28	16 39	9 23	M	7.87	8.05	−18	−1198
June 29	16 42	9 22	K	8.06	8.07	− 1	−1192
July 5	17 17	9 33	M	8.13	8.05	+ 8	−1156
	17 21	9 37	K	8.11	8.03	+ 8	
July 6	16 53	9 5	M	8.28	8.23	+ 5	−1150
	16 58	9 10	M	8.18	8.20	− 2	
July 12	16 38	8 27	K	8.36	8.55	−19	−1115
	16 46	8 35	M	8.29	8.57	−28	−1114
July 15	17 19	8 56	M	8.43	8.40	+ 3	−1096
	17 24	9 1	M	8.27	8.35	− 8	
July 16	17 7	8 40	M	8.44	8.57	−13	−1090
	17 17	8 50	M	8.51	8.48	+ 3	
	17 23	8 56	M	8.60	8.41	+19	
July 19	18 6	9 27	M	8.30	8.19	+11	−1072
	18 11	9 32	M	8.29	8.16	+13	
July 21	16 57	8 10	M	8.14	8.12	+ 2	−1061
	17 2	8 15	M	8.04	8.16	−12	
	17 7	8 20	M	8.21	8.22	− 1	
July 28	17 10	7 56	M	8.07	8.00	+ 7	−1019
	17 15	8 1	M	7.91	8.02	−11	
	17 30	8 16	M	8.09	8.11	− 2	
November 27	3 3	9 47	M	8.11	7.90	+21	− 287

Date	Sid. Time	G.M.T.	Obs.	Mag.	C.	O.-C.	Epoch
1902, December 11	$3^h 11^m$	9^h 0	K	8.18	8.04	+14	− 203
December 12	3 7	8 52	M	8.17	8.08	+ 9	− 197
	3 13	8 58	K	8.09	8.05	+ 4	
December 13	3 36	9 17	M	8.02	7.98	+ 4	− 191
	3 43	9 24	K	8.03	7.95	+ 8	
December 14	3 53	9 30	M	7.81	7.94	−13	− 185
	3 59	9 36	K	7.91	7.92	− 1	
1903, January 12	4 23	8 6	M	8.16	8.26	−10	− 12
	4 31	8 14	K	8.34	8.41	− 7	
January 13	3 26	7 5	M	7.90	7.93	− 3	− 6
	3 29	7 8	K	7.90	7.93	− 3	
	4 20	7 59	K	8.06	8.15	− 9	
	4 25	8 4	M	8.15	8.21	− 6	
	5 18	8 57	K	8.35	8.32	+ 3	− 5
	5 24	9 3	M	8.19	8.27	− 8	
	6 21	10 0	M	8.00	7.96	+ 4	
	6 28	10 7	K	7.99	7.94	+ 5	
	13 0	16 38	M	8.53	8.52	+ 1	− 3
	13 3	16 41	M	8.46	8.49	− 3	
	13 17	16 55	M	(8.63)	8.34	+29	
	13 28	17 6	M	8.34	8.25	+ 9	
	13 35	17 13	M	8.28	8.20	+ 8	
	13 47	17 25	M	8.04	8.13	− 9	
	13 59	17 37	M	8.05	8.06	− 1	
	14 12	17 50	M	8.03	8.00	+ 3	
	14 21	17 59	M	7.99	7.97	+ 2	
	14 33	18 11	M	7.86	7.93	− 7	
	14 44	18 22	M	7.93	7.91	+ 2	
January 14	0 20	3 56	M	8.27	8.12	+15	− 1
	0 30	4 6	K	8.20	8.22	− 2	
	0 35	4 11	K	8.35	8.29	+ 6	
	0 43	4 19	M	8.38	8.45	− 7	
	0 49	4 25	M	8.64	8.55	+ 9	
	0 55	4 31	K	8.57	8.58	− 1	
	1 4	4 40	K	8.55	8.51	+ 4	0
	1 8	4 44	M	8.55	8.47	+ 8	
	1 15	4 51	M	8.38	8.39	− 1	
	1 22	4 58	K	8.30	8.32	− 2	
	1 26	5 2	K	8.33	8.29	+ 4	
	1 35	5 11	M	8.16	8.23	− 7	
	1 40	5 16	M	8.31	8.19	+12	
	1 45	5 21	K	8.22	8.16	+ 6	
	1 51	5 27	K	8.09	8.12	− 3	
	1 57	5 33	M	8.00	8.08	− 8	
	2 3	5 39	M	8.09	8.05	+ 4	
	2 10	5 46	K	8.02	8.02	0	
	2 15	5 51	K	8.15	8.00	+15	
	2 22	5 58	M	7.99	7.98	+ 1	
	2 29	6 5	M	7.98	7.95	+ 3	
	2 36	6 12	K	7.90	7.93	− 3	
	2 44	6 20	K	7.89	7.91	− 2	
	2 51	6 27	M	7.83	7.90	− 7	
	2 59	6 35	M	7.82	7.90	− 8	
	3 6	6 42	K	7.91	7.90	+ 1	
	3 12	6 48	K	7.88	7.90	− 2	

Date	Sid. Time	G.M.T.	Obs.	Mag.	C.	O.-C.	Epoch
1903, January 14	3h 22m	6h 57m	M	7.95	7.91	+ 4	
	3 28	7 3	M	8.00	7.92	+ 8	
	3 34	7 9	K	7.94	7.93	+ 1	
	3 43	7 18	K	8.06	7.95	+11	
	3 49	7 24	M	8.04	7.97	+ 7	
	3 55	7 30	M	8.03	7.99	+ 4	
	4 1	7 36	K	8.05	8.02	+ 3	0
	4 7	7 42	K	7.96	8.04	− 8	
	4 14	7 49	M	8.10	8.08	+ 2	
	4 19	7 54	M	8.21	8.11	+10	
	4 23	7 58	K	8.10	8.14	− 4	
	4 38	8 13	M	8.42	8.33	+ 9	
	4 42	8 17	M	8.43	8.41	+ 2	
	4 44	8 19	K	8.45	8.45	0	
	4 52	8 27	K	8.59	8.56	+ 3	
January 17	0 31	3 55	K	8.02	8.09	− 7	17
	0 38	4 2	K	8.16	8.14	+ 2	
	0 44	4 8	M	8.19	8.19	0	
	0 50	4 14	M	8.40	8.28	+12	
	0 54	4 18	K	8.35	8.35	0	
	1 0	4 24	K	8.41	8.48	− 7	
	1 5	4 29	M	8.50	8.55	− 5	
	1 12	4 36	M	8.56	8.58	− 2	18
	1 15	4 39	K	8.66	8.56	+10	
	1 22	4 46	K	(8.74)	8.49	+25	
	1 25	4 49	M	8.37	8.45	− 8	
	1 31	4 55	M	8.37	8.39	− 2	
	1 35	4 59	K	8.36	8.35	+ 1	
	1 41	5 5	K	8.35	8.30	+ 5	
	1 46	5 10	M	8.24	8.26	− 2	
	1 52	5 16	M	8.24	8.22	+ 2	
	1 55	5 19	K	8.05	8.20	−15	
	2 2	5 26	K	8.12	8.15	− 3	
	2 5	5 29	M	8.10	8.13	− 3	
	2 11	5 35	M	7.90	8.10	−20	
	2 15	5 39	K	7.99	8.07	− 8	
	2 20	5 44	K	8.09	8.05	+ 4	
	2 25	5 49	M	8.00	8.03	− 3	
	2 31	5 55	M	8.09	8.00	+ 9	
	2 36	6 0	K	7.97	7.98	− 1	
	2 42	6 6	K	7.94	7.96	− 2	
	2 48	6 12	M	7.91	7.94	− 3	
	2 55	6 19	M	7.88	7.92	− 4	
	3 3	6 27	K	7.89	7.91	− 2	
	3 10	6 34	K	7.90	7.90	0	
	3 17	6 41	M	7.83	7.90	− 7	
	3 23	6 47	M	8.00	7.90	+10	
	3 31	6 55	K	7.87	7.90	− 3	
	3 39	7 3	K	7.96	7.91	+ 5	
	3 45	7 9	M	8.07	7.93	+14	
	3 50	7 14	M	7.95	7.94	+ 1	
	3 56	7 20	K	8.01	7.95	+ 6	
	4 2	7 26	K	7.93	7.96	− 3	
	4 14	7 38	M	8.15	8.01	+14	
	4 19	7 43	M	8.11	8.03	+ 8	

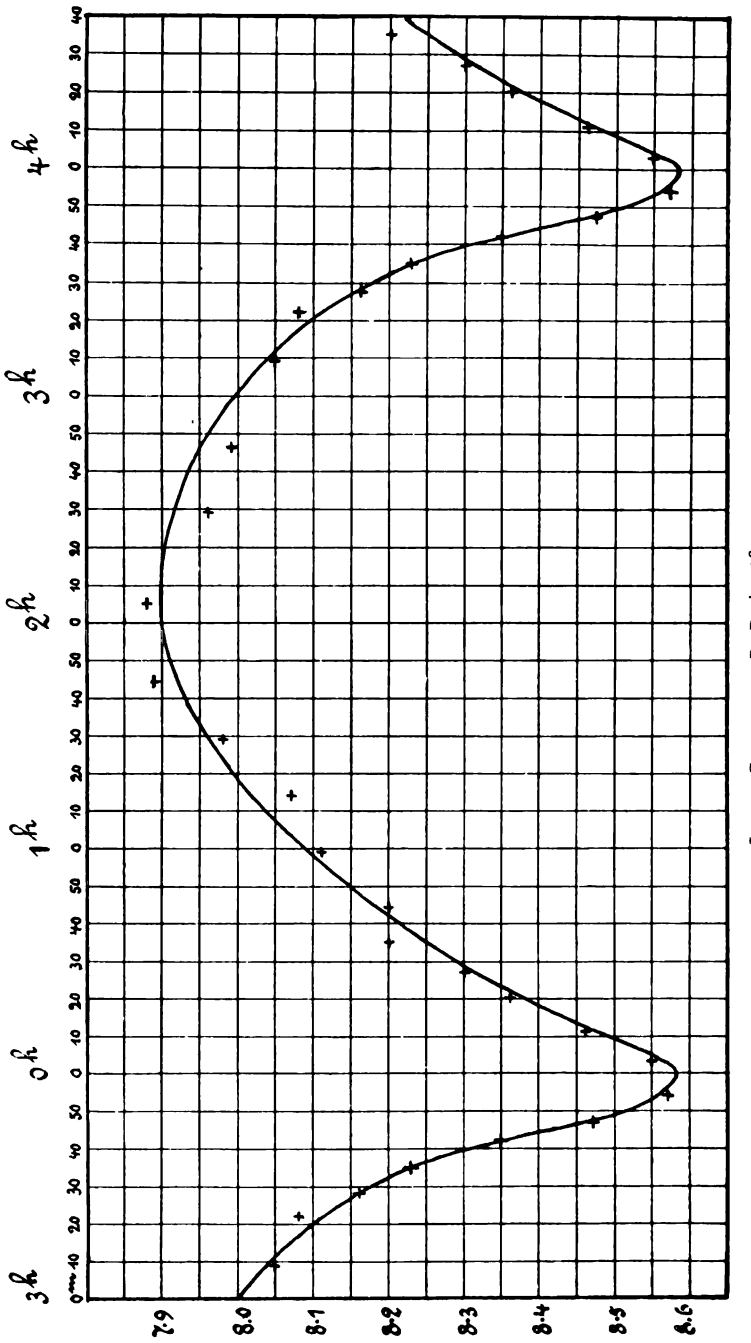
Date	Sid. Time	G.M.T.	Obs.	Mag.	C.	O.-C.	Epoch
1903, January 17	$4^h\,26^m$	$7^h\,50^m$	K	7.99	8.06	− 7	
	4 32	7 56	K	8.14	8.10	+ 4	
	4 37	8 1	M	8.08	8.13	− 5	
	4 43	8 7	M	8.28	8.18	+10	
	4 46	8 10	K	8.17	8.22	− 5	
	4 52	8 15	K	8.27	8.29	− 2	
	4 55	8 18	M	8.34	8.35	− 1	
	5 0	8 23	M	8.60	8.45	+15	
	5 3	8 26	K	8.58	8.52	+ 6	
	5 9	8 32	K	8.64	8.56	+ 8	
	5 13	8 36	M	8.61	8.58	+ 3	19
	5 18	8 41	M	8.37	8.54	−17	
	5 20	8 43	K	8.54	8.52	+ 2	
	5 26	8 49	K	8.34	8.45	−11	
	5 30	8 53	M	8.33	8.41	− 8	
	5 36	8 59	M	8.28	8.35	− 7	
	5 38	9 1	K	8.24	8.33	− 9	
	5 44	9 7	K	8.19	8.28	− 9	
	5 51	9 14	M	8.15	8.23	− 8	
	5 56	9 19	M	8.17	8.20	− 3	
January 18	4 30	7 50	M	8.01	8.06	− 5	24
	4 36	7 56	M	7.87	8.09	−22	
	4 39	7 59	K	7.99	8.11	−12	
	4 44	8 4	K	8.27	8.15	+12	
	4 48	8 8	M	8.07	8.18	−11	
	4 52	8 12	M	8.31	8.23	+ 8	
	4 55	8 15	K	8.24	8.28	− 4	
	5 1	8 20	K	8.36	8.37	− 1	
	5 4	8 23	M	8.45	8.43	+ 2	
	5 9	8 28	M	8.57	8.53	+ 4	
	5 12	8 31	K	8.45	8.56	−11	
	5 18	8 37	K	8.57	8.58	− 1	25
	5 21	8 40	M	(8.28)	8.56	−28	
	5 26	8 45	M	8.51	8.51	0	
	5 29	8 48	K	8.42	8.48	− 6	
	5 34	8 53	K	8.41	8.42	− 1	
	5 38	8 57	M	8.37	8.38	− 1	
	5 43	9 2	M	8.25	8.33	− 8	
	5 45	9 4	K	8.26	8.31	− 5	
	5 51	9 10	K	8.14	8.27	−13	

The graphical representation of the observations of January 14, 17, and 18 gave the following four minima, the uncertainty of which can be estimated at 10 minutes at most:

Jan. 14 - - - - $4^h\,34^m$ G.M.T.
17 - - - 4 40
17 - - - - 8 31
18 - - - 8 34

The combination of these data yielded the first provisional elements of the variable:

Min. = 1903 Jan. $14^d\,4^h\,34^m$ G.M.T. + $4^h\,0^m.0$ E.



LIGHT-CURVE OF *B. D.* + 56°1400.

The uncertainty of the period cannot in any case be assumed to be greater than 1 minute, whence the true value must lie between $3^h\ 59^m$ and $4^h\ 1^m$.

The observations of the year 1902 can be utilized for correcting the first approximation of the period. Since the brightness of the star at minimum is approximately 8.6, it is at once evident that the observations of April 22 and July 16 were made very nearly at the time of a minimum; the uncertainty could hardly amount to more than from 20 to 25 minutes. Combining these two dates together and with the minimum of January 14, it can easily be proved that only the four following periods can come into consideration, each of which can have a play of not more than ± 0.03 minutes:

$$4^h\ 0^m.65\ ;\ 4^h\ 0^m.21\ ;\ 3^h\ 59^m.77\ ;\ 3^h\ 59^m.33.$$

It may be further shown, with the employment of a light-curve provisionally derived by comparison with the remaining observations of 1902, that the first and the last two of the above four values are to be rejected within their whole range, because they yield inadmissibly large deviations between the computed and observed brightness. The second value only remains, and its application is limited to the range from $0^m.20$ to $0^m.22$. We therefore assume as the second approximation, after a slight displacement of the epoch, the elements:

$$\text{Min.} = 1903\ \text{Jan.}\ 14^d\ 4^h\ 32^m\ \text{G.M.T.} + 4^h\ 0^m.21\ \text{E.}$$

This formula was now used for constructing the light-curve of the variable from the measures of January 12–18, the epochs of minima being computed by it, and the differences of time of the separate data of observation as compared with the previous mimimum were formed. In all 143 measures were employed for the light-curve; three of these (distinguished in the table of observations by parentheses) were excluded, as they were clearly affected by somewhat large errors of observations, and would have influenced the result too strongly. The remaining 140 measures were arranged in order of their distance from a minimum and were finally combined in twenty mean values, each from seven measures. These normal values are contained in the following table:

NORMALS.

Distance from Minimum	Observed Magnitude	Curve	O.—C.
$0^h\ 3^m$	8.55	8.56	−1
0 11	8.46	8.48	−2
0 20	8.36	8.38	−2
0 27	8.30	8.31	−1
0 35	8.20	8.25	−5
0 44	8.20	8.19	+1
0 59	8.11	8.10	+1
1 14	8.07	8.02	+5
1 29	7.98	7.96	+2
1 44	7.89	7.92	−3
2 5	7.88	7.90	−2
2 29	7.96	7.92	+4
2 46	7.99	7.95	+4
3 9	8.05	8.04	+1
3 22	8.08	8.11	−3
3 28	8.16	8.15	+1
3 35	8.23	8.23	0
3 42	8.35	8.35	0
3 47	8.47	8.46	+1
3 54	8.57	8.56	+1

The accompanying light-curve was drawn with the aid of these values. The magnitudes read off from the curve for every five minutes are collected in the following table:

TABLE OF MAGNITUDES.

Distance from Minimum	Magnitude	Distance from Minimum	Magnitude	Distance from Minimum	Magnitude	Distance from Minimum	Magnitude
$0^h\ 0^m$	8.58	$1^h\ 0^m$	8.09	$2^h\ 0^m$	7.90	$3^h\ 0^m$	8.00
0 5	8.54	1 5	8.06	2 5	7.90	3 5	8.02
0 10	8.49	1 10	8.04	2 10	7.90	3 10	8.04
0 15	8.43	1 15	8.02	2 15	7.90	3 15	8.07
0 20	8.38	1 20	8.00	2 20	7.90	3 20	8.10
0 25	8.33	1 25	7.98	2 25	7.91	3 25	8.13
0 30	8.29	1 30	7.96	2 30	7.92	3 30	8.17
0 35	8.25	1 35	7.94	2 35	7.93	3 35	8.23
0 40	8.22	1 40	7.93	2 40	7.94	3 40	8.31
0 45	8.18	1 45	7.92	2 45	7.95	3 45	8.41
0 50	8.15	1 50	7.91	2 50	7.96	3 50	8.52
0 55	8.12	1 55	7.90	2 55	7.98	3 55	8.56
1 0	8.09	2 0	7.90	3 0	8.00	4 0	8.58

The magnitudes taken from this table are given, along with the observed magnitudes, in the above table of the normal values in the column entitled "Curve." The differences between observation and computation are given in the last column.

As may be seen from the table of magnitudes, and still better from the drawing, the light-variation proceeds very rapidly around the time of minimum, the curve at minimum almost forming an acute angle. The decline to the least brightness is somewhat steeper than the rise thereafter, the two branches not being entirely symmetrical. The maximum is far less sharply pronounced than the minimum, but the observations appear to exclude the possibility that the star remains at its greatest brightness without change for some time; wherefore it cannot be regarded as of the *Algol* type. It is somewhat striking that the normal values at about an hour before the maximum, and similarly some time afterward, lie in general below the curve. The impression is given of a short pause at these times in the increase or decrease of the light, and as if the curve ought to be drawn with two depressions. It cannot be proven without a much greater amount of observed data whether these irregularities are actually real, or are to be assigned to uncertainty or prepossession during the observations. We have paid no attention to them at present.

It should be remarked further that the observations up to this time give no indication of a different brightness at the even and the odd minima. Any irregularity in the intervals between every two successive minima is equally impossible of recognition.

The definitive table of magnitudes was further employed for more closely limiting the second approximation of the period. Here the first two observations of 1899 and 1901 could be employed, the one of which must lie at the time of maximum and the other not far from a minimum. Different trials showed that the most probable value of the period is included between $4^h\ 0^m.210$ and $4^h\ 0^m.220$, and in fact the sum of the squares of the residuals was least for the values $0^m.212$ and $0^m.214$. We adopted the mean from these values, and assume as the most probable elements of the new variable at the present time:

Min.$=$ 1903, January $14^d\ 4^h\ 32^m$ G. M. T.$+ 4^h\ 0^m\ 12^s.8$E.

The error of the period can hardly be more than $0^s.5$, and the correction to it cannot be expected for a number of months. The last columns in the table of measures show how all the

observations are represented by the above period. The magnitudes as taken from the light-curve are there given, together with the deviations between observation and computation. The last column contains further the number of the epoch of the minimum preceding the observations in question, reckoned from the initial epoch 1903, January 14. On the whole, the representation may be considered as satisfactory — among the 181 observations there is no deviation greater than $0^m.29$.

The most rapid oscillations in brightness among the variables hitherto known are exhibited by two stars in the star cluster ω *Centauri*, which is rich in variables; their periods are $7^h\ 11^m.4$ and $7^h\ 42^m.8$. *S Antliae* follows next with a period of $7^h\ 46^m.8$. Periods between 8^h and 9^h are found for several variables in that cluster. *U Pegasi* should be finally mentioned, the period of which is given as $5^h\ 32^m.2$ in Chandler's third catalogue, but which exhibits secondary minima according to Pickering's investigations (*Harvard Circular* No. 23), and has a period of $8^h\ 59^m.7$.

The discovery of the new variable raises the question of the cause of this exceedingly rapid light-variation. We might first think with Zöllner of a rotating body whose surface possesses a very unequal distribution of brightness as a consequence of an advanced stage of cooling. This view is opposed by the white color of the star, for we may assume a yellowish or reddish color for all stars which are strongly cooled off. Another natural supposition would be that of a figure widely departing from a sphere, perhaps a long ellipsoid or a body similar to one of Darwin's figures of equilibrium, which rotate about one of the lesser axes. This explanation, however, would meet with difficulties, because it can hardly be possible to represent the particular form of light-curve observed, especially the very rapid changes of brightness at minimum and the very slow changes around the time of maximum.

We may finally consider the hypothesis that the light-variation is produced by two celestial bodies almost equal in size and luminosity whose surfaces are at a slight distance from each other, and which at times almost centrally occult each other in their revolution. In this case the observed light-curve can be

almost exactly represented by computation. The fact that the difference of brightness between maximum and minimum is somewhat less than $\frac{3}{4}$ mag. would indicate that one body is a little smaller than the other, or that the occultation is not quite central. On this hypothesis we have only one difficulty, and the not inconsiderable one, as to whether such a system is mechanically possible and can remain stable for any length of time. But in spectroscopic binaries we have already come to know systems the existence of which formerly had to be considered as doubtful on similar grounds, and it would perhaps be possible by more exhaustive theoretical investigations to demonstrate also the permissibility of the assumption of still closer double stars.

POTSDAM,
February 1903.

THE SPECTROSCOPIC BINARY *o PERSEI*.[1]

By H. C. VOGEL.

THE ASTROPHYSICAL JOURNAL for April 1902 (**15**, 214) contains a communication by Mr. W. S. Adams as to certain spectroscopic binaries recently found at the Yerkes Observatory. The following five observations were given of the star *o Persei* ($\alpha = 3^h\ 38^m$; $\delta = 31^\circ\ 58'$):

1902, February 19	+ 134km
February 21	− 77
March 4	+ 128
April 2	− 117
April 3	− 4

A few plates of the spectrum of this star, taken by Dr. Eberhard with Spectrograph IV attached to the photographic refractor (32.5 cm) of the Potsdam Observatory, showed that *o Persei* could also be successfully observed with this telescope with the employment of the large dispersion given by Spectrograph IV. Thus far eighteen spectrograms have been obtained by Dr. Eberhard, with the assistance of Dr. Scholz, and I have undertaken their measurement and discussion.

The star is of the fourth magnitude, and has an ill-defined spectrum of Class I*b*, on earlier plates of which, taken with less dispersion, I was able to measure sixteen lines, chiefly due to hydrogen and helium.[2] Even with low dispersion the lines were very weak and obscure, particularly those of hydrogen. With the higher dispersion of Spectrograph IV there was visible, in the part of spectrum investigated (from λ 4315 to λ 4495), the hydrogen line $H\gamma$ as a faint brightening in the continuous spectrum. The helium line λ 4472, which was also visible and measurable, was very weak and broad; the helium line λ 4388, which was measurable on most of the plates, was similar to

[1] Translated from advance proofs, furnished by the author, of a paper to appear in the *Sitzungsberichte der K. Akad. zu Berlin*.

[2] *Publ. des Astrophys. Obs.*, **12**, 33.

λ4472. On some of the plates there were indications of the presence of a line somewhat less refrangible than λ4388, and of the *Mg* line at λ4481. This *Mg* line was indeed distinctly visible on several plates when they were viewed with a magnifying glass, but it disappeared under the higher magnification of the microscope. On some plates the helium line λ4388 (and also 4472) appeared fringed with bright edges, similar to the *Mg* line λ4352 and the hydrogen lines in certain stellar spectra. The measurements could therefore be made only upon the very broad, diffuse *Hγ* line and upon the two weak and broad lines of the clèveite gas at λ4388 and 4472. On account of the faintness of the *Hγ* line in the spectrum of *o Persei*, I could not here employ the method I had previously used with advantage of increasing the accuracy of the measurements of the broad and ill-defined *Hγ* line of most spectra of Class I by covering the line with a somewhat narrower pointer, which was then moved back and forth until the diffuse edges of the *Hγ* line were equally distant to the right and left of the pointer. For the same reason the double thread could never be used during the measurements.

Dr. Eberhard took the greatest pains to suit the exposure time to the atmospheric conditions, and gave especial care in the development of the plates. The exposures varied from 30 to 60 minutes, averaging 40 minutes. The slit-width was always 0.02mm.

I should further state that I derived the displacement of the lines in the stellar spectrum from measures of the distance of the three above-mentioned lines from neighboring lines of the *Fe* comparison spectrum. At least six settings were made on each star line, usually with a power of 20; the measurements were also often repeated with the use of different magnifications—from 10 to 35. The precaution was also always taken of not making the settings of the micrometer thread on the star line one after another, but with interruptions, in order that the eye should not be too greatly fatigued by the difficult measments, and in order to be free from any distinct *Auffassung* due to peculiarities of the photographic film. The smallest irregularities in the silver deposit may have a great effect on the weak

lines of the spectrum, and it is often difficult to obtain a correct *Auffassung*. To illustrate this further, I would say that with a power of 10 one of the lines on a certain plate appeared to be quite oblique to the direction of the length of the spectrum, while with higher powers, which clearly brought out the grain of the plate, it turned out that the impression of an oblique position of the line was produced by a small cross-mark which had been formed within the stellar line by the running together of several silver grains.

The table on p. 215 gives the velocities resulting from the displacements of the line $H\gamma$ (*a*), λ 4388 (*b*), and λ4472 (*c*).

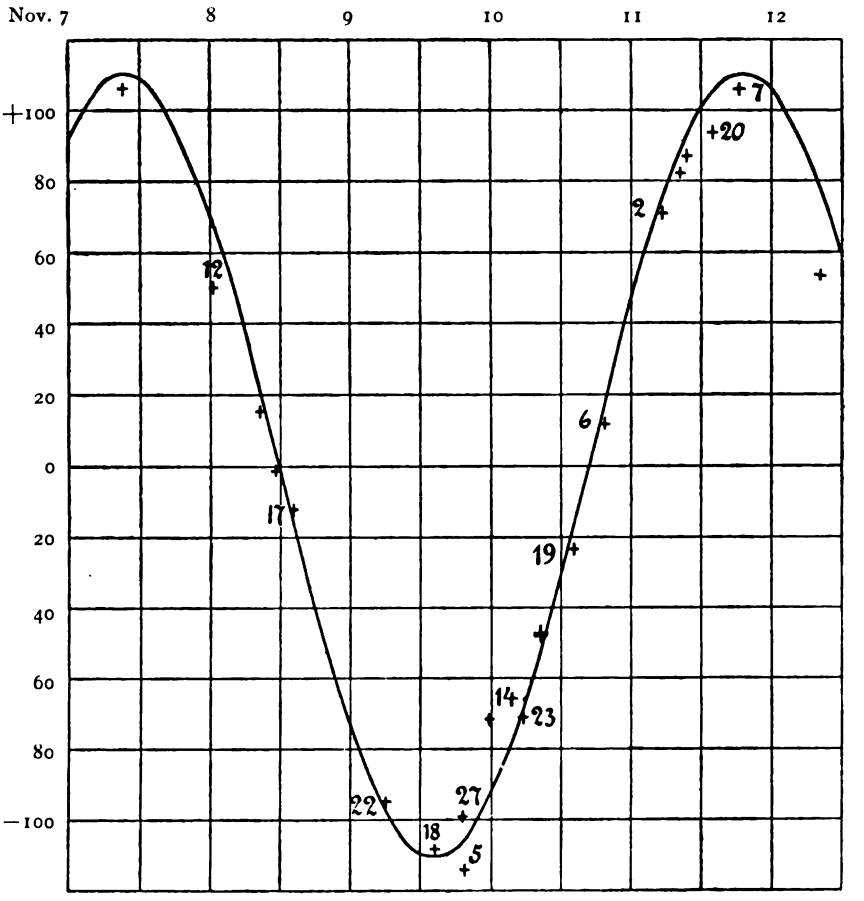
The arrangement of the table requires no further explanation. I have rounded the velocities off to kilometers, in conformity to the low degree of certainty of the measurements. A low relative weight was given to some of the measurements in the record of measures, which has been added in parentheses and was taken into account in forming the means. A: indicates less accuracy. In the last column 1 denotes very good, 4 bad.

On Plate No. 1269 a very fine double line is visible approximately at the position of the *Mg* line λ 4481; it would yield a velocity of -74 km. I regard this fine double line, however, as a defect on the plate, and have therefore not included that measure in the table. Something similar appears within the helium line λ 4472 on Plate 1271, where a fine line is visible somewhat less refrangible than the center of the line; but I must regard this also as a defect in the silver deposit. A velocity of -64 km would be derived from its measurement.

I now sought to combine the observations by a curve, and found that the period may be taken as 4.39 days. Its more accurate derivation will not be possible without further observations extending over a longer time. The assumption of 4.39 days has been sufficient, however, for the reduction of the different observations to the duration of the period, for the purpose of a more obvious graphical representation. It appeared further that there is no noticeable departure from a circular orbit, and the curve in the figure is therefore drawn, not so as to best include the observations, but to represent a circular orbit

Date (Central European Time)	Plate No.	Velocity Relative to Earth	Mean Values of the Velocity		Quality of Observation
			Rel. to Earth	Rel. to Sun	
1902, November $2^d.440$	1213	*a* + 51 km *c* + 71	+ 61 km	+ 71 km	2
5.431	1224	*a* −119 *b* −115 *c* −131	−122	−114	2
6.428	1228	*a* + 2 *b* + 12 *c* − 3	+ 4	+ 12	1–2
7.378	1234	*a* + 97 *b* + 87 *c* +113	+ 99	+106	1–2
8.374	1237	*a* + 14 *b* + 21 *c* − 9	+ 9	+ 16	2
8.462	1239	*a* 0 *b* + 17 *c* − 41	− 8	− 1	2
10.361	1242	*a* − 44 *c* − 65	− 55	− 49	2
11.354	1244	*a* + 85 *c* + 61(½)	+ 77	+ 82	3–4
11.401	1245	*a* + 85 *b* + 84 *c* + 77	+ 82	+ 87	1
12.413	1246	*a* + 47	+ 47	+ 52	3
14.355	1252	*a* − 70 *b* − 74 *c* − 80	− 75	− 71	2
17.363	1253	*a* − 28 *b* − 12 *c* − 1	− 14	− 12	1
18.379	1257	*a* −113 *b* −111 *c* −105	−110	−108	1
19.378	1262	*a* − 19 *b* − 24 *c* − 32(⅔)	− 24	− 23	1–2
20.364	1265	*a* + 77 *b* +104 *c* + 98	+ 93	+ 94	2
22.415	1269	*a* − 79 *b* −104: *c* −103	− 95	− 95	2–3
23.408	1271	*a* − 58 *b* − 79(½) *c* − 77	− 70	− 71	1–2
27.351	1282	*a* − 92 *b* −103 *c* − 93	− 96	− 99	1

computed with the above period and a maximum velocity of 110 km. The date of the transition from positive to a negative velocity is taken as November 8.50. There appears to be a



VELOCITY-CURVE OF o *Persei*.

The date is given for those observed points which were reduced to the portion of the curve from November 7 to November 12 with the period of 4.39 days.

slight difference indicated between the greatest positive and the greatest negative values, from which a negative value of several kilometers would follow for the motion of the system.

The accuracy of the observations, though indeed very small,

nevertheless turns out better than I had originally expected. On comparing the values for the single lines on a plate with the mean, the probable error of the velocity derived from one line on one plate comes out as ± 9 km; accordingly the probable error of the mean of the measures on one plate will be ± 5 km.

On a closer examination of the velocities derived from three lines on a plate the fact must nevertheless attract attention that, with the exception of those plates on which the lines exhibit a slight displacement, the velocities derived from the displacement of $H\gamma$ are almost always smaller, considered absolutely, than the mean value from the measures of the lines in the spectrum of the clèveite gas.

The derived velocities lie within 20 km on the four plates 1228, 1237, 1239, and 1253.

The velocities derived from the two helium lines are almost equal to that found from the $H\gamma$ line on the three plates 1234, 1245, 1257.

The $H\gamma$ line gives distinctly less values than the other lines on the nine plates, 1213, 1224, 1242, 1252, 1262, 1265, 1269, 1271, and 1282. The deviation averages 13 km.

The first exposure on November 11, No. 1244, may be excluded as too weak, and on Plate 1246 the motion is derived solely from $H\gamma$. The departure of the value from the curve indicates again, however, that it is too small.

If we accept these deviations as real it follows first that after the origin of this difference has been discovered, and it has been taken into consideration, the probable error of the measures would come out still smaller than is given above. But it is of much more importance that the discovery of the occurrence of this anomaly permits a further insight into the binary system under investigation, as I shall show below.

The fact that a periodic doubling of the lines is not perceptible might lead us to assume that one of the bodies of the system is dark, but the marked weakness of the lines of the spectrum of the clèveite gas leads us to conclude, however, that a second spectrum is superposed upon it. The broad and diffuse hydrogen lines, moreover, do not correspond to the typical spectrum of Class I*b* and lead to the further assumption that the superposed spec-

trum must belong to Class I*a*2.[1] The very delicate metallic lines which appear in this class of spectrum along with the broad and diffuse hydrogen lines, disappear completely as the result of the superposition of this spectrum by the spectrum of Type I*b* of the rapidly revolving body. On these assumptions it is now very easy to explain why the measures on $H\gamma$ yield smaller velocities than those on the cleveite lines. The narrower and less diffuse line of the I*b* spectrum broadens and strengthens the maximum of absorption of the very broad and diffuse $H\gamma$ line of the I*a*2 spectrum when the superposition is absolute. When the two spectra are relatively displaced to each other the line of the I*b* spectrum remains within the broad line of the other spectrum, but the intensity curves of the two lines are so added together that there results a broad maximum, unsymmetrically placed with respect to the center of the combined image of the two lines. The measure of the $H\gamma$ line of the periodically displaced I*b* spectrum is therefore affected by the $H\gamma$ line of the I*a*2 spectrum, and this occurs in such a manner that the measures of the displacement, regarded absolutely, come out too small. The extent to which this occurs naturally depends wholly on the relative intensity of the absorption lines of the two spectra and on the quality of the spectrogram. In spite of the faintness and breadth of these absorption lines it is to be assumed that two maxima could be perceived in the compound $H\gamma$ line at the time of the greatest displacement if both components of the double star are strongly displaced. Even with a less motion of the second star the effect on the *Auffassung* of the compound $H\gamma$ line would have to be stronger than was actually the case. The assumption therefore seems admissible from the observations that the center of gravity of the two bodies must lie very close to the star with the I*a*2 spectrum, or even within it.

After being led to these considerations by the observations I re-examined all the plates of the spectrum of *o Persei* and found

[1] The width of the hydrogen line $H\gamma$ is about 1.0 rev., that of the helium line 0.3 rev. One revolution of the screw of the measuring machine is 0.25 mm, and the displacement of this amount corresponds to a velocity of 290 km at $H\gamma$, or 340 km at λ 4472.

a very good confirmation of the suspicions above expressed. With a strong positive velocity the brightest point (on the negative) lay unsymmetrically to the center of the *Hγ* line, and the line was more diffuse on the side toward the violet, as was clearly seen on Plates 1213, 1234, 1245, 1265. With strong negative motion *Hγ* was more diffuse toward the red, distinctly on Plates 1224, 1252, and 1282, and not quite certainly on Plates 1257 and 1271. With the small motion the brightest point (on the negative) in the *Hγ* line was exactly symmetrical to the center of the line; the intensity of the absorption was distinctly greater than on plates at other times, particularly on Plates 1228, 1237, 1239, 1253, and 1262; also with a motion of −40 km on Plate 1242, the *Hγ* line appeared as on the above-mentioned plates.[1]

Taking 110 km as the greatest velocity, and assuming that the center of gravity of the system lies within the one body, and adopting the period 4.39 days, the distance of the two bodies is computed to be $\frac{6640000 \text{ km}}{\sin i}$, and the mass of the system to be $\frac{0.6 \odot}{\sin^3 i}$, where i is the angle included between the normal to the orbital plane to the system and the line of sight. I have taken for the greatest velocity the value obtained from the curve and also that directly observed. According to the considerations cited above, the mean values from three measured lines might be increased by about 5 per cent., which would increase the maximum values by about 5 or 6 km. If we base our computation upon a maximum velocity of 115 km instead of 110 km we obtain for the distance of the two bodies $\frac{6940000}{\sin i}$, and for the mass of the system $\frac{0.7 \odot}{\sin^3 i}$.

[1] I ought not to omit to mention that in my previous investigation of the motion of *α Virginis* I was led to similar considerations. *Pub. des Astrophys. Obs.*, 7, 139.

THE MASS OF 85 *PEGASI.*

By GEORGE C. COMSTOCK.

THIS is an interesting binary star ($\alpha = 23^h\ 56^m.8$; $\delta = +26°\ 33'$), discovered by Burnham in 1878, and soon recognized as possessing an unusually rapid orbital motion. Despite the large difference of magnitudes between its components, 5.7 and 11.3, and their small angular separation — the distance is always less than a second of arc — the star has been well observed, and fairly accordant orbits have been obtained for it by four different computers. The latest of these, by Burnham, assigns to the star a periodic time of revolution of 25.7 years and a semi-axis major of $0''.78$. There is a ninth-magnitude optical companion to the binary whose relative position was first measured by Otto Struve in 1851 and which has been well observed since that time, with the exception of one serious gap between 1852 and 1865, as shown in the following table.

I have recently discussed the comparisons of this star with 85 *Pegasi* to determine the relative proper motion of the stars and have been thus led to consider the possible motion of the bright component of 85 *Pegasi* with reference to the center of gravity of its system. The data of the problem are the observed position angles and distances, θ and ρ, of the ninth magnitude star, C, referred to the bright star, that I shall call A, and similar data, p and s, for the faint companion, B, of the binary system. These latter co-ordinates I have derived from Burnham's apparent orbit[1] for the dates of the available observations of C. I desire here to emphasize the great interest and value of these apparent orbits and to express the wish that a cut showing the apparent orbit might always be published in connection with orbit determinations of binary stars.

For the discussion of the observation we may put

μ = the difference of declination, at an assumed epoch, between C and the center of gravity of A and B.

[1] *Publications of the Yerkes Observatory*, 1, 270.

ν = the relative proper motion of C and the center of gravity above defined.

τ = the time interval between the given observation and the epoch 1885.54.

k = the fractional part of the distance A — B included between the center of gravity and the star A.

Transforming polar to rectangular co-ordinates, we find for the co-ordinate parallel to the hour circle,

$$\mu + \tau\nu + s \cos p \cdot k = \rho \cos \theta,$$

with a similar equation for the co-ordinate perpendicular to the hour circle,

$$\mu' + \tau\nu' + s \sin p \cdot k = \rho \sin \theta.$$

For the determination of the five unknowns in these equations I have employed the following observations, in which the posi-

OBSERVED CO-ORDINATES, A, C.

Date	Nights	θ	ρ	W	Observer
1851.96	1	114°3′	33.03	0.5	OΣ
52.67	1	113 51	32.60	0.5	OΣ
65.91	1	92 9	18.89	0.5	OΣ
68.77	1	82 24	17.03	0.5	OΣ
70.00	44	77 1	16.06	2	Brünnow
74.66	1	54 24	13.92	0.5	OΣ
76.77	1	40 18	14.02	0.5	OΣ
78.54	4	33 36	14.40	1	β
78.74	1	32 48	14.76	0.5	Dembowski
79.27	8	30 24	14.96	1	β
80.57	4	25 0	15.41	1	β
81.54	4	20 48	16.29	1	β
81.88	1	19 48	16.54	0.5	Bigourdan
82.62	1	15 12	16.98	0.5	OΣ
82.77	3	17 6	17.34	1	β
83.54	1	11 18	17.34	0.5	Seagrave
86.24	3	7 36	19.84	1	HΣ
86.99	3	6 6	21.15	1	Englemann
88.67	5	0 54	21.71	1	β
89.50	4	358 42	22.66	1	β
89.82	2	358 24	22.70	1	Leavenworth
90.52	3	356 42	23.59	1	β
91.56	3	354 42	24.58	1	β
91.94	8	354 18	25.02	1	β
95.06	1	350 0	28.86	0.5	Lewis
95.69	3	348 42	29.27	1	Aitkin
96.75	2	347 48	30.48	1	Aitkin
97.56	2	346 6	31.49	1	Aitkin
97.82	2	345 42	31.74	1	β
98.49	3	344 24	32.53	1	β
98.69	2	344 27	32.90	1	Aitkin
1902.73	3	341 19	37.64	1	Comstock

THE NEW GASES NEON, ARGON, KRYPTON, AND XENON IN THE CHROMOSPHERE.

By S. A. MITCHELL.

FROM a historical point of view, D_3 is one of the most important lines in the spectrum. When examining in April 1895 the spectrum of a specimen of clèveite, Ramsay announced the discovery of a substance that gave the characteristic helium line, it was felt that a great triumph had taken place for the methods of the "new astronomy." About the same time Rayleigh and Ramsay discovered another new element, argon, and in the early summer of 1898 Ramsay found two more elements, krypton and neon, and subsequently a heavier gas to which the name xenon was applied. Making use of the extremely low temperatures of liquid air and liquid hydrogen, it was found that these five new gases were present in atmospheric air. Investigations of the properties of these gases[1] seem to indicate that the atomic weights are: helium, 4; neon, 20; argon, 40; krypton, 82; xenon, 128; and that they form a series in the periodic table between that of fluorine and that of sodium.

The lines of helium are such prominent ones in the chromospheric spectrum that it would be interesting to see if the other new gases are also present in the chromosphere, and accordingly comparisons have been made between the spectra of the "flash" and of the new elements whose wave-lengths have lately been published.

The flash spectrum was photographed by the writer at the Sumatra eclipse,[2] with an apparatus consisting of a Rowland plane grating of 15,000 lines per inch, having a ruled surface of $3\frac{1}{2} \times 5$ inches, and a quartz lens, the whole mounted so as to give a normal spectrum. The photographs — like most of those at this eclipse — were made through clouds. These poor

[1] RAMSAY and TRAVERS, *Proc. R. S.*, **67**, 329, 1900.

[2] ASTROPHYSICAL JOURNAL, **15**, 97, 1902.

weather conditions, however, did not interfere with the spectrum as much as was expected, which is shown by the fact that 374 lines were measured between F and H. The dispersion employed was about one-fifth that of the largest Rowland concave gratings of 21½ feet radius, and 20,000 lines per inch, and about equal to the dispersion of the Lick and Yerkes spectrographs.

Comparisons of the *intensities* of the lines, and of the *numbers* of the lines due to the different elements in the flash and in the solar spectrum, as given by Rowland, led to the division of the elements into three groups (*loc. cit.*, Table V): those giving (1) lines strong in the flash and strong in the solar spectrum; (2) lines strong in the flash, weak in the solar spectrum; and (3) lines weak in the flash, but strong in the solar spectrum.

To the second group belong *H*, *He*, *Sc*, *Ti*, *V*, *Cr*, *Mn*, *Sr*, *Y*, and *Zr*. It has been shown[1] that helium, in consequence of its small density, ascends to great heights above the Sun's surface, and as its layer is covered up gradually by the Moon at the time of an eclipse, the resulting exposure is many times that given to denser but shallower layers, and consequently helium lines in the flash spectrum are very prominent. Taking into account the increasing atomic weights of the series of new gases, and also the behavior of the light and heavy vapors in the Sun's atmosphere, as found out by investigations of the flash, we should expect, as in the case of helium, none of these gases to be found in the ordinary solar spectrum. We should also expect lines of the more volatile gases of the Earth's atmosphere, neon and argon, of atomic weights 20 and 40 respectively, to be present in the flash, while those of the less volatile gases, krypton and xenon, of atomic weights 82 and 128 respectively, are most probably not to be found there.

No lines of these gases appear in the ordinary solar spectrum, but if we make a detailed comparison of their spectra with that of the flash, there seem to be certain lines of the latter that are undoubtedly due to these new gases of the atmosphere.

The most volatile of these were obtained[2] from their solution in liquid air by fractional distillation at low pressure, in

[1] *Ibid.*, p. 117. [2] RAMSAY and TRAVERS, *Proc. R. S.*, 67, 329, 1900.

this way removing the greater portion of the helium and neon from this mixture of gases, leaving the argon behind. Many attempts were made to separate the helium from the neon, which were not successful until these gases were subjected to the temperature of liquid hydrogen, when neon was liquified and perhaps solidified, while the helium remained gaseous.

The more volatile gases of atmospheric air uncondensed at this temperature have been examined spectroscopically by Liveing and Dewar, and wave-lengths have been published.[1] In this spectrum appear lines due to neon, to helium, and to free hydrogen in the Earth's atmosphere. These wave-lengths, given, however, only to the nearest Ångström unit, have been compared with those of the lines of the flash spectrum.[2] Although it is difficult to identify with certainty when wave-lengths have no greater accuracy than this, it seems highly probable that nearly all the stronger lines of the most volatile of the new gases are found in the flash, some of them agreeing with lines already identified as corresponding to Fraunhofer lines (in this case it being impossible to separate the lines), other lines identified with lines that have no counterpart in the ordinary solar spectrum. None of the lines of the flash due to these more volatile gases are strong lines like those of helium, their intensities being 0 on a scale where 0 denotes a line seen with certainty and 10 is the intensity of the strongest line. Lines in the flash which seem to belong to these gases and to no others are those at λλ4047, 4398, 4422, 4431, 4540, and 4844.

Several tables of wave-lengths of argon have been published. Kayser employed a large concave grating and published[3] his results to thousandths of a tenth-meter. Most of the strong lines of argon are found in the flash spectrum—but again as weak lines only. Argon lines appear at λλ4180.3, 4200.8, 4259.5, 4266.8, and 4430.3.

The most accurate wave-lengths of krypton are those of Runge, measured with a concave grating and given[4] to thousandths of a tenth-meter. The strongest lines are in a part of

[1] *Ibid.*, p. 467.

[2] ASTROPHYSICAL JOURNAL, **15**, 103, 1902.

[3] *Ibid.*, **4**, 1, 1896.

[4] *Ibid.*, **10**, 73, 1899.

the spectrum not covered by the flash photographs, and there seem to be no krypton lines in the flash.

The only wave-lengths of xenon are those of Liveing and Dewar,[1] given only to the nearest tenth-meter. Some of the strongest of the xenon lines do not appear in the flash, while the wave-lengths of some less strong seem to agree with those of flash lines; but as the wave-lengths are not accurate enough, it is impossible to say more than that the presence of xenon in the Sun's atmosphere is doubtful.

Consequently, it seems that the more volatile gases of atmospheric air uncondensed at the temperature of liquid hydrogen, together with hydrogen, helium, neon, and argon, are present in the chromosphere, while the evidence in regard to krypton and xenon is inconclusive.

The finding of these gases in the Sun and the undoubted presence of free hydrogen in the Earth's atmosphere have an importance for cosmical physics that can hardly be overestimated. According to Liveing and Dewar, "if the Earth cannot retain hydrogen nor originate it, then there must be a continued accession of hydrogen to the atmosphere (from interstellar space), and we can hardly resist the conclusion that a similar transfer of other gases must also take place,"[2] as has been shown by these distinguished physicists, and again by Dewar in his presidential address before the British Association for the Advancement of Science, these new gases, and particularly the more volatile gases of atmospheric air, play an important part in the spectra of the aurora, of nebulæ, and of the corona. Of more than a hundred auroral rays observed by Stassano, more than two-thirds appear to belong to the more volatile gases of atmospheric air, while the majority of the remainder seem to belong to argon, krypton, and xenon. We are also told by Dewar that of a "list of 339 lines photographed by Humphreys, during totality" (this, however, was called the spectrum of the corona, whereas it was the spectrum of the chromosphere), "only 55 do not differ by more than one unit on Ångström's scale from lines measured in the most volatile gases of the atmosphere, or in

[1] *Proc. R. S.*, **68**, 389, 1901. [2] *Ibid.*, **67**, 468, 1900.

krypton or xenon." It seems rather to the present writer that the great majority of these lines more closely correspond to Fraunhofer lines than to the lines of these rare gases.

These gases may take their origin from the Earth itself; in fact, helium and neon are occluded from the waters of the Bath spring in England. The presence of free hydrogen in the atmosphere cannot be explained in this way. It is more likely that hydrogen comes to us in small ionized particles from the Sun, being sent hither, as has been shown by Arrhenius,[1] by the pressure of light, and likewise helium and the more volatile gases are present in the atmosphere through being repulsed from the Sun by the ionization of small particles of these gases.

It seems therefore that the finding of these new gases in the Sun's chromosphere is an independent verification of the truth of the theory of Arrhenius, which states that particles of matter are being continually scattered throughout the universe, starting from one sun and reaching another, with the result that all bodies of the universe are gradually becoming more and more alike.

COLUMBIA UNIVERSITY,
New York City, February 1903.

[1] *Physikalische Zeitschrift*, November 1900; see also COX, *Popular Science Monthly*, January 1902.

ON THE OCCURRENCE OF SPARK LINES IN ARC SPECTRA.[1]

By J. HARTMANN and G. EBERHARD.

IN November of last year Dr. H. Konen published in the *Annalen der Physik* (**9**, 742 ff., 1902), some very valuable researches which he had made on the spectra of electrical discharges in water. This induces us to communicate briefly in what follows the results of investigations in the same field which we carried out last autumn.

In the first place it should be remarked that our observations confirm those of Konen even to details. We have, however, observed some additional phenomena not described by Konen, which seem to be of importance in the interpretation of the processes of electrical luminosity.

Our experiments at first were confined to a thorough study of the spectra of magnesium and silicon, which are of special importance in astrophysics. While we were varying as much as possible the conditions under which these elements were made luminous, we made use, among other things, of the electric arc under water. Here we noticed with surprise that in the spectrum of the arc there appeared lines which had hitherto been regarded as peculiarly characteristic of the spark spectrum.

Konen also has shown for a number of metals that lines in the spectrum of the metallic arc under water are not superposed on a strong continuous spectrum like the spark spectrum produced under the same conditions, but appear only as bright and generally sharp lines; the appearance in the arc under water of lines characteristic of the spark spectrum, however, has not to our knowledge been known hitherto.

When the arc is passed under water between two electrodes of metallic silicon, the spark lines $\lambda 4128$, and $\lambda 4131$ appear with almost the same intensity as does the chief arc line $\lambda 3905$. Under the same experimental conditions magnesium shows the

[1] Translated from advance proofs, furnished by the authors, of a paper to appear in the *Sitzungsberichte der K. Akad. zu Berlin.*

line at λ 4481, hitherto regarded as characteristic of the spark spectrum, and indeed it makes almost the strongest line in the whole spectrum; at any rate, its intensity considerably exceeds the arc line of magnesium at λ 4352. Both elements further exhibit the noteworthy phenomenon that the lines mentioned, which are known only as very broad and weak lines in the spark discharge in air, are considerably narrower and become more sharply defined, despite their intensity, when produced by the arc under water.

In order to ascertain whether the phenomenon described above was general, the experiments were also extended to other metals, and we were able to produce results entirely analogous with zinc and cadmium. With zinc the very diffused spark lines λ 4911 and λ 4924, when produced by the arc under water appeared strong and relatively sharp. The lines λ 5339 and λ 5379, which Kayser and Runge failed to find in the arc spectrum of cadmium, together with λ 4416, have also been easily obtained by us with the arc under water. We found the aluminium lines λ 4513 and λ 4530 less strong.

Carbon alone remained essentially anomalous, giving, with the arc under water, as Konen also found, no lines, but a band spectrum only.

We now suspected, in connection with a remark by Schenck[1] that the temperature of the electrodes and metallic vapors, no doubt much reduced by the water, might have an influence on the nature of the spectrum, and we thereupon greatly cooled zinc electrodes with liquid air, without obtaining, however, any material change in the arc spectrum. We then allowed sparks from an iron electrode to pass over to zinc placed in a crucible. While the zinc was heating to the melting-point and beyond, several observers (Vogel, Müller, Kempf, Hartmann, Eberhard) estimated the relative intensity of the pair of lines at λ 4911 and λ 4924 as compared with the three lines at λ 4680, 4722, 4809, and it appeared in every case that the intensity of these three lines was greatly augmented in comparison with the pair at λ 4911 and λ 4924 as soon as the zinc was heated.

[1] ASTROPHYSICAL JOURNAL **14**, 131.

In the success of this experiment we might see a confirmation of the above-mentioned conjecture that with ascending temperature of the electrodes the spectrum of the spark approached that of the arc. But it is not improbable that the observed phenomenon occurred merely through the increase of the metallic vapor due to heating the zinc, whereby, on account of the increase of these vapors, the resistance in the path of the spark became less and the spark lines accordingly gave place to the arc lines.

Since an answer to the question entirely free from objection cannot therefore be obtained by these means, we have endeavored to find an explanation in another direction. From former researches (Crew, Basquin) it is known that an atmosphere of hydrogen influences the arc spectrum of magnesium in such a manner that the spark line $\lambda 4481$ stands out prominently. Accordingly we have photographed the arc spectra of a series of metals in a current of hydrogen. It now appeared that spectra obtained in this manner are in fact almost identical with the arc spectra under water, and we are therefore of the opinion that hydrogen, released by electrolysis around the electrodes in water, causes the transformation of the arc spectrum into the form observed by us.

It should be observed that all lines mentioned by us in the foregoing are diffuse and hazy in the spark spectrum, and that it is also just these lines which can be made to disappear in the spark spectrum by the introduction of suitable self-induction, as we have convinced ourselves by our own experiments.

It follows from our investigations that it is not permissible to set apart individual lines of the spectra of metals as characteristic of the spark or of the arc respectively, and from their appearance to draw conclusions regarding the temperature of the corresponding processes of luminosity. The latter point applies particularly in the interpretation of stellar spectra, for which some have sought to make positive statements as to the temperature of stellar atmospheres, based upon the behavior of individual magnesium and silicon lines.

POTSDAM.
February 1903.

THE POSITION OF RADIUM IN THE PERIODIC SERIES ACCORDING TO ITS SPECTRUM.

By C. RUNGE and J. PRECHT.

THE spark spectrum of radium can be admirably observed with the radium bromide recently produced by Mr. Giesel. A few milligrams, which Mr. Giesel was kind enough to place at our disposal for this purpose, was sufficient with a low dispersion for obtaining the spectrum in a much more complete manner than has hitherto been observed, and with greater disperson the readily photographable lines could be investigated as to their behavior in a magnetic field. We thus found that the strongest radium lines are precisely analogous to the strongest lines of barium and the corresponding lines of the related elements *Mg*, *Ca*, and *Sr*. As has been shown by Runge and Paschen,[1] these lines can be grouped as three pairs, which, on account of certain analogies with the spectra of the alkalies, may be designated as the pair of the principal series, the pair of the first subordinate, and the pair of the second subordinate series. In the case of the pair of the first subordinate series there appears along with the line of the greater wave-lengths a fainter line on the side of longer wave-lengths, which Runge and Paschen designate as a satellite. The two lines of each of these three pairs have the same distance for each element, reckoned on the scale of vibration numbers, except that for the pair of the first subordinate series the satellite must be taken instead of one of the lines. But from element to element the distance is, on the contrary, different, increasing in a very regular way with increasing atomic weight, as will be discussed more particularly below. In the magnetic field these lines are resolved into components in a different way, as Runge and Paschen have shown, but so that, reckoned on the scale of vibration numbers, the separation of each line of one element is exactly the same as the separation of the *corresponding* line of each of the other elements.

[1] ASTROPHYSICAL JOURNAL, **16**, 123, 1902.

We have now found that precisely the same thing holds good for radium, so that *Ra* is to be placed with *Mg*, *Ca*, *Sr*, and *Ba* in a group of chemically related elements, as is also demanded by the chemical behavior of radium, in so far as this is known.

The corresponding lines are collected together in the following table:

	Mg	*Ca*	*Sr*	*Ba*	*Ra*
Principal series.....	2803	3969	4216	4934	4682
	2796	3934	4078	4554	3815
First subordinate series	....	3181	3475	4166	4436
	2798	3179	3465	4131	4341
	2791	3159	3381	3892	3650
Second subordinate series	2937	3737	4306	4900	5814
	2929	3706	4162	4525	4533

The separation of the radium line in the magnetic field is very well observable in case of the strongest lines; as yet we have been unable to resolve only the satellite and the line in the green at λ 5814.

As remarked above, the distances between the two lines of the pairs are the same for every element on the scale of vibration numbers. This is also true for radium, as is shown in the following table:

	λ	$\frac{10^8}{\lambda}$	Distance
Principal series.....	4682.35	21356.8	4858.3
	3814.59	26215.1	
First subordinate series.....	4436.45	22540.5	4858.5
	3649.77	27399.0	
Second subordinate series.....	5813.9	17200.2	4858.6
	4533.33	22058.8	

The deviations of the three distances from each other are sufficiently explained by the errors of observation. They correspond to very small errors in determinations of wave-length.

For *Mg*, *Ca*, *Sr*, and *Ba* the distances increase with the atomic weight from element to element:

	Atomic Weight	Distance
Mg	24.36	91.7
Ca	40.1	223
Sr	87.6	801
Ba	137.4	1691

It is natural to regard the atomic weight as a function of the distance and to extrapolate this function for radium. It has already been pointed out by Rydberg and by Kayser and Runge in their papers on "The Spectra of the Elements" that the distances of the pairs of lines within a group of chemically related elements increase in a regular way with the atomic weight. They state for the alkalies that the atomic weight is very nearly proportional to the square root of the distance. We would call attention to the fact that for the other group in which pairs of lines have been observed, the relationship between of the pair and the atomic weight may be represented by a simple formula, viz.:

In every group of chemically related elements the atomic weight is proportional to some power of the distance of the two lines of the pairs.

In general the exponent is not a whole number.

Expressed in other words this formula becomes:

If the logarithms of the atomic weight and distance be taken as co-ordinates, the corresponding points of a group of chemically related elements will lie in a straight line.

This law is shown in the two following figures. We see from Fig. 1 that among the alkalies, potassium is the only one which falls below the straight line through the remaining points. We would not here assert that the observed atomic weight of potassium is incorrect; but it seems to us interesting that this law of the straight line exhibits a larger deviation for just that element which is anomalous in respect to the periodic series, in so far as the atomic weight of *K* would have to be greater than that of argon in order to fit into the periodic series. As regards boron, gallium, and indium, the pairs of lines have not been investigated in a magnetic field. We cannot, however, doubt the

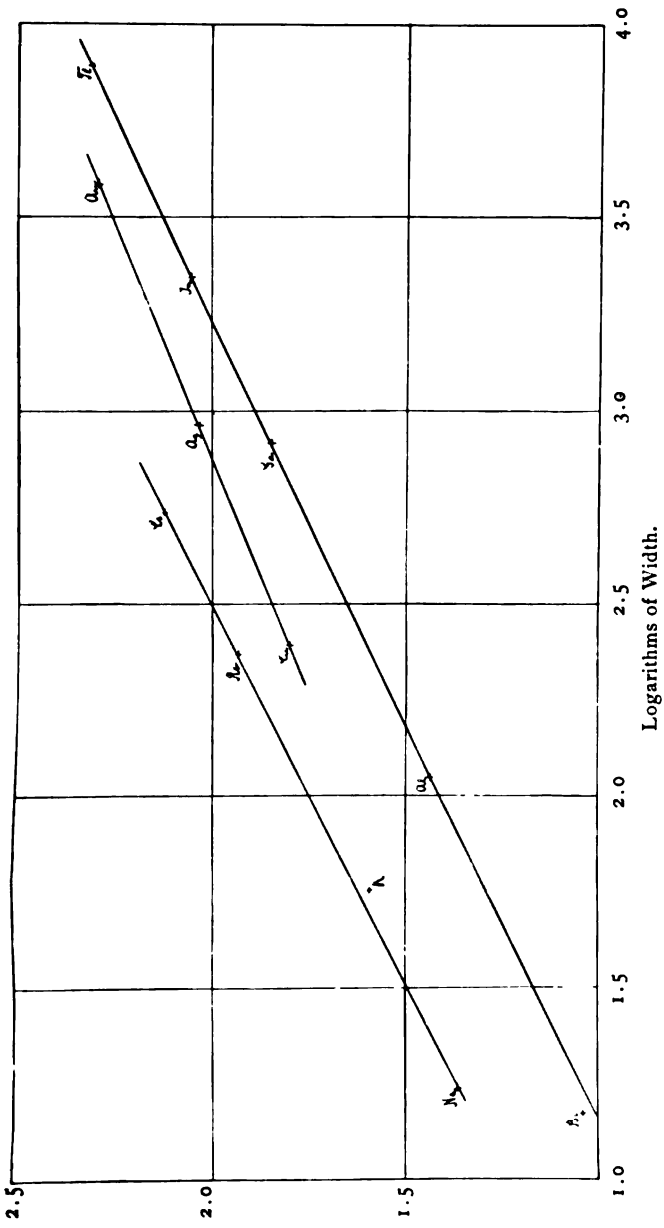

2.5
2.0
1.5
1.0
Logarithms of Atomic Weight.
4.0
3.5
3.0
2.5
2.0
1.5
Logarithms of Width.
(0.01 division of scale = 2.3 per cent.)

FIG. 1.

existence of the corresponding pairs. The same is true of the alkalies, where only the yellow lines of sodium have been investigated in a magnetic field.

Fig. 2 represents the same relationship for *Mg*, *Ca*, *Sr*, *Ba*, and *Ra*. Extrapolation gives 258 for the atomic weight of radium. We can, of course, somewhat displace and rotate the straight line without removing it too far from the points, but the figure clearly shows that the value 225 determined by Madame Curie is decidedly off the line.

In the following table the straight line is replaced by a formula and the extrapolation by computation. If x denote the separation of the pair on the scale of vibration numbers, $\times \frac{10^8}{\lambda}$, we have:

Atomic weight = No. whose log is $(0.2005 + 0.5997 \log x.)$

	Atomic Weight	
	By Formula	Observed
Mg	23.84	24.36
Ca	40.6	40.1
Sr	87.5	87.6
Ba	136.9	137.4

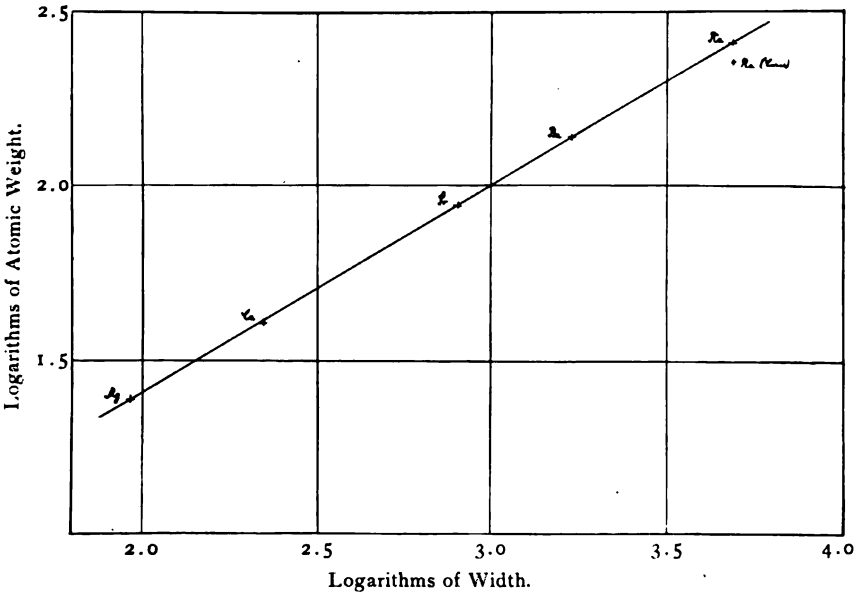
Extrapolation for radium gives:

Computed atomic weight of radium = 257.8.

We cannot risk the assertion that our figure deserves more confidence than the value determined by Madame Curie, but it should nevertheless be said that, in view of the close relationship between barium and radium, and of the small quantities of the substance with which the chemists are obliged to work, the complete separation of these two substances is very difficult; and further, that with an imperfect separation Madame Curie would necessarily obtain too small an atomic weight.

According to crystallographic observations soon to be published by F. Rinne, the bromides of radium and barium are isomorphous with each other, so that it is very probable that the two substances would crystallize together (in an isomorphous

mixture), similar relations as to solubility being assumed. The great difficulty of separating the two substances from each other by a process of crystallization is due to this, and it may happen that even with frequently repeated crystallization greater or less quantities of barium may be present in the corresponding radium compound. The number 223 fits the periodic system better in the respect that it fills the gap between bismuth and thorium in



(0.01 division of scale = 2.3 per cent.)

FIG. 2.

the correct column. For atomic weight 258 radium would have to be pushed two rows farther along in the column *Mg*, *Ca*, *Sr*, and *Ba*, and a number of new unoccupied places would arise in the periodic system.

On the other hand, we may adduce for the higher atomic weight the point remarked by Rutherford. The greater atomic weight suggests a complicated structure of the atom and hence a more easy breaking up into electrons. The element which most strongly emits electrons should therefore also have the greatest atomic weight.

The radium line at $\lambda 4826.14$, which is the most conspicuous in the Bunsen flame, is in respect to the separation in the magnetic field analogous to the strongest line in the Bunsen flame, *Ba* $\lambda 5535$, *Sr* $\lambda 4607$, *Ca* $\lambda 4246$. All these lines separate into three components which for all these elements are equally separated from each other on the scale of vibration numbers.

PHYSIKALISCHES INSTITUT DER TECHNISCHEN HOCHSCHULE,
Hannover, Germany, January 1903.

NOTE ON SOME EFFECTS OF RULING ERRORS IN GRATING SPECTRA.

By A. S. KING.

THE possibility that errors of ruling in diffraction gratings may produce "ghosts" near strong lines in the spectrum makes it desirable that we understand as fully as possible the forms that such ghosts may take. If they occur in the most frequent form, as two lines symmetrically placed with respect to the primary line and close to this line, they are readily recognized. The writer has observed, however, that a very good grating may give, in certain regions of the spectrum, ghosts of higher order at such distances from the primary line that they might easily be mistaken for real lines.

The theory of ruling errors is a complex one. The most complete treatment we have is that given by Rowland in his article entitled "Gratings in Theory and Practice."[1] In this Rowland establishes general relations as to position and intensity of ghosts, and develops in some detail the expressions for intensity of ghosts of different orders resulting from small periodic errors of ruling. He also calls attention to other effects which may result from special conditions.

The fundamental formulæ give the position of the ghost of nth order by the relation $\mu_n = 2\pi N/ba_0 \pm ne_1/ba_0$, and the intensity by the function $J_n^2(b\mu_n a_1)$, where the first term of μ_n gives the position of the primary line, and the second term the position of the ghost with respect to the primary line. N denotes the order of spectrum, a_0 the grating space, a_1 and e_1 the amplitude and period of ruling error, and $b = 2\pi/\lambda$. In accordance with these relations, we have the following characteristics of ghosts:

1. Ghosts of the same order are at equal distances on each side of the primary line, and of equal intensity.

2. The distance of a ghost from the primary line varies

[1] *Astronomy and Astro-Physics*, 12, 129, 1893.

directly with the wave-length, but is *independent of the order of the spectrum*, since the second term in the value of μ_n does not contain N.

3. The ghosts of different orders are equally spaced.

4. The intensity increases rapidly with the order of the spectrum and with the amount of ruling error.

5. The intensity of the primary line is diminished by an amount equal to the sum of intensities of the ghosts.

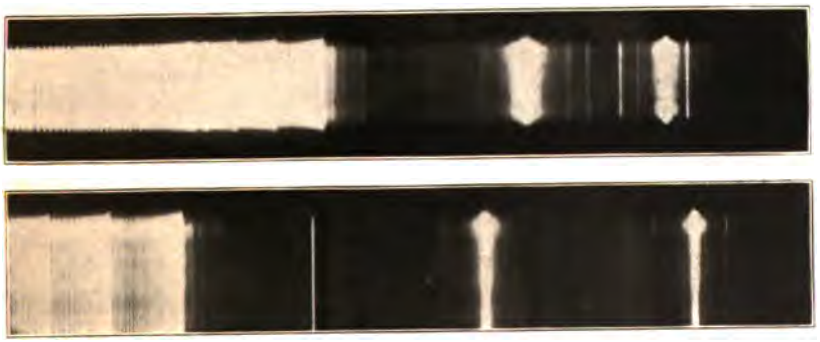
6. Since, for the same order of spectrum, the relative intensity of any ghost depends upon the value of a_1 in the expression $J_n^2(b\mu_n a_1)$, we may have the greatest intensity in a ghost of higher order than the first.

7. As $\mu = \sin\alpha$, these relations are constant only for a given position of the grating, and we have a new set of values when the angle of incidence is changed, as when we move the grating in order to observe another part of the spectrum.

The writer has recently obtained some photographs which show very clearly the effects of the last two determining elements given above. The concave grating used was one of 15 feet radius and 15,000 lines to the inch. In the first-order spectrum a prolonged exposure is required to show any ghosts on the photographic plate. On such over-exposed plates a faint appearance resembling the head of a band is seen at wave-length λ3888.75, which the writer tried in vain to identify with any known element of the spectrum, and which proves to be a first-order ghost of the λ3883 cyanogen band. Faint ghosts also appear beside very strong metallic lines, as the strontium line at λ4215.

In the second- and third-order spectra, however, remarkable series of ghosts appear above each of the cyanogen bands at λ3590, 3883, and 4216, and also near strong metallic lines in that region, as the H and K lines and the strontium line above mentioned. In each of these cases we have very distinct ghosts up to the fourth order, giving above each of the cyanogen bands the appearance of a second band with four equally spaced heads. Furthermore, the intensity of these ghosts does not diminish uniformly as they recede from the primary line. Those belonging to the head of the 3883 band occur at λ3888.75,

PLATE XVI.



GHOSTS IN GRATING SPECTRA.

3893.90, 3899.06, 3904.22, and have intensities in about the ratio 10, 5, 2, and 8. The ghost of fourth order is thus almost as strong as that of first order. The heads of the bands at λ3590 and λ4216 have ghosts of similar appearance, the spacing in each case being proportional to the wave-length. Photographs of these bands in the third-order spectrum show the ghosts much stronger than in the second order, but each series preserves the same relative intensities among the ghosts of different orders, and the spacing of corresponding series of ghosts in the two spectra is exactly the same; this last fact proving conclusively that they are ghosts.

The plate shows the second- and third-order spectra in the region of wave-length λ3900, giving the head of the 3883 cyanogen band and the H and K lines of calcium, with their attendant ghosts. The ghosts of the H and K lines are faint in the reproduction, the ghost of third order being scarcely visible, but the fourth-order ghosts of the K line, reversed like the primary line, can be seen at wave-length 3926.81 and 3940.85. A remarkable feature is that the fourth-order ghost of the K line below the primary line is considerably stronger than the corresponding ghost above the line. This difference shows very distinctly in the negative, and is not in accordance with the theory of equal intensity in ghosts of the same order. Whether such a difference also exists in the ghosts of the cyanogen bands cannot be determined on account of the dense structure below the head. The plate shows the equal spacing of ghosts and the unequal dispersion of true lines in the two orders of spectra.

The writer has been unable to obtain ghosts of higher order than the first on plates taken in the region of wave-length greater than 4600, which shows that a change in the angle of incidence may produce important changes in the character of the ghosts. Prolonged exposures in the upper part of the second- and third-order spectra give first-order ghosts of considerable intensity near all strong metallic lines and carbon bands, but no ghosts of higher order can be distinguished.

The simplest method of detecting ghosts, and perhaps the only really decisive test is to photograph the suspected lines in

spectra of different orders. If they are ghosts, the spacing will be the same in the two orders; while real spectral lines will suffer a dispersion proportional to the order of spectra used.

The theory of ruling errors as given by Rowland explains the action of only the principal causes of ghosts. Other causes, as periodic variation in the depth of ruling, periodic waves in the surface, and perhaps changes in the shape of the groove made by the ruling point, may all give effects whose exact nature is not easily determined. For example, the variation of intensity among the ghosts of different orders shown on my plates does not agree with any case in Rowland's table giving the distribution of intensity among the ghosts according to the amount of ruling error. It is probable that other causes have a part in producing the effects observed. However, the general effect is predicted by Rowland's theory, and the results show that ghosts need not be of any fixed type, nor of the same appearance throughout the spectrum. The fact that the grating may give ghosts of higher order at a considerable distance from the primary line (7.02 tenth-meters in the case of the K line) and more intense than those nearer the line, shows the need of care, especially in working with spectra of second or third order, that such widely separated ghosts are not mistaken for actual spectral lines. Photographs taken in spectra of different orders will at once decide this point.

UNIVERSITY OF CALIFORNIA,
February 1903.

ϵ *AURIGAE* A SPECTROSCOPIC BINARY.[1]

By H. C. VOGEL.

IN his investigations made a few years ago on the more refrangible parts of stellar spectra, Dr. Eberhard was struck by the fact that in the spectrum of the well-known variable ϵ *Aurigae*, which lies in the transition between types I and II, the series of hydrogen lines clearly extended farther beyond the H and K lines than is the case with stars of similar type. He suspected that the spectrum of the star should be regarded as the superposition of two spectra of different types.

Changes in the spectrum great enough to be recognized with the slight dispersion of the single prism spectrograph (D) used by Dr. Eberhard were not exhibited by plates taken at various times.

In the latter part of April and beginning of May, 1900, three spectrograms were obtained by Professor Hartmann with the large spectrograph (III) in connection with the 80cm refractor, but a comparison of these plates in the region from $\lambda 415$ to $\lambda 455$ showed nothing striking. On November 9, 1901, and November 18, 19, and 22, 1902, Dr. Eberhard further photographed the spectrum of ϵ *Aurigae* with the three-prism spectrograph (IV) designed by me three years ago for the photographic refractor of 32.5 cm aperture. A superficial comparison of the spectra taken in the latter year with those of the preceding year was sufficient to show that the spectrum had undergone a change. The thorough examination and measurement of the spectrograms which I at once began has so far furnished the result that the suspicions of Dr. Eberhard were well founded, and that the spectrum of ϵ *Aurigae* is in fact the result of the superposition of two spectra, the one similar to that of α *Cygni*, and the other lying between the first and second types, like α *Persei* or γ *Cygni*.

At present the first-named spectrum is the more intense and

[1] Translated from advance proofs, furnished by the author, of a paper communicated to the *Kgl. Akademie der Wiss. zu Berlin.*

it is displaced relatively to the other toward the violet by an amount which would correspond to a velocity of from 30 to 40km per second. The spectrum is now distinguished from that of the previous year principally by the fact that but few lines of the iron spectrum are recognizable in it. Most of the lines have disappeared, probably as a consequence of the relative displacement of the spectra, and practically the only lines recognizable are those of the spectrum similar to *α Cygni*. Most of these appear double and are characterized by the fact that the component lying toward the violet is with few exceptions the stronger, and the boundary on the violet side of the double lines, which are often difficult to separate, is extremely sharp. This is particularly striking in case of the hydrogen lines, as is shown by a very successful plate taken by Professor Hartmann at my request on November 22, 1902, with the one-prism spectrograph (I) in connection with the 80cm refractor.

There can accordingly be no doubt that ε Aurigae is a spectroscopic binary and probably one of very long period.

Considerable difficulties have been encountered in the comparison and measurements of the spectra because of the complications which result from dissimilarity of the superposed spectra, particularly in certain parts of the spectrum. I intend to communicate later more fully the very interesting details as to the spectrum of this star, which will be regularly observed here.

POTSDAM,
December 1902.

Minor Contributions and Notes.

TRANSPARENCY OF COMET 1902 *b*.[1]

THE statement is frequently made that comets are perfectly transparent, even faint stars being visible through them. The observations on which this statement is based appear to be very vague, as, even if careful comparisons were made, large errors might be introduced by the effect of the bright background formed by the light of the comet. The rapid motion of Comet 1902 *b* caused it to cover a large area, and therefore rendered it easier to find a star over which it would pass. After waiting for a suitable occasion, the observations given in Table I

TABLE I.

OBSERVATIONS.

G. M. T.	Difference	Residuals	Distance
h. m.			′
13 22.5	1.06	+.01	2.0
13 33.3	1.03	+.04	1.1
13 44.7	1.10	−.03	2.0
13 57.7	1.07	.00	4.0
14 10.8	1.06	+.01	5.5
14 26.7	1.06	+.01	7.9
14 46.9	1.08	−.01	11.0
15 2.3	1.12	−.05	13.1

were made by Professor O. C. Wendell with the polarizing photometer attached to the 15-inch equatorial. On the evening of October 14, 1902, the comet passed within about 1′ of the star +21°.3483, photometric magnitude 7.12. This star was compared with +21°.3484, magnitude 8.19. Each set of observations was the mean of sixteen settings. The Greenwich Mean Time is given in the first column, the difference in magnitude of the two stars in the second, and the deviation of this magnitude from the mean value, 1.07, in the third column. A positive sign indicates that the star +21°.3483 was faint, a negative sign that it was bright. The fourth column gives the distance of the nucleus of the comet from the star. The diameter of the coma was

[1] From *Harvard College Observatory Circular* No. 68.

about five or six minutes. The star was, therefore, covered by it in the first three observations. The nucleus resembled a star of about the tenth magnitude and the brightness of the coma was about that of a star of the ninth magnitude when spread over a circle one minute in diameter. The largest residual -0.05 is the last one, when the altitude of the comet was only 22°. The mean of all the residuals is ± 0.02. It appears, therefore, that the absorption of the light by the comet, if any, is insensible, and probably does not exceed one or two hundredths of a magnitude.

EDWARD C. PICKERING.

ADDITIONAL STARS OF THE *ORION* TYPE WHOSE RADIAL VELOCITIES VARY.

SPECTROGRAMS taken with the Bruce spectrograph since our communication in the March number of this JOURNAL show that the radial velocities of *τ Tauri* and *ψ Orionis* vary through a wide range.

In order to reach fainter spectra than can be photographed with the use of the cameras A and B, we have lately been experimenting with lenses of shorter focus (8 to 12 in. = 20 to 30 cm), with encouraging results. Several of the best types of modern anastigmat lenses have been loaned to us for trial by their manufacturers or agents. These show a great gain in the reduction of the exposure time, and it seems likely that with a camera lens of about 10.5 inches (267 mm) focus, especially constructed for our purpose, this result may be obtained without a greater loss of accuracy in the settings than would be justified under some circumstances. Pending the procurement of such a lens, the Bausch & Lomb Optical Co., of Rochester, N. Y., has most kindly allowed us to retain the "Unar" lens, Series I*b*, No. 7, which proved the most satisfactory for our purposes of all these lenses tried in connection with the spectrograph.

With this camera lens we have obtained plates of *τ Tauri* ($\alpha = 4^h\ 36^m$; $\delta = +22^\circ\ 46'$; Magnitude, *H. P.*, = 4.4) on three dates: 1903, February 25, 26, and March 5. The first plate gave a radial velocity of about +70 km, which, in view of the usually small values for stars of the *Orion* type, was strongly suggestive of a variation. The other two plates confirm this, and show a total range of about 75 km.

Three plates have also been obtained of *ψ Orionis* ($\alpha = 5^h\ 22^m$; $\delta = +3^\circ\ 1'$; Magnitude. *H. P.*, = 4.7), the first with the short focus

camera and the other two with camera A. The examination of the first A plate indicated a high positive velocity, which led to the measurement of the short focus plate previously obtained, on which accordant values were deduced from different lines, although the C plate should necessarily be given less weight than the A plate.

The data so far secured follow:

Plate	Date	Taken by	Velocity	No. of lines	Measured by
C 17	1903, February 4	F.	− 122	3	A.
A 406	February 18	F.	+ 148	4	F.
A 415	March 12	F.	− 31	2	F.

It would seem probable that this star will show a very large total range when observations have been secured of its maximum and minimum radial velocities. Plates of other stars of the *Orion* type taken with the short-focus camera suggest variation, but we are not at this time prepared to report as to them.

Several plates have been obtained with the longer cameras of the star η *Hydrae* (also of Type I *b*) which cause us to have suspicions as to the constancy of its radial velocity. The spectrum is very difficult of measurement, owing to the diffuseness of the lines, and we shall follow it further.

EDWIN B. FROST and WALTER S. ADAMS.

YERKES OBSERVATORY,
March 16, 1903.

HENRY A. ROWLAND MEMORIAL LIBRARY.

IN order to establish a permanent memorial of the late Professor Rowland, and to promote the efficiency of the Physical Laboratory of the Johns Hopkins University, one of its former students has given a most generous sum of money with which to found a special collection of books, pamphlets, and other publications in the field of radiation and spectroscopy. This is to be called the "Henry A. Rowland Memorial Library," and is to be placed in the Physical Laboratory of the University. Mrs. Rowland has given the library all those books and pamphlets belonging to Professor Rowland, which refer to spectroscopy.

To make the collection complete, and to maintain its usefulness, the co-operation of observatories, laboratories, and investigators is necessary. It is earnestly hoped that sets of official publications,

books, reprints of papers on spectroscopy or allied subjects, and photographs of spectra and of apparatus will be contributed to the library, both now and in the future.

They may be addressed to the care of

JOSEPH S. AMES,
Director of the Physical Laboratory,
Johns Hopkins University,
Baltimore, Maryland, U. S. A.

PLATE XVII.

MARS.

September 2, 1894. $\lambda = 100^{\circ}$.
36-inch refractor of Lick Observatory.

THE

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THE SOUTH POLAR CAP OF *MARS*.

By E. E. BARNARD.

HOWEVER much the general surface features of *Mars* may be misrepresented, and however much the canals, single and double, may be illusory, the polar caps seem to have escaped this general deluge of uncertainty and misrepresentation.

They stand for their face value, whatever may be their composition. There seems yet no definite proof that they are not as much ice and snow as that with which we have to deal in our own terrestrial winters. So much is at least suggested by the great seasonal changes they undergo from winter to summer.

The south polar cap is the better known of the two, because of its more favorable position for observation at the near approaches of *Mars*. The changes in its dimensions have been recognized since the earliest telescopic observations of the planet, and the early supposition that they were due to accumulations of snow during the winters of *Mars*, which melted away at the approach of summer, is as good a theory as any of those put forward today to explain the phenomenon.

During the close approach of *Mars* in 1892 and 1894 the writer had a good opportunity to study the planet at the Lick

Observatory with the 12-inch and 36-inch refractors. A carefu series of drawings was made at that time, but they have not yet been published.

Struck with the great changes that take place in the polar caps, I decided to make a series of micrometer measures of the south cap, instead of remaining content with estimates and drawings of the phenomenon, though a great number of drawings were made of the cap—some sixty of which were published in *Popular Astronomy* for June 1895 (**2**, 433–443).

It would seem that the changes in the dimensions of these caps have always been studied from drawings and never from a systematic series of micrometer measures. While these changes can be fairly studied from careful drawings, there must always remain a large percentage of uncertainty, especially if the drawings are by different observers; careful micrometer measures are therefore always to be preferred, as changes indicated by them can be relied upon.

Following are the measures of the cap at the oppositions of 1892 and 1894. The time is $8^h\ 0^m$ slow of Greenwich. The column headed "Days" is the number of days before or after the summer solstice of the southern hemisphere of *Mars*. The measured diameter is given under the heading "Observed, while the values under "MΔ" are the diameters reduced to the mean distance of *Mars* = 1.52369. Under the head "Residuals" the difference of each observation from the curve is given.

1892.

(Made with the 12-inch refractor.)

Date			Days	Observed	MΔ	Residuals
July	3d	15h 25m	−102	9″.31	2″.78	−0″.12
	8	13 11	97	10.08	3.06	+0.20
	15	12 55	90	9.97	2.67	−0.05
	22	13 35	83	9.90	2.52	−0.05
	31	12 40	74	10.02	2.49	+0.19
August	2	11 20	72	8.76	2.17	−0.10
	3	11 0	71	8.50	1.75	−0.13
	3	13 40	71	8.86	2.10	−0.42
	4	11 50	70	8.48	2.10	−0.11
	5	11 45	69	8.70	2.16	0.00
	7	11 25	67	8.17	2.02	−0.07

1892.—*Continued.*

(Made with the 12-inch refractor.)

Date				Days	Observed	M Δ	Residuals
	8	10	35	−66	8.75	2.17	+0.10
	9	11	30	65	8.38	2.08	+0.05
	10	10	30	64	7.91	1.96	0.00
	11	12	20	63	7.89	1.98	+0.01
	12	11	40	62	7.99	1.99	+0.06
	14	10	42	60	7.93	1.99	+0.11
	17	9	30	57	6.59	1.67	−0.05
	18	12	5	56	7.13	1.79	+0.07
	19	12	35	55	7.13	1.82	+0.12
	21	11	45	53	6.71	1.73	+0.17
	23	9	42	51	6.73	1.75	+0.20
September	2	10	10	41	3.62	1.01	−0.25
	4	7	50	39	3.80	1.08	−0.12
	6	9	30	37	2.81	0.81	−0.35
	8	8	6	35	4.44	1.11	0.00
	9	8	22	34	3.54	1.05	−0.06
	9	11	0	34	3.65	1.08	−0.05
	11	8	25	32	3.45	1.04	+0.05
	16	7	40	27	3.75	1.19	+0.22
	19	8	20	24	2.95	0.96	+0.08
	22	6	50	21	2.94	0.99	+0.15
	23	7	30	20	2.12	0.72	−0.14
October	7	7	27	− 6	1.82	0.71	+0.07
November	6	6	20	+24	0.60	0.31	−0.07

1894.

(Made with the 36-inch refractor.)

Date				Days	Observed	M Δ	Residuals
May	21^{d}	16^{h}	1^{m}....	−103	3′.84	2′.79	−0′.02
	28	16	0	96	3.66	2.58	−0.10
June	11	16	0	82	3.81	2.46	+0.07
	18	15	20	75	3.69	2.28	0.00
	24	15	50	69	3.55	2.10	−0.02
	25	15	25	68	3.71	2.18	+0.06
July	1	16	0	62	3.66	2.07	+0.15
	8	16	30	55	3.56	1.92	+0.22
	15	15	5	48	2.25	1.15	−0.35
	30	13	50	33	2.99	1.37	+0.19
August	5	16	50	27	2.16	0.94	−0.11
	6	13	40	26	2.24	0.97	−0.05
	13	17	55	19	2.95	1.21	+0.28
	26	13	55	− 6	1.65	0.61	−0.10
September	2	12	30	+ 1	2.17	0.76	+0.12
	9	12	10	8	1.52	0.50	−0.09
	16	14	45	15	1.75	0.55	+0.02
	30	11	10	29	1.14	0.33	−0.02
October	7	9	40	36	1.43	0.41	+0.05
	8	10	30	37	1.49	0.42	+0.06
November	11	9	35	71	0.33	0.11	−0.10

In recently looking over these measures of the south polar cap, it occurred to me that if they were plotted with respect to the summer solstice of the Southern Hemisphere of *Mars*, and if curves were drawn through them, the results might be very instructive. The two curves that accompany this paper have therefore been prepared in that manner.

The measures are all reduced to the mean distance of *Mars*, and hence are strictly comparable. In the diagrams the vertical scale represents the diameter of the cap in seconds of arc, while the horizontal scale gives the time in days before and after the summer solstice of the southern hemisphere of the planet.

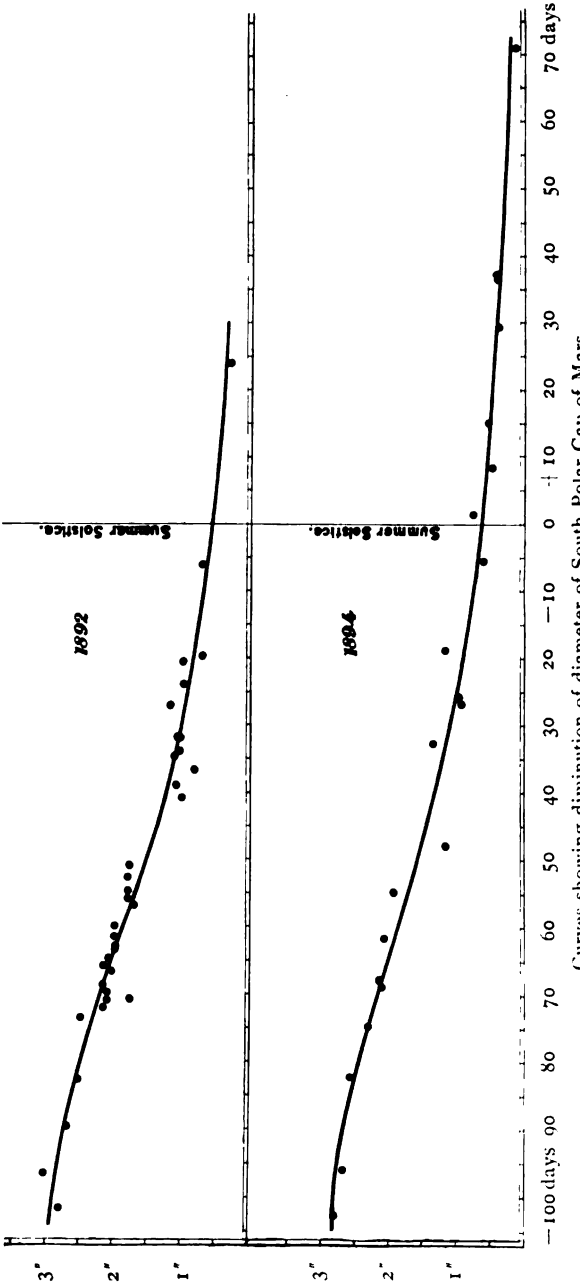
At first sight the striking fact is apparent that these two curves are parallel throughout. The cap, therefore, at both oppositions, followed the same law of decrease with surprising closeness.

Another important feature, to which I have already called attention in the number of *Popular Astronomy* referred to, is that the cap continues to diminish after the summer solstice, or beyond the time when the greatest heat is received from the Sun, showing, as in the case of the Earth, that the highest temperature is not reached until several months after the maximum of solar heat. This may have considerable bearing upon the presence of a Martian atmosphere.

There seems to be a general belief now that *Mars* certainly has an atmosphere. This atmosphere seems to be very much less dense than our own, and yet it is of sufficient density to produce the phenomena of the polar caps by condensation and evaporation, and also to produce, though rarely, some form of clouds.

The maximum diameter of the cap is perhaps shown by my measures, as the Sun had only risen on the south pole some four or five weeks earlier, and the curves do not seem to indicate that there was any sensibly larger diameter for the short interval between the rising of the Sun and the beginning of the measures.

It would be very interesting to watch the growth of the south cap to see if the increase follows the same law as the decrease.



Curves showing diminution of diameter of South Polar Cap of Mars.
Ordinates: Measured diameters of cap. Abscissae: Days before and after Martian Summer Solstice.

It seems, however, that the increase can never be seen. Though the planet was carefully watched in 1894, nothing was visible of the cap after November 19. It seemed to have entirely disappeared. Records of its non-visibility were kept as late as May 12, 1895. After the middle of February, however, it would have been immersed in the shadow, as the Sun, after that date, would pass too far north to shine on it. Hence, if we take these observations as a criterion, it will not be possible to watch the growth of any portion of the cap at any time.

On several occasions a portion of the outline of the polar cap was lost by a dusky obscuring medium which would eventually clear away and the outlines would then again become distinct. There seemed to be good reason for believing this temporary obscuration to be of the nature of clouds.

In May of 1894 the polar cap covered an area of about 365,000 square miles. By the last of November it had all disappeared, having apparently melted entirely away.

The rapid decrease of the polar cap shows that the snow, if snow it be, cannot be of great depth. It is probably a more or less thin sheeting, just as in winter here on the Earth the ground will be sometimes covered with snow down beyond middle latitudes, which may in the course of a few days melt away, producing a rapid change in the apparent extent of the Earth's polar cap as seen from, say, *Venus*.

The following table gives the diameter of the south polar cap for every five days before and after the Martian summer solstice, taken from the curves of 1892 and 1894. It will be seen that the difference is very small, not exceeding 0".08. Whether the change of sign is due to a real difference in the diminishing of the cap in 1892 and 1894 it is not possible to tell—the quantity involved in the apparent change is too small to deal with in measures of this kind, and can well be attributed to errors in drawing the curves. From the strikingly close agreement of the curves at the two seasons, it would appear that the mean curve indicated in the last column would serve as a means of predicting the mean diameter of the cap with great exactness—indeed, with greater accuracy than it can be measured. Of

course, this is a statement resting on only two Martian years, but I believe the changes in these caps, apparently uninterrupted by freaks of weather, and depending probably on the position of the Sun alone, follow a perfectly regular law which is shown by these curves, and which permits an accurate prediction of the mean diameter of the cap.

DIAMETER OF THE SOUTH POLAR CAP FOR EVERY FIVE DAYS, TAKEN FROM THE CURVES OF 1892 AND 1894.

No. Days before Summer Solstice	1892	1894	Difference	Mean Curve
100	2″.90	2″.85	+0″.05	2″.87
95	2.83	2.77	+0.06	2.80
90	2.74	2.70	+0.04	2.72
85	2.61	2.57	+0.04	2.59
80	2.50	2.42	+0.08	2.46
75	2.35	2.31	+0.04	2.33
70	2.20	2.17	+0.03	2.19
65	2.04	2.01	+0.03	2.02
60	1.86	1.87	−0.01	1.87
55	1.69	1.73	−0.04	1.71
50	1.53	1.56	−0.03	1.54
45	1.37	1.43	−0.06	1.40
40	1.24	1.30	−0.06	1.27
35	1.12	1.20	−0.08	1.16
30	1.02	1.09	−0.07	1.06
25	0.94	0.98	−0.04	0.96
20	0.84	0.91	−0.07	0.88
15	0.77	0.82	−0.05	0.80
10	0.70	0.75	−0.05	0.73
5	0.64	0.68	−0.04	0.66
0	0.58	0.63	−0.05	0.60
After 5	0.52	0.60	−0.08	0.56
10	0.48	0.56	−0.08	0.52
15	0.44	0.52	−0.08	0.48
20	0.41	0.49	−0.08	0.45
25	0.37	0.45	−0.08	0.41
30	0.35	0.42	−0.07	0.39
35		0.39		
40		0.35		
45		0.32		
50		0.30		
55		0.27		
60		0.25		
65		0.24		
70		0.23		

The outline of the polar cap was often irregular. Details were frequently visible upon its surface in the form of bright spots, broad bright lines, and changing dark spots. A large temporary dark spot was visible near the middle of the cap at both

oppositions when the cap was at its largest. This seemed to disappear as the cap became smaller. On one occasion this central marking on the cap was of a reddish hue, like the general surface of the planet, and on several dates the cap was distinctly double.

The most striking phenomenon connected with the cap, however, and perhaps the most important, was the recurrence of a projection from the edge of the cap, which appeared at the same point in both years, and which was left behind as a bright strip as the cap diminished, showing it to be due to some local peculiarity of the surface of *Mars*. It was suggested at the time that this might be caused by a group or range of mountains at that point on *Mars*. If these were mountains, the snow on their summits would remain stationary as a white strip, after the receding cap had left it, until it melted. Thus the decay of the polar cap probably reveals to us the existence of mountains on *Mars*, which would otherwise remain unknown.

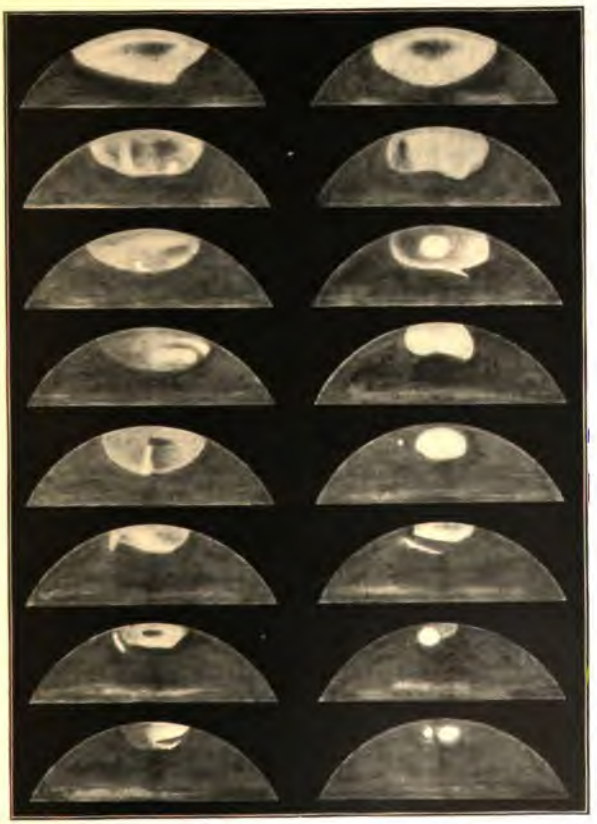
In tracing back this phenomenon through former oppositions it was found that this very thing had been seen by Mitchel at Cincinnati in the year 1845 at the same point on the surface of *Mars*. It doubtless repeats itself every Martian year when the cap is diminishing.

It is possible to predict when it will be visible from the Earth. It begins to appear about sixty days before the Martian summer solstice, and is passing through its changes for about a month. It ought to begin to be visible when the heliocentric longitude of *Mars* is about 320°. When more careful and continuous observations have been secured, its reappearance ought to be predicted very closely.

Since writing the above I have consulted the classical paper by Mr. N. E. Green in the *Memoirs of the Royal Astronomical Society*, Vol. XLII, where he has given a series of twelve admirable drawings of *Mars*[1] (made at Madeira in 1877), with a map

[1] Mr. Green mentions that these twelve are selected from forty-one drawings of the planet made by him in 1877. These splendid drawings, which so faithfully represent the telescopic appearance of *Mars*, should all be made available to those interested in the planet. In these days of half-tone work their accurate reproduction would be inexpensive.

PLATE XVIII.



THE SOUTH POLAR CAP OF MARS.

of the surface of the planet. In his paper Mr. Green calls attention to the repetition of the projection from the south polar cap (the same shown in my drawings) and offers the same explanation for its cause, *i. e.*, a mountainous surface. One of these bright spots he has named "Mitchel Mount," in honor of Mitchel, of the Cincinnati Observatory, to whose drawings we have referred.

The north cap in 1895 was well seen, clear, and distinct on May 6. Evidences of it had been seen several months earlier. A note on December 23, 1894, at 6^h says: "there is a strong bluish white glow at the north limb, but no definite polar cap." This glow was frequently noticed thereafter until the true cap made its appearance.

To show some of the peculiarities of the outlines of the cap and of thedet ails upon its surface, eight drawings for each of the seasons, 1892 and 1894, have been selected for reproduction here (Plate XVIII). No effort has been made to retain any of the details of the surface of the planet near the caps, as such details would more properly belong to the drawings of the planet itself.

One drawing of the planet is also given (Plate XVII), as a sample of the details visible in 1894. In many of the drawings secured at that opposition, when the planet was well seen, the details were so abundant and complicated, especially in the dark regions, that it was impossible to correctly delineate them.

YERKES OBSERVATORY,
April 2, 1903.

CATHODO-LUMINESCENCE AND THE NEGATIVE POLE SPECTRUM OF NITROGEN.

By PERCIVAL LEWIS.

I. A STUDY OF THE NEGATIVE GLOW.

IN a previous paper[1] it was shown that the vapors of a number of metals can be made luminous by the action of cathode rays alone, precautions being taken to prevent the passage of a conduction current through them. It has been suggested by several investigators that the characteristic negative glow observed in vacuum tubes containing certain gases, notably nitrogen, is likewise the direct effect of the energy absorbed from the cathode rays by impact or otherwise. In the case of volatile metals it is easy to isolate their heavy vapors by heating the metals at one end of a long vacuum tube. Their slow diffusibility and rapid condensation on passing beyond the heated region prevents them from raising the pressure of the highly rarified atmosphere around the distant cathode, from which the rays are projected into the mass of vapor. Such a method is obviously impossible in the case of light and rapidly diffusible gases, such as nitrogen. In such cases complete isolation from the region traversed by current can be secured only by a very thin metal partition, as in Lenard's experiments,[2] and then, as shown by Lenard, the luminosity produced is too small to be examined spectroscopically. Such luminosity does exist, however, although it cannot be positively identified with the negative glow. The only practicable method of showing that the negative glow is due altogether to cathode rays seems to be an indirect one — by comparing those parts of the negative glow through which it is likely that little or no current flows with those parts through which the greater part of the current flows. If no difference is found in the luminosity, either qualitative or quantitative, it would seem

[1] P. LEWIS, ASTROPHYSICAL JOURNAL, 16, 31, 1902.

[2] *Wied. Ann.*, 51, 229, 1894.

PLATE XIX.

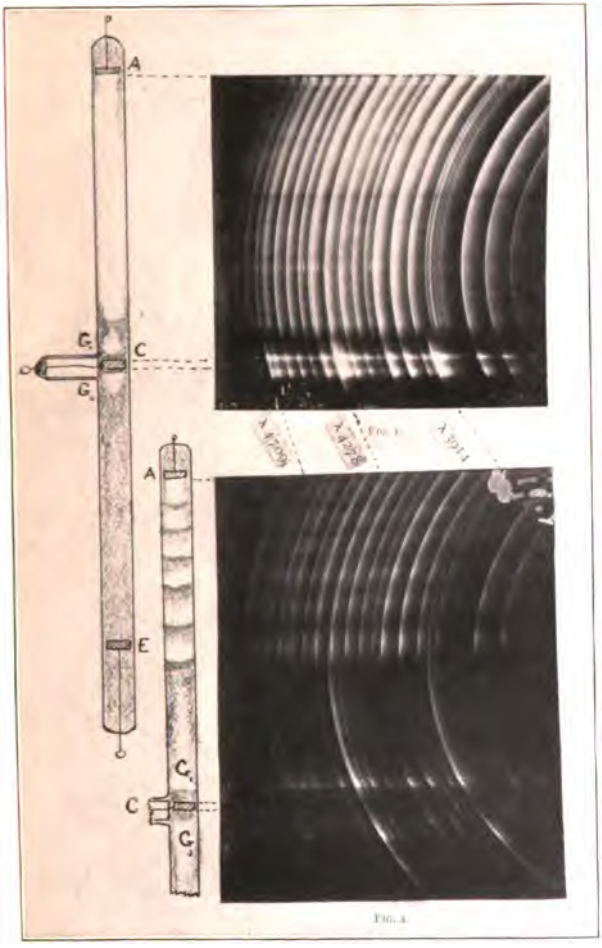


FIG. 1.

exceedingly probable that the luminosity in this part of the tube is due to cathode rays solely.

Since there is convincing evidence that cathode rays consist of negatively charged particles, it may be asked whether the continuous projection of such particles does not in itself constitute a convective electric current; for it is generally taken for granted that the current through the positive column is also transported convectively, by some process akin to electrolytic conduction. There are, however, at least two grounds for differentiation between the two processes: (1) the cathode discharge at low pressures takes place in all directions normal to the cathode, with absolute indifference to the position of the anode, while the positive column, like the ordinary electric current, takes the shortest or least resistant path between the electrodes; and (2) the cathode rays seem to consist of subatomic corpuscles, while such evidence as we have indicates that the positive electricity travels with particles of atomic magnitude, as in the case of electrolysis.

A straight tube of about 6mm internal diameter was used. (See Plate XIX.) The cathode *C* was in the middle, the anode *A* near one end, and an idle electrode *E* (usually earthed through a high resistance) at the other end. The distance between the electrodes was about 7cm. They were made of thick disks of aluminum, and nearly filled the cross-section of the tube. A Leeds induction coil giving a very heavy spark about 6cm long was used. The nitrogen was prepared by heating a solution of ammonium sulphate and sodium nitrite in molecular proportions, and was freed from oxygen, carbon dioxide, and water vapor by passing it in succession through pyrogallic acid, concentrated potassium hydrate solution, and a train of drying tubes.

It is, of course, well known that the direction of the negative glow is independent of the position of the anode. It has not, perhaps, been sufficiently emphasized that when the cathode has sufficient free space on both sides the glow is perfectly symmetrical, and is just as large and intense on the side opposite the anode as on the same side. This is true when the cathode almost closes the tube, but not when it entirely closes it. The

glow G_2 resembles G_1 in form, size, and intensity, whether all three electrodes are insulated, A alone earthed, or A insulated and C and E both earthed. The only difference observed was that at very low pressures in the latter case there was a division of the negative discharge between C and E, with a short independent positive column between them; but even in this case the discharge from C was perfectly symmetrical.

The luminosity of G_2 was apparently equal to that of G_1 at all pressures. This was tested by throwing an image of each in succession on the slit of a Glan spectro-photometer and measuring the intensities of the three bright negative pole bands λ 5228, λ 4709, and λ 4278 at equal distances on each side of C through a wide range of pressures. At low pressures and at considerable distances from the electrode the luminosity was small and measment difficult; but up to distances of 2 cm from C at a pressure of about 1 mm there seemed to be equality of intensity on each side, within the range of experimental error. This symmetry is also clearly shown on photographs to be described later.

With this form of tube it would certainly be incorrect to say that no conduction current passes from the anode to the farther side of the cathode, and consequently through G_2. This is shown by the fact that when C completely closes the tube, except for a small tortuous channel to permit equalization of gas pressure, there is no negative glow on the farther side, as there should be if the negative glow were independent of the existence of the anode within the same inclosure, as well as of its direction. The greatest resistance to the discharge is at the cathode, and at low pressures we may expect that every available part of its surface will be equally brought into requisition to facilitate the discharge, even though the diverted part of the conduction current be forced to take a very roundabout and constricted path to the rear of the cathode. It seems likely, however, that the greater part of the current will pass directly to the nearer side, and that the diverted part, after passing around the cathode, will take as short a path as possible to its surface; and yet the intensities of the negative glow on the two sides is the same not only near the cathode, but at distances as great as 2 cm. The matter is com-

plicated by the fact that the canal rays discovered by Goldstein[1] and shown by Wien[2] to consist of projected positively charged particles, coming probably from the anode, pass readily through openings through and around the cathode, and may take some unknown part in the processes going on in that region. The canal rays, however, have not been observed at pressures of more than a few tenths of a millimeter.

In order further to vary the conditions, a completely closed wire-gauze cylinder was inserted between E and C, touching the former and ending about 5 mm from the latter. The negative glow penetrated the meshes of this cylinder, without any apparent changes of either intensity or length. In most cases C was earthed by a wire and E by contact with a wooden table, so that static discharges might escape from the latter without any appreciable conduction current.

The effects of the electrostatic forces acting between C and the gauze cylinder and the walls of the tube must not be overlooked, and there can be little doubt that some discharges through this region must have resulted from them. They were never sufficiently strong to cause visible luminosity.

Thus the currents through G_2 must have been small compared with that through G_1; furthermore, the conditions were so varied that there must have been great relative changes in the intensity of these small currents—and yet G_2 remained exactly like G_1. There seems to be no escape from the conclusion that the negative glow is independent of current and dependent upon some other form of discharge, and no alternative to fixing the responsibility upon the cathode rays.

An arrangement was next devised for photographing the spectrum of all parts of the tube simultaneously, in order that any local differences might be more clearly shown and recorded. A metallic slit about 15 cm long was placed immediately in front of the discharge tube and parallel to it. By means of two lenses of about 30 cm focal length, one simple and the other achromatic, and a large Rutherfurd compound prism, a spectrum was thrown on a photographic plate. The arrangement was

[1] *Berichte d. k. Akad. Berlin*, 1886. [2] *Wied. Ann.*, 65, 440, 1898.

crude, and not capable of accurate or permanent adjustment; nevertheless some of the results, especially those dependent upon altering the nature of the discharge, are, perhaps, of sufficient interest to warrant the reproduction of a few of the plates.

The principal characteristics of the spectrum of the simple induction discharge at different pressures are shown in Figs. 1 and 2, Plate XIX, corresponding to pressures of about 10 and 1 mm respectively. The wave-lengths of the principal negative pole bands are indicated, and an iron comparison spectrum is also given. Alongside of Fig. 1, is a diagram of the vacuum tube, showing the corresponding appearance of different parts of the discharge. In Fig. 1 the first dark space extends less than a millimeter from the cathode, the second is nearly a centimeter long, and the negative bands barely stretch across it. The almost perfect symmetry of the glow is well shown. It is to be noted that all the positive pole bands appear at the cathode, and are about as intense as in the positive column. They extend faintly across the second dark space, as well as the first, so that these spaces are by no means absolutely dark. Sufficient exposure would always bring out all the positive bands throughout the dark spaces.

In Fig. 2 the first dark space is several millimeters long, while the second is driven up to the positive column by the negative glow; but the strong negative pole bands run across to the anode, with uniformly diminishing intensity. Here also the negative pole bands appear to be symmetrical on both sides of the cathode, while the positive bands are somewhat stronger toward the anode. This is probably due to the fact that the discharge was not strictly uni-directional, so that feeble positive alternated with strong negative discharges from *C*. Similar effects were observed on several plates at low pressures, but not at high pressures. Several striations of the positive column are shown, with marked variations in the intensity of the positive bands, but so little in that of the negative bands that it is suggested that the latter run through the positive column without any variations except a gradual decrease of intensity, the small differences observed being due to the underlying weak

positive bands. Similar effects were observed on several plates, and show that the negative glow is independent of the changes of potential gradient found in the striations and the second dark space. In the first dark space, however, the bands are somewhat weaker than they are at the beginning of the glow. It is interesting to note that the *F* line, which frequently appeared, and two strong lines which appear to be λ3962 and λ3944 the longest lines of aluminum, are strongest in the first dark space. These aluminum lines appeared on nearly all the plates, increasing in intensity as the pressure diminished, and extend, not the entire length of the negative glow, but only about as far as the "spluttering" of aluminum on the walls of the tube could be observed. This shows that the particles of metal detached from the electrode assist in the discharge, and can give a spectrum, although the contrary is sometimes assumed, and that this spectrum is probably due to aluminum in the atomic state, not in the form of cathode corpuscles, in which case the lines would extend throughout the negative glow.

II. EFFECTS DEPENDING UPON NATURE OF DISCHARGE.

It was found that some interesting changes resulted from altering the nature of the discharge, as in passing from the simple discharge to that with a spark-gap in the circuit and a condenser in parallel, or in adding self-induction to the above. Some of these effects, such as the transition from the line to the band spectrum, are well known, but others seem to have previously escaped observation.

These changes are described in detail below, with references to figures in some cases.

Pressure 100mm (Plate XX, Fig. 3).—Simple discharge (*A*): The negative pole bands are faint and very short. The aluminum pair is absent. Self-induction (*B*):—There is a remarkable change in the appearance of the spectrum. The negative pole bands are strong across the entire field. The positive pole bands are very weak. The aluminum pair appears at both electrodes, and some faint aluminum lines at one electrode. Condenser (*C*):—The well-known line spectrum

appears, with some weak bands. The aluminum pair appears, but is not so strong as with self-induction. The appearance of the condenser discharge was dazzling white; that with self-induction was a light pink.

Pressure 40 mm (Fig. 4). — Simple discharge: As in previous case, except that the negative bands are longer. Self-induction (*A*): The negative pole bands are strong across entire field, but the contrast with the positive bands is not so strong as at 100 mm. The aluminum lines were weak. Condenser: Line spectrum as before. Tesla current (*B*): Very much like spectrum with self-induction, but the negative bands are relatively a little weaker. Note that this spectrum appears identical as far as relative intensities is concerned with the condenser spectrum (*B*) of Fig. 5.

Pressure 10 mm (Fig. 5). — Simple discharge: See Fig. 1, Plate XIX. Self-induction (*A*): Like (*A*), Fig 4, except that the positive bands are possibly relatively a little stronger. Condenser (*B*): The lines have changed to bands, but the negative bands are relatively not quite so intense as with self-induction. The spectrum is like that with the Tesla current at 40 mm. The condenser current was far more intense than the Tesla current; the self-induction current was of intermediate intensity. The relative intensities of the different discharges may be inferred from the fact that usually the times of exposure with condenser, self-induction, simple, and Tesla currents were about 10 minutes, 15 minutes, 20 minutes and 1 hour respectively. Where the relative differences in the spectra are small, perhaps the best estimate may be obtained by comparing the head of λ 3914 with the head of the neighboring positive band on the left. With self-induction in every case the former is more intense; with condenser or Tesla current they are about equal.

Pressure 2 mm (Fig. 6). — Appearance about as before, except that negative glow is much longer with simple discharge. Spectra with self-induction and with condenser are shown in (*A*) and (*B*). (*C*) and (*D*) will be described later.

Pressure 0.7 mm. — Simple discharge. — Negative bands run across to anode, and positive bands are very weak. Positive

PLATE XX.

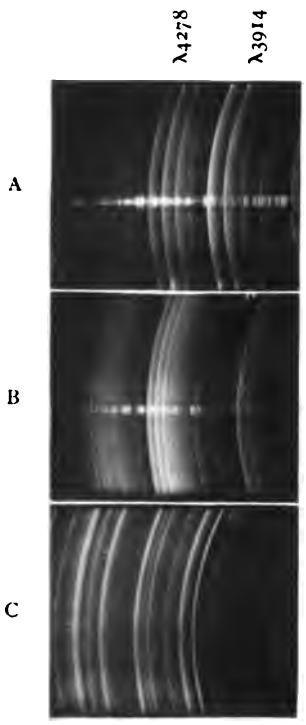


FIG. 3. P = 100 mm.

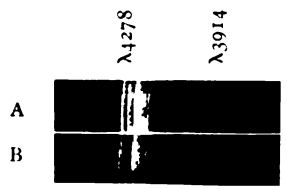


FIG. 4. P = 40.

FIG. 5. P = 10.

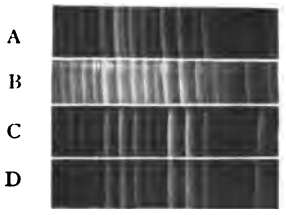


FIG. 6. P = 2.

column is about 4mm long. The spectra with self-induction and with condenser are hardly distinguishable, and the negative bands, while present throughout tube, are hardly so intense as one would expect from their appearance with the simple discharge and the low pressure. They by no means predominate over the positive bands.

With a slowly oscillating current through a vacuum tube, such as can be obtained when a condenser, without spark-gap, is in series with the tube (in which case the frequency is twice that of the primary break) both ends of the tube give the negative glow, which is limited to the ends. The question arises: why, with very rapid oscillatory discharges, do the negative bands appear from one end to the other? Goldstein[1] noted that with very intense discharges the negative bands appeared in all parts of the tube, and he seems to have associated this effect with the intensity of the discharge. We see, however, that the spectrum of the feeble Tesla discharge at 40mm is identical with that of the strong condenser discharge at 10mm. (At 40mm the condenser discharge gave a line spectrum, and at 10mm the tube was broken before a Tesla exposure could be made, so comparison at the same pressure was not made.) In all cases there was an enhancement of the negative bands by self-induction, although this effect became small at low pressures; but the self-induction discharge is weaker than that of the condenser. The electromotive intensity of the Tesla current is greater than that of the condenser, its intensity (energy) is less, and its oscillatory frequency is of the same order. The self-induction discharge is intermediate in energy, its frequency is much less than that of either condenser or Tesla discharge. It seems that we should look for the cause of the observed differences, not to electromotive intensity or energy (although they may take a part), but in altered frequency of the discharge. Just how this time element affects the problem is next to be considered.

A clue to the cause of the presence of the negative bands through the tube, and also to the effect of self-induction, is found in two important papers by Spottiswoode and Moulton.[2] They show that,

[1] *Berichte d. k. Akad. Berlin*, 1876 p. 281; *Wied. Ann.*, 15, 280, 1882.

[2] *Phil. Trans.*, 1879, 1880.

in general, the time required for the discharge of negative electricity from an electrode is greater than that required by positive, and that in some cases discharges may take place from parts of the inner walls of the tube under the action of strong electrostatic stresses. Later J. J. Thomson[1] showed that the positive luminosity starts at the anode, travels at a finite rate toward the cathode, and even in a long tube will arrive at a point near the cathode before the negative glow. On account of the lag in the negative discharge, the walls of the tube, on which electricity is separated inductively, may assist the cathode in neutralizing the positive electricity, and this general discharge may account for the general presence of the negative bands when intense discharges are employed. If, however, the rate of oscillation is very rapid, as in the condenser discharge, the interval between two oscillations may be too short to allow complete discharge of the negative electricity—in other words, the discharge may be, as Spottiswoode and Moulton found in some cases, almost uni-polar, taking place only from the anode, and the negative bands will be weak. On introducing self-induction the oscillations become slower and a more complete discharge of negative electricity may result, thus enhancing the negative bands. The intensity or temperature of a discharge may determine the production of a line or band spectrum, but it seems at least possible from these considerations that the appearance of the negative bands throughout the tube is only an indirect effect of the intensity, depending directly upon the cathode rays emitted from the walls of the tube under the action of electrostatic stresses. It might be anticipated that the metallic screen with slit near the tube would magnify these effects, but the same results were obtained when a slit ruled on a developed photographic plate was substituted.

If this hypothesis be correct, the action of a cathode or X-ray tube with spark-gap and condenser in parallel should be stimulated by adding self-induction. It is probably known that this is the case, although previously unknown to the writer. A test substantiated this expectation; self-induction added to the cir-

[1] *Recent Researches in Electricity and Magnetism*, p. 116.

cuit of a cathode ray tube increased the fluorescence several times, although the spark-gap showed plainly that the intensity of the discharge was thereby diminished.

K. Wesendonck[1] showed that the walls of a vacuum tube seem to facilitate the negative discharge.

Another test was made by placing the electrodes previously used in a large glass bulb about 15 cm in diameter. The discharge through this tube was very unsteady except at pressures under 10 mm, so that it was difficult to get photographs at higher pressures. At about this pressure both the condenser and self-induction gave the continuous negative bands; but they were very feeble compared with those obtained with the small tube. At a pressure of about 2 mm the heads of the $\lambda 4278$ and $\lambda 3914$ bands can barely be detected in the spectrum of the condenser discharge (Fig. 6, *D*), but with self-induction they are seen faintly at the electrodes, while elsewhere there is not a trace of them to be seen (Fig. 6, *C*).

This is strong corroborative testimony, but is, of course, not absolutely conclusive as excluding the effects of intensity until we know how much the latter is modified by the size of the tube. With the large tube, however, the discharge confined itself within a limited space, and while not so intense as at the same pressures in the small tube, the difference in intensity did not seem sufficient to account for the difference in spectrum. In the large tube the discharge seemed more noisy and disruptive, just as though the cathode rays discharged from the walls of the smaller tube might have had some effect in facilitating the discharge. With both condenser and self-induction the large tube fluoresced very strongly, as though vigorous actions of some sort were going on at its surface.

All of these facts are suggestive, if not conclusive, and show that the effects of the walls of vacuum tubes cannot safely be neglected in the study of spectra.

It seems possible that the peculiar effects of self-induction noted by Hemsalech[2] may be accounted for in a similar way, not as a result of modified intensity of discharge or temperature,

[1] *Wied. Ann.*, 41, 473, 1890. [2] G. HEMSALECH, *Thèse*, Paris, 1901.

but as dependent upon the time allowed for the discharge of negative electricity. Spottiswoode and Moulton showed that the time required for the discharge of "molecular streams" from the cathode was of the same order as that for the discharge of negative electricity. If these molecular streams consist of the particles which splutter from the cathode, the initial spark of a rapid oscillatory discharge must pass through air because there has been no time for sufficient metal to pass off; but by the time the second oscillation begins some metal has passed off and carries the discharge. When the oscillations are made slower by self-induction, there will be enough metal present from the beginning to carry the discharge. Why the self-induction brings out the negative-pole bands of air with some metallic electrodes is not so clear; but perhaps a difference in the time of discharge of molecular streams with different metals as compared with that of the discharge to air may account for it.

It seems likewise possible that some vacuum tube effects which have been held to suggest electrolysis may also depend upon the relative times required for negative electricity to discharge itself into different gases and metallic vapors. The spectrum of traces of hydrogen and of metallic vapors in vacuum tubes may appear stronger at the cathode simply because the discharge to them may take place more easily and more rapidly than to the gas, so that they can act as stepping-stones between the cathode and the latter. At the anode the converse may be true.

It seems obvious that this and other important questions regarding the division of the current in mixed gases, differences at the electrodes, and the effect of different forms of electric discharge can be most satisfactorily dealt with by photographing the spectrum of all parts of the tube simultaneously. An effort will be made to carry these preliminary results farther with improved methods which will allow accurate adjustment, greater dispersion, and an extension of the observations to the ultra-violet.

The conclusions suggested by this preliminary work may be summarized as follows:

1. The negative pole spectrum of nitrogen is due to cathode

rays. The intensity of the negative bands with simple discharge is dependent solely upon distance from the cathode, and is independent of changes of potential gradient in the tube.

2. The positive bands are present in the spectrum of all parts of the tube, but are of variable intensity, which seems to depend in large part upon the potential gradient.

3. The presence of the negative pole bands in the spectrum of all parts of a tube when strong discharges are used is due to the discharge of cathode rays from the walls of the tube.

4. The enhancement of the negative bands by self-induction is due to the slower rate of oscillation, permitting a more complete discharge of negative electricity, which requires more time than the positive.

UNIVERSITY OF CALIFORNIA,
March 1, 1903.

ON A NEW RELATIONSHIP BETWEEN ARC AND SPARK SPECTRA.[1]

By J. HARTMANN.

IN collaboration with Dr. Eberhard[2] I recently pointed out that in the spectrum of the arc under water the lines otherwise observed only in the spark spectrum, appear in the case of different metals—a phenomenon also previously noted by other observers with the arc burning in a hydrogen atmosphere. I have now succeeded in finding a way by which the arc spectrum can be transformed into the spark spectrum in atmospheric air also, hence without any alteration of the dielectric. I desire briefly to communicate in what follows these experiments, which give us a new insight into the processes involved in electric discharges.

My experiments were at first concerned with the spectrum of magnesium, which is of such great importance in astrophysics. As early as 1888 Liveing and Dewar[3] had pointed out that the line at $\lambda 4481$, which otherwise occurs only in the spark spectrum, appears also in the arc spectrum if this is made to pass between two magnesium rods, and not between carbon poles into which the metal was introduced, as was the general practice. I had photographed the spectrum of the arc between metallic poles several times, and was surprised that I always found only faint traces of the line $\lambda 4481$, while this had been obtained on the spectrograms published by Crew[4] in 1895, taken with the "rotating arc,"[5] as one of the strongest lines, and at least ten times as strong as the neighboring "arc line" $\lambda 4352$. Basquin[6]

[1] Translated from advance proofs, furnished by the author, of a paper to appear in the *Sitzungsberichte der K. Akad. der Wiss. zu Berlin*.

[2] ASTROPHYSICAL JOURNAL, **17**, 229–231, 1900. [3] *Proc. R. S.*, **44**, 241, 1888.

[4] *Normal Spectrum of the Magnesium Arc.* Evanston, 1895.

[5] HENRY CREW and ROBERT TATNALL, "On a New Method for Mapping the Spectra of Metals," *Phil. Mag.* (5), **38**, 379, 1894.

[6] "The Spectrum of Hydrogen Given by the Metallic Arc of Tin, Copper, Silver," etc., ASTROPHYSICAL JOURNAL, **14**, 13, 1901.

also reached the conclusion that the line 4481 appears in the rotating arc, but not in the arc between stationary metallic poles in the air.

In the method of the rotating arc employed by Crew, numerous interruptions of the current occur, and we might suspect that the extra currents associated with the formation of the spark might have produced a spectrum similar to that of the spark, since the circuit contained coils of wire when the direct current from the dynamo was used. To exclude this possibility I have always used the current from a storage battery of 60 cells, and have avoided all coils of wire in the circuit, aside from a few turns of a resistance coil from which no self-induction could be feared. The observations described below were made on photographs taken with stellar spectrograph No. I of the Astrophysical Observatory.

In order first to determine whether the line at λ 4481 appears only at the instant of lighting or extinguishing the arc, or whether it appears only at definite positions in the arc, a photograph was made of the spectrum of the metallic arc when burning quietly, those two instants being excluded while the images of the electrodes were projected on the slit. The current was of about 6 ampères at a voltage of 120. This plate shows the line λ4481 to be actually present in the spectrum of the quiet metallic arc, but also that it appears principally at the electrodes, as in case of the spark discharge, while it is hardly visible at the center of the arc of about 8 mm length. But even close to the electrodes the intensity of the line was not great, certainly very much less than that of the line λ4352. If we denote with J the intensity of the spark line λ4481, that of the arc line λ4352 on the same plate being taken as unity, we may summarize the result of this plate as follows:

METALLIC ARC IN AIR 8 mm LONG, AT 120 VOLTS AND 6 AMPÈRES.

At the electrodes - - - - -	$J = 0.1$
At the center of the arc - - - -	$J = 0.01$

This experiment would therefore be insufficient to explain the high intensity ($J = 10$) of the line on Crew's plate, for even if it were desired to use light only from the immediate neighbor-

hood of the electrodes, the line at λ4481 would still appear decidedly fainter than the arc line λ4352. I then began experiments with different current strengths at a constant voltage of 120, since I suspected that the current strength might have an influence on the intensity of individual lines. I employed incandescent lamps for resistance in order to cut down the current sufficiently without introducing resistance coils.

Since the occurrence of the line in the quiet metallic arc was proven by the above-described experiment, I no longer projected the arc upon the slit and no longer excluded from the plate the instant of starting and extinguishing the arc. Hence J denotes the intensity of the line in the total light radiated by the arc. These experiments now gave the quite astonishing result that the 4481 line becomes the stronger as the current strength is lessened. Repeated photographs gave the following results:

METALLIC ARC IN AIR AT 120 VOLTS.

At 8 ampères	- - - - -	$J = 0.03$
6 "	- - - - -	0.05
3 "	- - - - -	0.5
0.8 "	- - - - -	3
0.4 "	- - - - -	10

These figures clearly show how the intensity of the spark line relatively to the arc line continuously increases with decreasing current strength; and indeed so regularly that it would be easy, for instance, to specify the current strength at which the two lines would appear of equal intensity; this would occur at about 2 ampères.

In view of the result of these experiments it would be difficult to maintain the assertion that the line λ4481 is an evidence of a very high temperature of the metallic vapors, for the development of heat is certainly not greater in the arc at 0.4 ampères than at 8 ampères with the same voltage. With the latter current strength the electrodes were so greatly heated as readily to melt, and the vaporization of the metal was so great that the poles could be separated by more than 10mm; while at 0.4 ampères the electrodes were entirely cool and the arc could not be lengthened beyond 0.5mm at most, and then it burned for

hardly more than a second, and had to be very frequently started up.

It might now perhaps be asserted that close to the electrodes the temperature of the vapor is so much higher than at other parts of the arc that, as a direct consequence of the small separation of the poles required by the low current, the *average* temperature of the whole arc was higher in the latter case than if the arc had a stronger current and greater separation of the poles, and could perhaps cool off more near its center. But were this assumption correct, then in the first-described arrangement of the experiment with the projection of the portion of the arc close to the pole, the intensity of $\lambda 4481$ should have been correspondingly high—in fact, much higher than the intensity of the arc line $\lambda 4352$. This, however, was not most remotely the case, since the intensity in that case also was only 0.1. There remains, therefore, nothing else to assume than that the conditions for the development of those molecular vibrations to which the line $\lambda 4481$ corresponds were much more favorable in the small arc in spite of its lower, or at least certainly not higher, temperature than in the larger arc.

It cannot at present yet be stated with certainty what constitutes these "conditions," but we might hope to find an explanation of the observed phenomenon in the following two facts. As already stated, the small arc went out very frequently, and had to be started several hundred times during the exposure, which lasted for about half an hour on account of the small brightness of the point of light, so that only a comparatively short time of quiet burning followed each rekindling. The observed changes of intensity could be reasonably explained, if we assume that at the moment of kindling or of extinction another spectrum occurred in which the line $\lambda 4481$ was strongly predominant. The correctness of this explanation could be determined by successive exposures of the arc with a rotating mirror or on moving films, which have recently been carried out in a very complete manner, for instance, by Crew and Baker.[1]

[1] "On the Thermal Development of the Spark Spectrum of Carbon," ASTROPHYSICAL JOURNAL, 16, 61, 1902.

Should the explanation prove to be correct, it is to be noted that this introductory spark should by no means be confused with the commonly observed induction spark; with the precautions that I have described it can possess no elevated tension and temperature. We should rather conclude only that at the moment of the first passage of the electricity from one electrode to the other particles of metal are torn off which even at low temperature execute vibrations corresponding to the line λ4481.

The second fact which could be adduced in explaining the phenomena I have observed would make the line λ4481 appear, not as a high-temperature line, but even as one of low temperature, if such a designation were indeed at all permissible. According to an observation by Schenck,[1] which I was also able to confirm, the intensity of the line 4481 decreases in the *spark spectrum* if the electrodes are heated to the point of melting, so that the resistance in the spark gap is diminished by the active vaporization of the metal. I found further in another experiment that the intensity of this line decreases if the resistance in the dielectric is reduced by exhaustion. If we transfer this to the arc spectrum, we shall obtain a very good explanation of the phenomena in question. We may assume, according to Schenck's investigations, that the line λ4481 results from the vibrations of particles highly charged with electricity, and therefore it can appear only of reduced intensity, if at all, if such a charge is in any way prevented. Now, as soon as the electrodes are heated by the stronger current there occurs a vigorous vaporizing of the metal, the conductivity in the arc becomes greater, and the separate particles take on only very slight charges, so that the vibrations corresponding to the line λ4481 must always become weaker with increasing current strength.

As I have already remarked, the magnesium electrodes very soon melted when only moderate currents were used, and thus they immediately set a limit to further heating and vaporization. When using metallic electrodes the experimenter is therefore compelled by the purely incidental process of melting to main-

[1] "Some Properties of the Electric Spark and its Spectrum," ASTROPHYSICAL JOURNAL, **14**, 116, 1901.

tain the poles in a cool condition in some way, whether by artificial cooling, by the employment of a very small current, by the frequent interruption of the current, or by the continual introduction of new and cold parts of the metal into the arc. Crew employed the latter process in his rotating arc, and in consequence obtained so powerful a 4481 line, as above stated. The vaporization was similarly very slight in the small arc with 0.4 ampères, and the line accordingly was very strong, becoming more and more faint with the increasing vaporization of the electrodes.

The absence of this line from the carbon arc can be very simply explained in this way. If magnesium is introduced into the arc burning between carbon poles, the above mentioned limit is no longer set for the greater heating and vaporization of the metal, and the arc becomes a very good conductor, as might already be inferred from its greater length, whereby, in accordance with what has been said, the disappearance of the arc line is occasioned.

The ingenious experiments of Basquin[1] who simultaneously caused the arc and the spark to pass between the same electrodes, are also in full agreement with the view here developed. When a second spark gap was introduced into the circuit of the spark discharge, there appeared in the arc simultaneously the arc and the spark lines. If, however, the second spark gap was made very short, the spark lines disappeared completely in the arc, although the spark discharges passed through the arc as before. If the arc current was now interrupted, there was a duration of about a second before the spark began to pass in the ordinary noisy manner, and then the spark spectrum also first appeared. This experiment proved, first, that in the condition of the vapors prevailing in the electric arc those molecular vibrations are also possible to which the spark lines correspond; and second, that these vapors, even up to a second after the extinction of the arc, conduct so well that a high-tension current can pass between the electrodes without forming sparks, therefore without producing the spark lines.

[1] *Loc. cit.*, p. 15.

I would also revert here to the experiments which I made, in common with Dr. Eberhard,[1] using zinc electrodes. We succeeded, in the first place, in showing that the spark spectrum of this metal approaches the arc spectrum on artificial heating and vaporization of the zinc electrodes, in a manner analogous to the process observed by Schenck. But *per contra*, we did *not* succeed at that time with the reverse experiment, namely, of transforming the arc spectrum into the spark spectrum by cooling the electrodes with liquid air. This failure may, however, be simply explained if a very high temperature still prevails at the points of the greatly cooled electrodes; we might infer that this was the case from the fact that the arc could be drawn out to a length of several millimeters without going out. It is possible that a photograph would have shown a slight alteration in the relative intensity of the arc and spark lines, but on account of its difficulty this experiment was not made. But we did succeed in obtaining very distinctly the two zinc spark lines at $\lambda 4912$ and $\lambda 4925$ on reducing the current to 0.5 ampères and by using very large poles which conducted the heat well, although we had found hardly a trace of these lines in the arc with a current of about 6 ampères.

I have obtained a further confirmation of my observation on magnesium in the spectrum of bismuth. The very strong spark line at $\lambda 4260$, which does not appear in the carbon arc, appears in the arc between metallic poles only slightly fainter than the neighboring arc lines at $\lambda 4254$ with 6 ampères current; but at 0.3 ampères it still appears very distinctly, while the arc line has wholly disappeared.

With lead electrodes in the arc I succeeded even at 6 ampères in obtaining the two principal spark lines, at $\lambda 4245$ and $\lambda 4387$, although they do not appear at all in the carbon arc. In order to test still further whether the small amount of vapor between the electrodes is in fact a sufficient cause for the appearance of spark lines in the arc, I made the experiment with this metal in such a manner that the one electrode consisted of melted lead, heated to active incandescence and vaporization. When the

[1] *Loc. cit.*, p. 41.

same current of six ampères was passed across from this vaporizing mass of metal to a cold lead pole there no longer appeared any trace of the two spark lines in the arc spectrum. This observation accordingly also furnished a proof of the correctness of the view expressed above.

Since we might be inclined to assume that a high voltage is necessary for the production of the vibrations corresponding to the spark lines, which is indeed always present in the ordinary spark discharges, I made the further attempt of also bringing out the spark lines, which are under ordinary conditions lacking in the arc, merely by diminishing the tension while the current strength remained constant. This experiment was also completely successful. For this I again used magnesium poles, and employed the current of 20 volts and 4 ampères from ten storage cells. The development of heat was so slight that there was no longer any appreciable vaporization of the metal, whence the formation of an actual arc was excluded. The very small introductory spark, certainly hardly 0.1 mm long, which appeared every time the poles were separated from electrical contact, showed the line 4481 decidedly stronger than did the longer arc, in spite of the fact that it surely had no higher temperature than the arc with equal current strength and sixfold greater voltage. My photograph gave

at 20 volts and 4 ampères - - - $J = 10$

while from the figures above we get

at 120 volts and 4 ampères - - - $J = 0.3$

Although the consumption of energy is in this case reduced to one-sixth, the relative intensity of the spark line nevertheless has risen about thirtyfold.

All of my observations which I have here described, as well as the results of numerous other observers, of which I have mentioned only a few above, indicate that the spark lines do not correspond to a thermal radiation but rather to electroluminescence. This idea is not new, but was expressed in the following very precise manner in 1888 by Liveing and Dewar,[1] when they discovered the occurrence of the $\lambda 4481$ line in the

[1] *Proc. R. S.*, **44**, 241–242, 1888.

arc spectrum: "The observations, however, render doubtful the correctness of the received opinion that the temperature of the spark discharge is much higher than that of the arc. Heat, however, is not the only form of energy which may give rise to vibrations, and it is probable that the energy of the electric discharge as well as that due to chemical change, may directly impart to the matter affected vibrations which are more intense than the temperature alone would produce."

This clearly expressed view unfortunately received too little attention at the time, and in later years hypotheses have been founded on the wholly unproven assumption, rendered dubious by the above remark by Liveing and Dewar, that the spectrum of an incandescent gas is a function of only the *one* variable, "temperature," which hypotheses should render possible a direct conclusion as to the temperature of the luminous gas from the occurrence of individual lines. This would have been not without significance, especially for astrophysical research, although there exist other methods, free from objection, for determining the temperature of celestial bodies; but, as I have shown, such a conclusion is not permissible, at least in so far as the magnesium lines $\lambda 4481$ and $\lambda 4352$ are in question, since so far a proof is entirely lacking that the line $\lambda 4481$ always corresponds to a higher *temperature* than the line $\lambda 4352$. We should be inclined from my experiments described above to reach the contrary conclusion, but such a conclusion is also unpermissible, since, as is confirmed by all previous observations, the production of the spark lines probably is not related to the temperature. I do not regard it to be impossible that those molecular vibrations which cause the occurrence of the spark lines, whose excitation has hitherto been solely possible by the use of electric discharges, may occur exclusively by the introduction of energy in the form of heat; but for the correctness of any such assumption no laboratory demonstration can yet be adduced.

In view of what has been said, it may well be difficult further to maintain the hypothesis advanced by Scheiner[1] in 1894.

[1] "Die Temperatur an der Oberfläche der Fixsterne und der Sonne, verglichen mit derjenigen irdischer Wärmequellen," *Sitzungsberichte der k. Akad. der Wiss. zu Berlin*, 1894. p. 257.

Inasmuch as the line λ4481 appears very strong in the spectra of numerous stars belonging to Vogel's first type, while λ4352 is very faint or not present, Scheiner reached the conclusion that on these stars the temperature of the absorbing layer was approximately that of the electric spark, while the temperature of the stars of the second and third type, in whose spectrum the line λ4352 more strongly occurs, the temperature approximated more closely that of the arc. The above demonstration that the intensity ratio of the two lines denoted by J by no means increases with rising temperature, indeed probably stands in no direct relation to the temperature, causes these conclusions to lose their foundation as well as their force.

But there is still a further circumstance to be considered. As has been shown by the observations of Crew,[1] Basquin,[2] and Porter,[3] as well as by the above-mentioned investigation by Eberhard and myself, the dielectric has a great influence on the spectrum of electrical discharges. Now since all stars of the first spectral type are characterized by a great preponderance of hydrogen in their atmospheres, it would seem that only observations which have been made in an atmosphere of hydrogen should be adduced in interpreting the spectra of those stars. It was already known from the publications of the first-named observers that the hydrogen atmosphere so favored the appearance of the line 4481 that it also came out very strong in the arc, and I therefore suspected that my experiments in a hydrogen atmosphere discussed above would furnish still larger values of J. This expectation was most fully confirmed, for a photograph of the arc burning in a hydrogen atmosphere at 120 volts and 0.3 ampères showed *no trace of the arc line*, while the line 4481 came out perfectly sharp and as the strongest line of the whole spectrum; in addition to this line the spectrum further contained only the three lines λ3830, λ3832, and λ3838, and

[1] "On the Arc Spectra of Some Metals, as Influenced by an Atmosphere of Hydrogen," ASTROPHYSICAL JOURNAL, 12, 167, 1900.

[2] *Loc. cit.*, p. 1.

[3] "The Influence of Atmospheres of Nitrogen and Hydrogen on the Arc Spectra of Iron, Zinc, Magnesium, and Tin, Compared with the Influence of an Atmosphere of Ammonia," ASTROPHYSICAL JOURNAL, 15, 274, 1902.

very faintly the *b* group. Accordingly the strength of the 4481 line and the absence of the arc lines of magnesium in stellar spectra of the first type can no longer be regarded as striking. It would seem to be more important to give some attention to the occurrence of the two above-mentioned magnesium triplets. These are the most permanent lines in the portion of spectrum I have investigated, occurring with every form of luminosity, even in the spectrum of burning magnesium, and we may well assume that the corresponding molecular vibrations are of a purely thermal origin. Nevertheless, they do not occur in many star spectra which contain a strong 4481 line, as Keeler[1] has already remarked of the *b* group.

The last-mentioned spectrum photograph shows a further peculiar appearance which I should not leave unmentioned. While those two triplets and the line 4481 are all that remain of the whole magnesium spectrum, there further appear on the plate as impurities of the metal, the three calcium lines λ 3934, λ 3969, and λ 4227, as well as the lead line λ 4058—all of them lines which, like the triplets, are easily reversible and make their appearance at low temperatures. Just as the method of oscillating discharges first employed by Schuster and Hemsalech, which permits a gradual transition from the spark spectrum to the arc spectrum, so also the method of the "small arc" described here, which represents the inverse transition from the arc to the spark spectrum, should be suitable for furthering the discovery of related groups of spectral lines, and hence for the study of the processes of luminosity.

ASTROPHYSICAL OBSERVATORY,
Potsdam, February 26, 1903.

[1] "The Magnesium Spectrum as an Index to the Temperature of the Stars." *Astronomy and Astro-Physics*, 13, 660, 1894.

DISCUSSION OF A QUESTIONABLE TYPE OF TEMPORARY STARS.

By J. G. HAGEN, S. J.

IT is understood that by a "temporary star" is meant a star which has been seen to vary in brightness a single time, although this variation may have extended over months or years, and may have been composed of fluctuations of various amplitudes. If two or more variations of this kind have been observed, the star is usually called a periodic variable, regular or irregular.

When the outburst of a temporary star was so short in duration that it was seen by one observer only, without independent confirmation by another, the record has generally been received with suspicion and accepted with explicit reserve. This is the case with two stars that have found a place in the catalogues of variable stars. There are three more instances of an earlier date, one of which has never been published. A simultaneous inspection of these five cases and of the authorities on which they rest will probably remove all reasonable doubt as to the existence of short variations of this kind. The astrographic charts, which give three successive exposures to the stars, may reveal some more variations of this character by abnormal intensities in the points of the small stellar triangles.

The five temporary variations of short duration will be related in the inverse order of time, beginning with the most recent. They can very conveniently be put in three groups.

I. TWO KNOWN TEMPORARY STARS OF SHORT VARIATION.

1. The star *U Scorpii* is No. 73 in *Schönfeld* I, No. 87 in *Sch.* II, and No. 5860 in *Chandler* I-II. When Pogson observed its variation in 1863, he was government astronomer in Madras, and had a ten years' experience on variable star work. He saw the star decrease from 9th mag. to invisibility in twelve days. The star has never been recognized by anyone else, but is called by Pogson a well-assured, rapidly changing star. The chance for

a second observer to see the variation was probably confined to one or two days. When the star was below mag. 10 it would have been very difficult to identify it without a chart.

2. The other star is *T Bootis*, No. 58 in *Sch.* I, No. 70 in *Sch.* II, and No. 5097 in *Ch.* It is almost of exactly the same character as the former, and was observed in England only three years previously, in 1860, by Pogson's brother-in-law, J. Baxendell. On April 9 it was of magnitude 9¾, on April 11 of mag. 10, on April 20 of mag. 12.8, and on April 23 it was invisible (in Baxendell's own scale). The variation was observed during twelve days, but the chance for independent confirmation could hardly have extended over more than two days.

II. TWO INSTANTANEOUS VARIATIONS OF LIGHT.

1. An instantaneous fluctuation of light was observed by Pogson in the star *U Geminorum* on March 26, 1856, while he was preparing a chart for this variable. Pogson was then assistant astronomer at the Radcliffe Observatory in Oxford, where he had commenced working on his intended atlas of variable stars three years previously. His observation has lately been published in the *Astronomical Journal* (**22**, 127, 1902), from which we quote what is essential: "The variable subject to strange fluctuations at intervals of 6 to 15 seconds. . . . . when the adjacent stars were quite steady. At times it surpassed the star 8.9, at others it quite vanished. . . . . Watched it for half an hour, with powers 54, 65 and 95 on the equatorial." The comparison star 8.9 is probably No. 5 in the *Atlas Stellarum Variabilium* (Series II, 2815). The range of these fluctuations seem to have been the same as that of the periodic changes of this variable.

2. A similar fluctuation of light was observed by Heis in the star ζ *Lyrae* on September 26, 1850. Heis observed the variable star β *Lyrae* from 1841 to 1868, and used as comparison stars, among others, ϵ and ζ *Lyrae*. Each of these has the magnitude 4½, but appears double to the naked eye on very clear nights. Heis found the relative brightness of these two comparison stars slightly changeable. On the date mentioned the observing book (which will soon appear in print) has the following note: ζ *Lyrae*

became for a ***moment very bright***, and then again faint" ("*ζ Lyrae* wurde einen *Moment sehr hell* und hierauf wieder dunkel").

The star probably did not reach the brightness of the neighboring first-magnitude star *α Lyrae*, else Heis would hardly have omitted the comparison. The time of the fluctuation must have been too short to be measured, because Heis was in the habit of recording the time in whatever he observed.

The short note deserves the more confidence, as Heis had been familiar with this comparison star for nine years; further, as his eyes saw the stars as distinct points without false rays (see E. Heis, *De Magnitudine relativa*, etc. Monasterii, 1852, page 4), and finally as his observations of variable stars show a rare precision in estimating steps.

III. AN OLD OBSERVATION OF CHRISTOPH SCHEINER, S.J., IN 1612.

This observation is described by Scheiner in his second letter to Welser, dated April 14, 1612, and printed in his book: *De Maculis solaribus*, etc. Augustae Vindelicorum, 1612. As this book is rare, Winnecke republished the Latin text in the *V.J.S.* (13, 283–288, 1878).

1. What Scheiner *observed* may be summarized as follows: He gives nine drawings of the field of view in his telescope, which he calls the best he ever saw for observing *Jupiter's* satellites. They were made when the sky was very clear and without moonlight. They give the relative positions of the planet *Jupiter* and its satellites, on March 29, 30, 31; April 1, 3, 5, 6, 7, 8. In the same field a star denoted by the letter E is drawn, about 6′ south of the planet. Scheiner's argument that this star had a motion of its own is evidently not conclusive. It is based on eye-estimates only of its motion relative to the planet, which became stationary a few days later, about April 10. Scheiner was prejudiced on this point by the hope of discovering a fifth satellite of *Jupiter*.

What he says about the light of this star is of more importance. The first drawing, on March 29, shows nothing of the star in question. On March 30 Scheiner saw it for the first time (*primus illius contuitus mihi obtigit*), and, as is usual with tempo-

rary stars, at once very bright and large (*lucentissimam et maximam*), like any of *Jupiter's* satellites (*quanta . . . . hactenus quaevis conspecta est stella Jovialis*), and, as he adds a little later, much brighter and larger than the fainter satellites, which he describes as much inferior in light and size (*lucis et corporis multo quam potiebatur stella E minoris*). During the following days it fell below the fainter satellites, until, on April 8, it was seen with great difficulty (*ægerrime*) in a very clear sky. After that date it was never seen again, although carefully looked for under favorable conditions. Scheiner had several smaller telescopes with which the star was seen at its greatest brilliancy. He also showed the star to his friends.

2. *The identification* of this star was made by Winnecke.[1] With the help of Küstner he found from Leverrier's tables of the Sun and *Jupiter* that the position of Scheiner's star was identical with the place of *B.D.* + 15°2083, $8^m.5$, within the residuals 0^s, $1'.4$. This star is not a periodic variable. It has been constantly recorded as of magnitude 7–8 or 8 or 8.5, by Lalande (1796), Weisse (1825), Struve (1826), Preuss (1833), in the *B.D.* and the *H.P.* Winnecke himself watched it for seventeen years. Among the thirteen observations in the *H.P.* (Vol. 24, page 220) there is one large residual from the mean magnitude 8.0, amounting to +2.4, which would make the star $10^m.4$; yet the series 991, in which this residual occurs, has the accompanying remark: "thin clouds noticed in various parts of the sky" Vol. 23, Part I, page 39). Clouds will explain any positive residual, and also any negative residual, if the pole star, which served as comparison star for the meridian photometer, was covered by them.

3. For the *range* of the light variations we can deduce from Scheiner's description an approximate upper limit. The literature on the photometry of *Jupiter's* satellites is put together in Dr. Müller's *Photometrie der Gestirne*, pp. 385–393. The mean

[1] Winnecke makes no reference to the possibility that Scheiner's star might have been a planet, stationary near *Jupiter*. *Uranus* and *Vesta* reach the brightness of *Jupiter's* satellites, but would hardly show variations as those described. We leave this question open and base our further discussion on Winnecke's opinion that the star in question was a fixed star.

results of Engelmann's, Pickering's, and Spitta's photometric measures for the faintest and brightest satellites are the following: satellite IV: $6^{m}.5$, and satellite III: $5^{m}.5$, the other two ranging between. These figures hold for "mean opposition," *i. e.*, for the opposition of Earth and *Jupiter* at their mean distances, and for the mean albedo or reflecting power of the satellites. The reduction to quadrature, at which Scheiner observed, would diminish these magnitudes only $0^{m}.1$ or $0^{m}.2$, as may be concluded from Engelmann's reductions of Dr. Auwers' visual observations (*Helligkeitsverhältnisse der Jupiterstrabanten*, Leipzig, 1871, page 67), while the varying albedo may change these figures by $\pm 0^{m}.5$ (Engelmann, *loc. cit.*, page 69).

If we now remember that Scheiner calls his star E as bright as any satellite, and much brighter than the fainter satellites, we are safe in saying that his star had reached the sixth magnitude. In ten days it faded away to invisibility. This limit, in a telescope which was considered good and showed the fainter satellites well, should be at least as low as eighth magnitude, the normal brightness of star E. What the lower limit of the fluctuation actually was cannot be gathered from the data on hand.

Winnecke was familiar with Scheiner's writings and judges him as "thoroughly trustworthy in communicating what he had seen."

In conclusion these five instances seem to show that short variations in the light of stars, although noticed by only one observer, cannot be rejected when assured by good authority. Should Heis' remark on ζ *Lyrae* be considered too short or too casual to inspire full confidence, it seems that Scheiner's star in the constellation *Leo* has as much claim for enlistment among the temporary stars as *U Scorpii* and *T Bootis*. The star *U Geminorum* will always remain among the periodic stars, but its extraordinary fluctuation in 1856 should be borne in mind.

GEORGETOWN COLLEGE OBSERVATORY,
February 1903.

ON THE EFFECT OF CIRCUIT CONDITIONS UPON THE WAVE-LENGTHS OF SPARK LINES.

By NORTON A. KENT.

IN the summer of 1901 Professor Charles E. St. John, of Oberlin college, and the writer of this article, at the suggestion of Professor Hale, undertook to verify certain results obtained by Haschek[1] in regard to the relative position of the arc and spark lines of titanium.

Haschek had found that in some cases the displacement of spark lines toward the red from the position of the corresponding arc lines exceeded 0.1 tenth-meter, *e.g.*, 0.13 for *Ti* λ 3900.68. These were unusually large displacements.

After continuing the investigation alone for a short time[2] the writer was obliged to defer its completion to take part in a more important study (that of the spectrum of the electric discharge under liquids), and only recently has it been possible to return to the former subject.

APPARATUS USED.

The apparatus used was, in its general features, the same as that employed in the investigation of the spectrum of the electric discharge in liquids, and a description of the same will appear in the *Publications of the Yerkes Observatory* soon to be issued. For the present purpose a brief description will suffice.

A 5-horse-power motor drove a 3.3 kilowatt alternator (125 cycles per second, potential 110 volts, current 30 ampères), which fed a 1 kilowatt Lakon transformer. An adjustable condenser gave a capacity of from 0.0066 to 0.0560 microfarads. The arc and spark apparatus were set upon the circumference of a circle at the center of which was a plane silver-on-glass mirror, capable of being rotated about a vertical axis. The light could then be thrown from either piece of apparatus to a concave

[1] ASTROPHYSICAL JOURNAL, 14, 181, 1901.

[2] *Ibid.*, 14, 201, 1901.

mirror fitted with slow-motion screws which gave motions about horizontal and vertical diameters. From this latter mirror the light passed to the slit of the spectroscope—Rowland mount: radius of curvature of grating 10 feet; 15,000 lines per inch. The first spectrum alone was used; the dispersion was 5.63 t. m. per mm. The arc (direct current) was so arranged that after contact the carbons could be moved apart to a stop; this gave an arc of constant length. A current of 18 ampères was used in all cases. Before every exposure a small piece of metal, several milligrams in weight, was placed in a shallow hole in the lower carbon which formed the positive pole of the arc. These precautions were taken to avoid any possible change in the position or intensity of the standard lines of the comparison spectrum. Suitable shutters made possible the method of double exposure of the comparison spectrum. The arc was exposed for ½ second; then the spark for various lengths of time; then the arc again for ½ second, this third exposure being superimposed upon the first.

It will be noticed that the mirror method—suggested by Professor Hale—furnishes an easy means of avoiding the possibility of any false shifts due to unequal illumination of the grating. The short time of exposure employed for the arc might introduce such shifts unless all parts of the beam were exposed an equal time. For this reason, and also to facilitate obtaining arc spectra of the same intensity upon every plate, the following arrangement was used: a camera shutter, set less than an inch in front of the slit and operated pneumatically by a long tube and bulb, was fitted with a horizontal rectangular window. This window was so arranged that when the shutter was set the slit of the spectroscope was covered. When the bulb was pressed the beam was disengaged from the left side, and subsequently covered from the left side. Then the shutter automatically returned to its former position, repeating the above process in the reverse direction. Thus each of the four sets of exposures of ¼ second duration was complete in itself. As previously stated, the shutter was close to the slit, and on this account as well it seemed improbable that any error due to unequal illumination of the

grating would enter. The device worked most satisfactorily. The arc was thrown upon the shutter by setting the plane mirror properly and making final adjustments (always small) with the concave mirror. During this procedure the image was viewed with a telescope. Then the bulb was pressed and the exposure made.

The spark terminals were formed of pieces of titanium carbide or "cast titanium," 85 per cent. *Ti* and 15 per cent. carbon, purchased of Eimer & Amend, of New York. The shape of the terminals was not regular; but rough edges were presented to each other and the pieces shifted often to obtain new edges. It is to be regretted that time did not permit greater care in this respect, for it is well known that the form of the surface alters the potential of discharge, and this *may* affect the position of the line. Still it may be stated that the results show no errors which can be traced directly to this cause.

The displacements were measured with a Zeiss comparator of double microscope pattern, under a magnification of about eight diameters. Double parallel wires were found to give most accordant results. The plates were placed violet left and four settings were made on the comparison, or arc spectrum with three on the internal or spark spectrum; then the same set of measurements was repeated with the plate placed violet right.

RESULTS.

The results of the investigation are given in Tables I to IV. Table I shows the agreement between different plates, those in each group of two or three plates being taken under similar conditions as far as known, and the groups differing among themselves by reason of change of spark-length, capacity, etc. The average deviation from the mean of the measurements made upon the individual plates of the different groups is 0.005 tenth-meter, and the average deviation from the mean of the measurements of the four lines on the same plate (chosen at random) is under 0.003 tenth-meter.

Table II covers the results of six short series, upon which series the following brief remarks may be made:

A. *Changing the length of the primary or main gap*[1] apparently causes a change in the position of the spark line. The spark was vertical and its length was varied from 1 to 20 mm. The slit of the spectroscope was cut down to 6 mm; the concave mirror was so mounted that it gave an image 2.4 times as large as the source. In the three cases, then, the images were 2.4, 24, and 42 mm in length. Therefore the whole image of only the shortest spark fell upon the slit. With the 10 and 20 mm sparks the central 2.5 mm were used. This point will be discussed later. In Plates 11 and 12 a strong air-current was directed upon the spark by an electric fan—this to produce greater disruptiveness in the discharge. It may appear that the true series should be composed of Plates 3 and 4, 13 and 14, and 1 and 2; for in none of these was the fan used. But it may be well to remark that in 3 and 4, and 1 and 2, the spark was highly disruptive, in 13 and 14 it was not. (At this seemingly critical spark length it so much resembled the uncondensed spark that the exposure time had to be increased to 4 minutes, from 15 seconds in 1 and 2.) Therefore 11 and 12 may well lay claim to the right of being substituted for 13 and 14. *Probably* the fan will increase the displacement in all cases in which the spark is not inherently disruptive.

B. The work upon the spectrum of the spark under liquids (carried on in conjunction with Professor Hale), in which an *auxiliary air gap* was placed in the discharge circuit of the condenser in series with the liquid gap to produce sufficient disruption of discharge, at once suggested the trial of the effect of a second air gap in this case also. The results in the two cases, spark under water and spark in air, are identical as far as displacement is concerned: the introduction, or an increase in length, of the secondary gap produces an increase in displacement. The fan forced a current of air against both gaps.

C. *Increase of capacity* over the range employed seems to produce a small increase in displacement. For values of plates of the condenser in meters and microfarads see Table III.

[1] The difference between primary and secondary gaps will be made evident in section B.

D. *Introduction of self-induction*, even as small an amount as 0.000426 henries, into the discharge circuit of the condenser produces a great change: under such conditions the spark lines are more nearly coincident with the arc lines.

E. *Introduction of impedance*, of considerable but unknown amount[1] into the primary circuit of the transformer, has some influence on the position of the lines.

F. *Introduction of resistance* into the exciting circuit of the alternator, in addition to that of the field coils themselves, causes a decrease in displacement. Whether this change will always be in this direction—increased resistance, decreased displacement—is an open question. For certain reasons, which cannot be discussed here, but will be made evident in the *Publications of the Yerkes Observatory*, the writer is led to believe that under some conditions an increase of resistance may cause an increased displacement.

The effects of five[2] of the six sets of changes have their exact parallel in the water spark, and, indeed, the attempt to obtain such effects was suggested by the results obtained in that investigation. This collateral evidence goes far to compensate for the meagerness of data here presented. Moreover, in the spark in air at high pressure (210 pounds), introduction of self-induction decreased the displacement.[3]

It was stated above that in the primary spark-gap series only a 2.5 mm central section of the longer gaps was used. It seemed advisable to study further the 20 mm spark. Therefore the terminals were turned into a horizontal position, and a series of plates taken as shown in Table IV, to which the last portion (G) of Table I is preliminary.

The difference between the displacements given by plates 34 and 35, also by 36 and 37, Table I, is well accounted for by the fact that it was difficult to expose identically the same portions of

[1] Impedance coil: seven sections connected in parallel, each of 150 turns of No. 16 B. & S. copper wire; internal diameter of coil, 6.3 cm; external diameter, 9.4 cm; total length of coil, 17 cm; soft iron wire core; closed magnetic circuit.

[2] In the water-spark investigation the effect of impedance was not tried.

[3] ASTROPHYSICAL JOURNAL, 17, 157, 1903.

the spark. It is perfectly evident, however, that a large difference exists between the displacement given by that part of the spark which is near the terminals and that which lies between them. From plates 36, 37, 38, and 39 it appears that there is little difference between the positions of lines given by the central portions of either long or short gaps. But no special stress should be put upon this point, because of the difficulty of placing upon the slit the central portions of either long or short gaps.

Attention should be called to the fact that plates 15, 16, and 28 do not give results which are in absolute accord with the primary spark-gap series. Nor do 36 and 37 agree well with 3 and 4. In the writer's opinion, this should not count against an acceptance of the results as accurate and of the effects as true, because (1) the plates cited were not taken upon the same day nor upon succeeding days (it is possible that a change in atmospheric conditions alters the positions of the lines); and (2) the shape of the terminals may not have been the same.

GENERAL DISCUSSION OF RESULTS.

Such large displacements as Haschek gives have not presented themselves. He used a capacity of 625 meters or 0.0694 microfarads. The maximum capacity here employed was 504 meters or 0.0560 microfarads. By referring to Mohler's table, given in his article on "Pressure in the Spark,"[1] it will be seen

TABLE I.

Relative positions of spark and arc lines observed under various modifications of the conditions of the spark circuit.

Displacement of spark lines toward red from position of arc lines given in tenth-meters. M = mean displacement.

A. EFFECT OF CHANGE IN LENGTH OF PRIMARY SPARK-GAP.

Line	Displacements for Separate Plates, with Means											
	3	4	M	13	14	M	11	12	M	1	2	M
3900.68	+.003	+.004	+.004	+.012	+.021	+.017	+.011	+.012	+.012	+.030	+.039	+.034
04.95	−.002	−.006	−.004	−.001	−.005	.003	.....	+.010	+.010	+.011	+.034	+.022
13.58	+.009	−.001	+.004	+.011	+.009	+.010	+.007	+.013	+.010	+.032	+.044	+.038
98.77	+.003	−.011	−.004	−.009	−.018	−.014	.....	+.007	+.007	+.015	+.033	+.024

[1] ASTROPHYSICAL JOURNAL, 10, 204, 1899.

TABLE I.—*Continued.*

B. EFFECT OF CHANGE IN LENGTH OF SECONDARY SPARK GAP.

Line	Displacements for Separate Plates, with Means									
	15	16	28	M	17	19	M	20	21	M
3900.68.....	+.010	+.014	+.017	+.014	+.028	+.026	+.027	+.044	+.037	+.041
04.95.....	−.010	−.012	+.003	−.006	−.003	−.005	−.004	−.019	+.027	+.023
13.58.....	+.007	+.007	+.018	+.011	+.025	+.037	+.031	+.041	+.043	+.042
98.77.....	−.007	−.012	+.010	−.003	.....	+.009	+.009	+.023	+.027	+.025

C. EFFECT OF CHANGE OF CAPACITY.

Line	Displacements for Separate Plates, with Means												
	1	2	M	9	10	M	7	8	M.	42	5	6	M
3900.68....	+.030	+.039	+.034	+.030	+.042	+.036	+.040	+.041	+.040	+.050	+.043	+.036	+.043
04.95....	+.011	+.034	+.022	.....	.....	.....	.....	.....	.....	+.038	+.022	+.031	+.027
13.58....	+.032	+.044	+.038	+.034	+.043	+.038	+.034	+.049	+.042	+.054	+.040	+.030	+.041
98.77 .	+.015	+.033	+.024	.....	.....	.....	.....	.....	.....	.....	+.034	+.025	+.030

D. EFFECT OF INTRODUCTION OF SELF-INDUCTION.

E. EFFECT OF INTRODUCTION OF IMPEDANCE.

Line	Displacements for Separate Plates, with Means												
	23	29	M	42	5	6	M	26	27	M	1	2	M
3900.68.....	+.008	+.012	+.010	+.050	+.043	+.036	+.043	+.036	+.025	+.031	+.030	+.039	+.034
04.95.....	−.010	+.003	.004	+.038	+.022	+.031	+.027	+.008	+.005	−.007	−.011	+.034	+.022
13.58.....	−.010	−.014	+.002	+.054	+.040	+.030	+.041	+.026	+.018	−.022	+.032	+.044	+.038
98.77.....	−.002	+.001	−.002	.....	+.034	+.025	+.030	−.001	−.001	−.001	+.015	+.033	+.024

F. EFFECT OF CHANGE OF RESISTANCE IN FIELD CIRCUIT OF ALTERNATOR.

Line	Displacements for Separate Plates, with Means					
	24	25	M	1	2	M
3900.68.....	+.027	+.031	+.029	−.030	+.039	+.034
04.95.....	+.000	−.002	−.001	−.011	+.034	+.022
13.58.....	+.008	+.025	+.017	+.032	+.044	+.038
98.77.....	.004	+.010	+.003	+.015	+.033	+.024

G. EFFECT OF USING DIFFERENT PARTS OF SPARK, AND SPARKS OF DIFFERENT LENGTH, AS SOURCES OF LIGHT.

Line	Displacements for Separate Plates, with Means								
	34	35	M	36	37	M	38	39	M
3900.68.....	.....	−.058	+.058	+.026	+.019	+.023	+.029	+.022	+.026
04.95.....	+.017	+.023	−.020	+.021	+.007	+.014	.....	.....	.....
13.58.....	−.039	−.051	+.045	−.032	−.011	+.022	+.024	+.018	+.021
98.77.....	+.010	−.029	+.020	−.011	−.006	+.002	+.011	.....	+.011

TABLE II.

VERTICAL SPARK.

Mean displacements taken from Table I. Results of six series of plates in which length of primary gap, length of secondary gap, capacity, etc., are severally the variables. Displacement of spark lines toward red in tenth-meters: positive unless otherwise indicated.

CONDITIONS	Plate numbers		3, 4	13, 14	11, 12	1, 2	15,16,28	17, 19	20, 21	1, 2	9, 10	7, 8	4a,5,6	23, 29	4a,5,6	26, 27	1, 2	24, 25	1, 2
	Date of exposure		Dec. 29	Dec. 31	Dec. 31	Dec. 29	Jan. 3, 5	Jan. 3	Jan. 3	Dec. 29	Dec. 30	Dec. 30	Dec. 30	Jan. 5	Dec. 30	Jan. 5	Dec. 29	Jan. 5	Dec. 29
	Length of primary spark gap in mm.		20	10	10	1	5	5	5	1	1	1	1	1	1	1	1	1	1
	Length of secondary spark gap in mm		0	0	0	0	0	10	20	0	0	0	0	0	0	0	0	0	0
	Capacity in plates of condenser		9	9	9	9	9	9	9	9	19	38	77	77	77	9	9	9	9
	Self induction in henries		0	0	0	0	0	0	0	0	0	0	0.	0.000426	0	0	0	0	0
	Impedance in primary transformer circuit		0	0	0	0	0	0	0	0	0	0	0	0	0	2 4	0	0	0
	Field circuit resistance in ohms		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	75	0
	Fan running		No	No	Yes	No	Yes	Yes	Yes	No	No	No	No	No	No	No	No	No	No
	LINE [1]	INTENSITY	(1) LENGTH OF PRIMARY SPARK-GAP				(2) LENGTH OF SECONDARY SPARK-GAP			(3) CAPACITY [3]				(4) SELF-INDUCTION		(5) IMPEDANCE		(6) FIELD CIRCUIT RESISTANCE	
RESULTS	3900.68	5 [2]	0.004	0.017	0.012	0.034	0.014	0.027	0.041	0.034	0.036	0.040	0.043	0.010	0 043	0.031	0.034	0.029	0.034
	04.95	7	−.004	−.003	.010	.022	−.006	−.004	.023	.022	.....	.....	.027	−.004	.027	.007	.022	−.001	.022
	13.58	5 [2]	.004	.010	.010	.038	.011	.031	.042	.038	.038	.042	.041	.002	.041	.022	.038	.017	.038
	98.77	8	−.004	−.014	.007	.024	−.003	.009	.025	.024	.	.....	.030	−.002	.030	−.001	.024	.003	.024

[1] Wave-lengths and intensities from Hasselberg's table of the arc spectrum of titanium.

[2] In the spark spectrum lines 3900.68 and 3913.58 are stronger than 3904.95 and 3998.77.

[3] For values of condenser plates in meters and microfarads, see Table III.

[4] Impedance of unknown amount.

that the average displacement for two cadmium lines increases but slowly between 48 and 70.2 meters; and Mohler remarks that "this seems to indicate that the pressure in the spark does not increase directly with the capacity, but, as indicated by Haschek and Mache, the pressure approaches a maximum value."[1] (Haschek and Mache had inclosed the spark in a receptacle and measured the pressure developed when the discharge passed. They used a spark-gap of 2 mm between brass terminals 3 mm in diameter and produced the spark by a transformer which gave "5,200 volts effective" potential.)

TABLE III.

Condenser capacities in meters and microfarads.

No. of plates of condenser	Meters	Microfarads
9	59.4	0.0066
19	125.1	0.0139
38	249.3	0.0277
77	504.3	0.0560

TABLE IV.

HORIZONTAL SPARK.

Mean displacements taken from Table I. Part of spark exposed and spark length are severally the variables. Displacements of spark lines toward red in tenth-meters: positive unless otherwise indicated.

CONDITIONS	Plate numbers		34, 35	36, 37	38, 39
	Date of exposure		Jan. 16	Jan. 16	Jan. 16
	Length of spark-gap in mm		20	20	1
	Capacity in plates of condenser		9	9	9
	Self induction in henries		0	0	0
	Impedance in primary transference circuit		0	0	0
	Field circuit resistance in ohms		0	0	0
	Fan running		No	No	No
			PART OF SPARK EXPOSED		
	LINE	INTENSITY	Near terminal	Near center	Near center
RESULTS	3900.68	5	0.058	0.023	0.026
	04.95	7	0.020	0.014	.....
	13.58	5	0.045	0.022	0.021
	98.77	8	0.020	0.002	0.011

[1] ASTROPHYSICAL JOURNAL, 9, 351, 1899.

In view of this it seems extremely improbable that an increase in capacity from 504 to 625 meters would increase the displacement from 0.043 (the maximum shift obtained by the writer for λ 3900.68—see plates 4a, 5 and 6) to 0.13 tenth-meter.

Haschek states that the transformer and induction coil give different positions for the spark lines—which, indeed, is probably the case. Difference in energy consumption is assigned as the explanation[1] of the difference in spark pressure as found by Mohler, and by Haschek and Mache.[2] This will not explain the larger displacements given by Haschek, for the writer used quite as powerful apparatus, if not more so. In Plates 20 and 21, for instance, the potential difference on the terminals of the primary circuit of the transformer was 77 volts, the current flowing 21 ampères, and the resultant power by actual measurement 1000 watts. The secondary voltage was about 10,500. Haschek's "spark was produced by a high potential transformer supplied by a primary current of 12 ampères and 100 volts, which transformed up to about 10,000 volts." Therefore difference in transformer would not be likely to produce such a difference in displacement.

Another matter may be touched upon at this point. Table IV shows that near the center of the spark gap the displacement is less than near the terminals. After discussing the work of Haschek and Mache "On the Pressure in the Spark,"[3] Haschek writes:[4]

> An explanation of the origin of spark pressure may be found in the investigation of Schuster,[5] who observed that the small particles of the metallic vapor discharged from the electrodes move with great velocity, which diminishes sharply with increasing distance from the electrodes. From this it follows that the particles crowd together toward the middle of the path of the spark, so that the pressure increases with the distance from the electrode. A conception of how the pressure affects the wave-length of the emitted light may be obtained by considering that, upon a sufficiently close approach, the particles must exert mutual forces upon each other. These

[1] ASTROPHYSICAL JOURNAL, 14, 198, 1901.

[2] *Ibid.*, 12, 50, 1900. [3] *Ibid.*, 9, 347, 1899. [4] *Ibid.*, 14, 188, 1901.

[5] *Nature*, 57, 1899; *Phil. Trans.*, 193, A, 1899.

light-producing particles will then no longer vibrate freely, but under the influence of a dampening which causes an increase of wave-length.

This reasoning is not clear to the writer. If we are justified in assuming that the kinetic theory of gases is applicable to the metallic particles of the luminous source, we have:

$$p = \tfrac{1}{3}\, m\, n\, u^2,$$

where p is the pressure exerted by the conglomeration of the particles, m the mass of a particle, n the number of particles, and u^2 the "mean squared velocity."

Even if n increased toward the center of the spark gap, the conclusion could not be drawn that the pressure was greater at this point, for the decrease in the value of u^2 might more than compensate for the increase of n. The fact is that there are probably, in general, more glowing metallic particles near the terminals than in an equal volume at a point midway between them, as is shown (1) by the time of exposure necessary in the two cases to photograph the lines investigated—either with vertical or horizontal spark gap, this being especially true of the lines λ 3900.68 and 3913.58, which appear much enhanced near the terminals;[1] and (2) by the results obtained by Schenck[2] who states that "the principal spark lines show a tendency to shorten up close to the poles, while the arc lines go clear across the gap." Of course, we may not be justified in using the word "pressure" with reference to the spark, because in a phenomenon such as this the motion of the particles is so irregular that the term "mean squared velocity" may well be regarded as having no physical meaning. Moreover, in general, it may not be justifiable to base any line of reasoning on conclusions drawn from the kinetic theory of gases.

All that may be said is that, if increase of pressure in the spark produces an increase in the wave-length of the line, the pressure near the terminal is greater than that in the center of the spark.

[1] Schuster states that "the metallic vapors remain luminous in the center of the spark for a longer period than near the poles." However, the intensity of illumination appears—for the characteristic spark lines at least, if not indeed for the rest as well—greater near the "pole."

[2] ASTROPHYSICAL JOURNAL, 14, 129, 1901.

To study further the effect of inserting self-induction in the discharge circuit of the condenser there was taken somewhat later a series of plates covering the region λ4200 to λ4700, one set with 0.000426 henries self-induction and the other with none.

TABLE V.

Relative positions of spark and arc lines observed with and without self-induction in the discharge circuit of the condenser. M=mean displacement of spark line.

Line	Displacement for Separate Plates					
	41	42	M	45	46	M
4443.97.....	−0.015	−0.029	−0.022	−0.007	−0.008	−0.007
55.48.....	.051	.046	.048	.003	.006	.004
68.65.....	.030	.033	.031	000	.006	.003

TABLE VI.

Mean displacements taken from Table V. Self-induction is the variable. Displacements in tenth-meters: positive toward the red.

Conditions	Plate numbers	41, 42	45, 46
	Date of exposure	Jan. 17	Jan. 17
	Length of spark gap in mm	1	1
	Capacity in plates of condenser	77	77
	Impedance in primary transference circuit	0	0
	Field circuit resistance in ohms	0	0
	Fan running	No	No

	Line	Intensity	Self-Induction in Henries	
			0	0.000426
Results	4443.97	6	−0.022	−0.007
	55.48	6	.048	.004
	68.65	6	.031	.003

Table V shows the agreement of the measurements made upon the members of each set; and Table VI gives the condition under which the plates were taken. It will be noticed in the case of $\lambda 4443.97$ that the insertion of self-induction appears to place the line farther toward the red—an exception to the general rule found above.

The results of this brief survey of the questions considered may be summed up as follows:

1. When a spark spectrum is obtained for purposes of measurement, the conditions under which it is taken should be stated more fully than is customary at present, for the position of the lines is altered to more than a negligible degree by the condition of the electric circuit as to capacity, self-induction, resistance in field circuit of alternator, and impedance in primary circuit of transformer.

2. Different parts of the spark give spectra in which the lines have not only different relative intensities, but different wave-lengths.

That the wave-lengths depend upon the capacity and the apparatus used, whether induction coil or transformer, has been known for some time. That self-induction alters the position of the lines, and that the part of the spark exposed enters as important factors, are facts which had not come to the writer's notice.

3. Such large displacements, as are given by Haschek for the lines of the *Ti* spectrum have not been found here.

It must be clearly understood that the grating used is not one which possesses a dispersion fully adequate for an accurate quantitative study of the question in hand; and that, with the dispersion given, the limit of accuracy of measurement lies far too near the magnitude of the effects studied. However, the collateral evidence is so strong that the results here given are undoubtedly qualitatively correct. Moreover, it appears impossible to attribute the persistent and regular changes in the positions of the spark lines to mechanical motion in the apparatus, unequal illumination of the grating, heating of the slit, errors in measurement, or, in short, to any other causes than those given

above. The writer intends, as soon as possible, to continue the research with a larger grating.

In conclusion I wish to acknowledge my indebtedness to Professor Hale for proposing the investigation, to both Professor Hale and Professor Frost for their valuable suggestions, and to Messrs. Ellerman and Fox for their assistance in the laboratory during the latter part of the investigation.

WABASH COLLEGE,
Crawfordsville, Ind., April 7, 1903.

Minor Contributions and Notes.

THE NEW STAR IN *GEMINI.*

Telegraphic announcement of the discovery on March 16 of a new star in *Gemini* by Turner (presumably on the Oxford photographs) was received at the Yerkes Observatory on the afternoon of March 27. The same evening the star was observed with the forty-inch refractor. Its conspicuous red color was at once explained by the great intensity of the *Hα* line, as observed by Mr. Ellerman and the writer with a direct vision spectroscope. Very bright lines in the yellow and blue were clearly visible on a faint background of continuous spectrum. The star thus presented the familiar characteristics of *Novæ.*

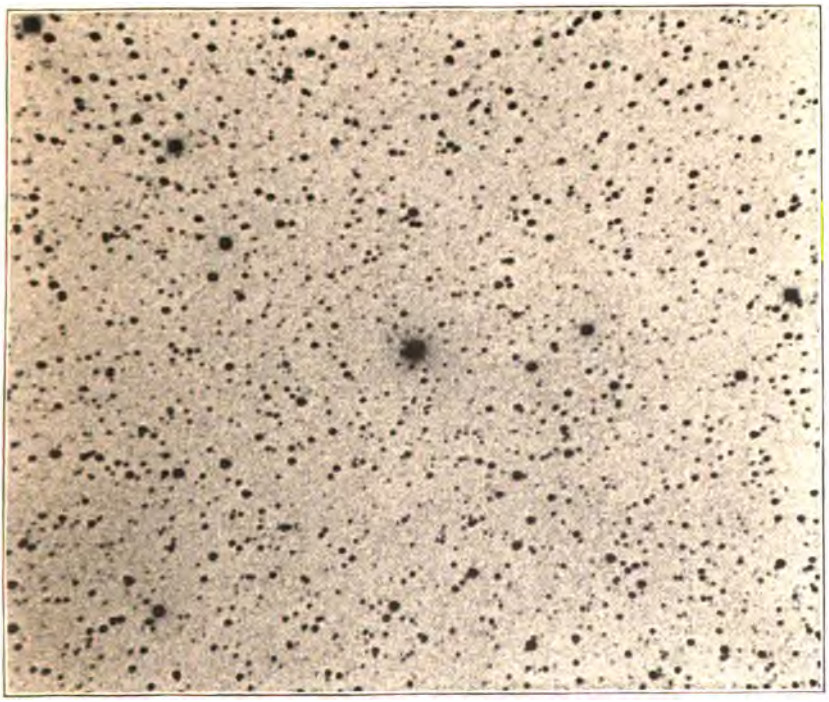
The region of the *Nova* had been photographed with the two-foot reflector by Mr. Parkhurst on February 21, 1903, for the purpose of charting the field around the variable star 2404 *X Geminorum*, which lies two-thirds of a degree east. The exposure time was 20 minutes, from $8^h\ 22^m$ to $8^h\ 42^m$ C. S. T. After the receipt of the telegraphic announcement of Professor Turner's discovery, Mr. Parkhurst carefully examined the region of the *Nova* on this plate. What appeared to be an image of the *Nova* was found. It is a little brighter than the fifteenth magnitude, the faintest stars visible in the vicinity being about half a magnitude fainter. Careful comparison with similar plates taken since the *Nova* brightened, and also with the sky as seen in the large telescope, indicates that the star photographed is either the *Nova* or a fifteenth magnitude star within three seconds of arc of the place. The comparison certainly shows that the *Nova* was not brighter than the fifteenth magnitude on February 21.

The region of the *Nova* was photographed with the two-foot reflector by Mr. Pease on March 28 and 29 with a total exposure of 8¾ hours (Plate XXI). As the conditions were fairly good during the exposure of this photograph, it seems probable that no bright nebulosity, like that surrounding *Nova Persei*, existed at this time in the vicinity of *Nova Geminorum.*

Position.— Micrometric measures with the 40-inch refractor by Professor Barnard gave the following positions for the new star with respect to *B. D.* 29°.1342 (8.2 mag.):

PLATE XXI.

N.



Scale: 1 mm = 20″. S.

PHOTOGRAPH OF THE REGION OF *NOVA GEMINORUM*.

Date	*Nova*—Star	
March 27....	$\Delta\alpha$ -2^m $21^s.84$	$\Delta\delta$ $+5'$ $57''.29$
March 30....	-2 21.76	$+5$ 57.25
	-2 21.80	$+5$ 57.27

In the observations of March 27, an intermediate star was used in both $\Delta\alpha$ and $\Delta\delta$, and on the 30th in $\Delta\alpha$.

Mean place of comparison star for 1900.0	Authority.
$\alpha = 6^h\ 40^m\ 10^s.79$; $\delta = +29^\circ\ 56'\ 42''.05$.	Leiden *A. G. C.* 2813 and Cambridge *A. G. C.* 3482.

It was observed twice at Leiden and three times at Cambridge; the declinations differ by $1''.7$, which does not appear to be due to motion. These two positions were weighted according to the number of observations.

The following position results for the *Nova:*

1900.0 $\alpha = 6^h\ 37^m\ 48^s.99$. $\delta = +30^\circ\ 2'\ 39''.3$.

The star is of a strong red color — redder than *B. D.* $+29^\circ.1342$.

Micrometric measures have been made of the small stars near the *Nova* to form an accurate chart for detection of possible motion in the *Nova*. The measures so far made are given below. The stars may be identified from the accompanying drawing (Fig. 1) showing the field immediately surrounding the *Nova*.

Nova and $12^m.3$ star following = No. 1.

1903, March 30.....	$104^\circ.82$	$32''.21$
April 4.....	105.22	32.52
	105.02	32.36

Nova and $12^m.6$ star South = No. 2.

1903, March 30.....	$169^\circ.21$	$45''.66$

Nova and $14^m.5$ star North = No. 3.

1903, March 30.....	$341^\circ.61$	$55''.69$

Nova and $12^m.5$ star preceding = No. 4.

1903, March 30.....	196°.15	84'.93
April 4.....	196.30	85.15
	196.22	85.04

Nova and $12^m.5$ star North = No. 5.

1903, March 30.....	313°.86	99'.87
April 4.....	313.90	99.93
	313.88	99.90

Nova and 12^m star North = No. 6

1903, March 30	16°.98	102'.81
April 4.....	17.45	103.00
	17.21	102.90

POSITIONS OF THE SMALL STARS, REFERRED TO THE MEAN EQUINOX OF 1900.0.

	R. A.			Dec.		
No. 5.....	6^h	37^m	$43^s.44$	+ 30°	3'	48".5
4.....	6	37	47.16	+ 30	1	17.6
3.....	6	37	47.64	+ 30	3	32.1
2.....	6	37	49.65	+ 30	0	54.5
6.....	6	37	51.33	+ 30	4	17.6
1.....	6	37	51.40	+ 30	2	30.9

Focus.—The following observations made by Professor Barnard show that the focus of the *Nova* with the forty-inch refractor did not differ appreciably from that for the comparison star:

		Inches	
	March 27 *Nova* focus	2.12	(4 obs.)
B. D.	+ 30°.1314 "	2.10	(5 obs.)
	Nova − star = + 0.02 = + 0.51 mm		
	March 30 *Nova* focus	2.13	(9 obs.)
B. D.	+ 30°.1314 "	2.15	(9 obs.)
	Nova − star = − 0.02 = − 0.51 mm		

When in the best focus there was a decided glow about the star for some 2" or 3", which was of a crimson color. This was not present with the other stars. By drawing the eyepiece out, a small, well-defined image of the star was formed of about the tenth magnitude and

about 0".1 in diameter. It was sharply defined and of a beautiful crimson color — far more vivid than Hind's celebrated crimson star. The *Nova* was surrounded by a pale, grayish blue halo 3".8 in diameter by measurement. This phenomenon was not present when other stars were thrown out of focus.

Five very accordant settings were made for the focus for this crimson image, giving focus = 2.51 inches. This, compared with the observations for the true focus for this star on the same date, showed that this second image, one and a half magnitudes less than the other, was formed 0.38 inch (9.6 mm) outside the proper focus for the *Nova*. This was undoubtedly due to the predominance of the bright *Hα* line in the light of the *Nova*.

Magnitude.—The photometric magnitudes of the *Nova*, as measured on seven nights by Mr. Parkhurst with an equalizing wedge photometer attached to the twelve-inch refractor and six-inch reflector, are as follows:

PHOTOMETRIC MAGNITUDES OF *NOVA GEMINORUM* ON HARVARD COLLEGE OBSERVATORY SCALE.

(To express on Potsdam scale, add 0.20 mag.)

Date.	G. M. T.	Mag.
1903, March	27.715	8.51
	28.636	8.71
	29.603	8.85
	29.650	8.91
	30.673	8.76
	31.588	8.89
April	3.564	8.91 ±
	4.583	8.96

Measured from four standard stars ; mean residual from separate stars, 0.03 mag.

Spectrum.—As neither the position nor the physical characteristics of the bright lines could be determined satisfactorily by visual observations, two of the prisms of the Bruce spectrograph were removed, and a short camera attached for use with a single prism. The camera lens employed was a Unar, series Ib, No. 7, kindly loaned by the Bausch & Lomb Optical Co. pending the construction of a special short focus doublet. A mounting for the camera was made in the instrument shop on March 28, and the spectrum of the *Nova* was photographed on that evening by Professor Frost and Mr. Adams. The

photograph, which is reproduced in Fig. 2, was taken at $15^h\ 40^m$ G. M. T. (middle of exposure), and was exposed for $3^h\ 12^m$. A Cramer isochromatic plate was used, without the correcting lens. Titanium was employed for the comparison spectrum, supplemented by helium, the spectrum of which was photographed on another plate immediately after the end of the exposure for the star.

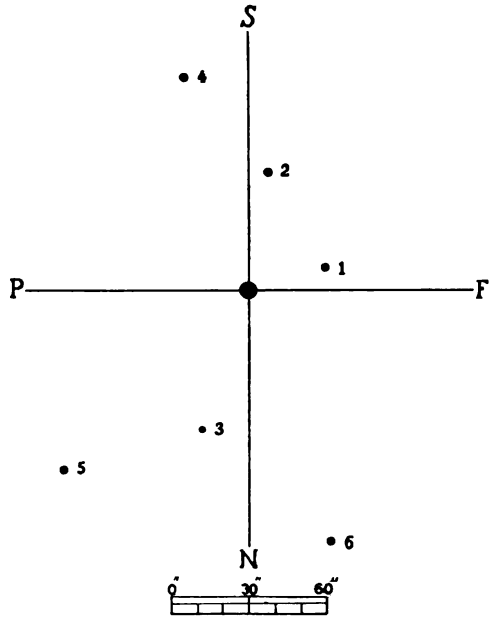


FIG. 1.—Stars near the *Nova*.

Professor Frost's study of this photograph shows that the most conspicuous features are the very strong bright band extending from about λ 4598 to λ 4696 (mean at λ 4647), and the very strong *Hβ* line, extending from λ4839 to λ4886 (mean at λ4862). There appeared to be two narrow bright maxima near the less refrangible end of *Hβ*, at λ4877 and λ4882. At the less refrangible end of the plate there are two fairly strong bands, one extending from λ 5647 to λ 5685 (mean at λ 5666), the other extending from λ 5729 to λ 5775 (mean at λ 5752). The less refrangible of these lines is rather sharply bounded on the violet side, suggesting the presence of a dark band. The lack of sensitiveness of the plate near λ 5000 may account for the absence of the band near λ 5016, which, though found in the spectra of *Nova Aurigae* and *Nova Persei*, is exceedingly faint in *Nova Geminorum*. *Hγ*, though hardly measurable, is visible on the plate as a very faint band merging into a brighter band which extends from λ 4347 to λ 4371 (mean at λ 4359). The intensity of this part of the spectrum was greatly reduced through the absence of the correcting lens, and the slit was not in the focal plane of the forty-inch telescope for this region. This probably accounts for the decided faintness of *Hγ* as

compared with $H\beta$, but in any case $H\gamma$ would appear to be distinctly fainter than $\lambda 4359$.

The spectrum of *Nova Geminorum* thus corresponds with the spectra of *Nova Persei* and *Nova Aurigae* in their later stages of development, particularly with respect to the relative intensity of the bands observed, all of which were present in both of these stars. After the transformation of the spectrum of *Nova Aurigae* into that of a nebula, in August 1892, the intensities of $\lambda\lambda 4358$, 4640, 4863, and 5750 were estimated by Campbell as 8, 7, 10, and 10 respectively, and were only exceeded by the chief nebular lines $\lambda 5007$ and $\lambda 4959$ with intensities of 100 and 30. On the negative of the spectrum of *Nova Geminorum* the very faint bright band in the region of the principal nebular lines is too weak for measurement.

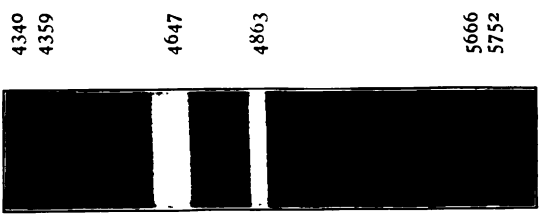


FIG. 2.—Spectrum of *Nova Geminorum*, March 28, 1903.

Visual observations of the *Nova* have been made with a single ocular prism, or direct vision train, on each available night since March 27, but no changes sufficiently obvious to be noticed with such small dispersive power have been observed.

GEORGE E. HALE.

YERKES OBSERVATORY,
April 6, 1903.

NOVA GEMINORUM BEFORE ITS DISCOVERY.[1]

ON March 27, 1903, a cable message was received from Professor Kreutz, of Kiel, stating that an object which was probably a new star, but was possibly a variable, had been discovered by Professor Turner. Also, that on March 16 it was of the magnitude 8.0, while on February 16 it had not been seen (presumably on a photograph). Its apparent place was R. A. $6^h\ 37^m\ 48^s$, Dec. $+30^\circ\ 4'$. The grant from the Carnegie Institution permitted an examination to be made of the early photographs of the Henry Draper Memorial, and furnished the history of this object from its first appearance to the present time. An excellent photograph of the region, taken 1903, March $1^d\ 15^h\ 3^m$, G. M. T., showed stars of the magnitude 11.9, but no trace of the *Nova* was

[1] *Harvard College Observatory Circular* No. 70.

visible. A similar result was found from sixty-seven plates, the first taken March 3, 1890, the last on February 28, 1903, although nearly all of these plates showed stars fainter than the twelfth magnitude. One or more of these photographs were taken on each intermediate year. It did not therefore seem necessary to examine the other early plates of this region, a hundred or more in number. A plate taken 1903, March 2^d 13^h 19^m, showed stars of the ninth magnitude, but no trace of the *Nova*. The evenings of March 3, 4, and 5 were cloudy, but on a plate taken March 6^d 14^h 28^m, an object of the magnitude 5.08 appears in the given place. Plates taken on several later nights showed that the magnitude was gradually diminishing, as appears in Table I, which gives in successive columns the date, the Greenwich Mean Time, the photographic magnitude of the *Nova*, and the initial of the observer. F. denotes Mrs. Fleming and L. Miss Leland.

The photograph of March 6 has especial value, since, so far as is known, it contains the first photograph of the *Nova*. The image is on the very edge of the plate, and accordingly was compared with fifteen other stars at about the same distance from the center of the plate. The *Nova* was compared twice with each star by each observer. The value of the grade was much larger than usual, and equaled 0.21 and 0.33 for the two observers. The mean result for all was magnitude 5.08, with an average deviation, for the separate stars, of $\pm$ 0.26. The magnitude on each of the other dates given in the table was found by

TABLE I.

PHOTOGRAPHIC MAGNITUDES.

Date	G. M. T.	Mag.	Obs.	Date	G. M. T.	Mag.	Obs.
	h. m.				h. m.		
1903 March 1	15 3	< 11.4	L.	1903, March 14	14 14	7.42	F.
2	14 31	< 9.5	F.	14	14 14	7.34	L.
6	14 28	4.97	F.	14	16 20	7.32	F.
6	14 28	5.20	L.	14	16 20	7.34	L.
11	15 18	6.76	F.	15	13 44	7.27	F.
12	14 25	7.06	F.	15	14 53	7.57	F.
12	15 25	7.16	F.	15	14 53	7.46	L.
13	14 52	7.12	F.	25	13 26	7.94	L.
13	14 52	7.17	F.	25	13 39	8.08	F.

comparison with six stars whose brightness was nearly equal to that of the *Nova*. These magnitudes were derived assuming that the photometric and photographic magnitudes were nearly the same for stars whose spectra were of the first type.

The evening of March 27 was cloudy and also the early part of March 28. One plate, however, taken on the latter date, gave the magnitude 8.34. Several photographs were taken on March 29, 31, and April 1, and gave the mean magnitudes 8.24, 8.24, and 8.25. These magnitudes are based upon the results given in Table II, which contains the designations, *Durchmusterung* numbers, rectangular coordinates expressed in seconds of arc with the *Nova* as an origin, and provisional photographic magnitudes of a sequence of comparison stars. It is probable that the fainter stars are really fainter than these magnitudes indicate, but the latter will serve to determine the relative changes in the *Nova* as it grows fainter, and thus render the results of different observers comparable. All the magnitudes can later be reduced to an absolute scale. They also serve to compare the faintest stars shown on early plates. Thus, the photograph taken March 1, 1903, shows star *t*, and also stars at least a tenth of a magnitude fainter. Star *u* does not appear. Hence this plate shows stars of the magnitude 11.9 and brighter.

The last plate in Table I is of interest, since it was taken with an objective prism, and accordingly shows the spectra of the *Nova* and of the adjacent stars. Six bright lines are shown in the spectrum of the *Nova*, whose designations, assumed wave-lengths, and intensities, calling the intensity of the line $H\gamma$, *10*, are as follows: $H\zeta$, λ 3889, *1*; $H\epsilon$, λ 3970, *3*; $H\delta$, λ 4102, *8*; $H\gamma$, λ 4341, *10*; —, λ 4643, *11*; $H\beta$, λ 4862, *9*.

TABLE II.

COMPARISON STARS.

Des.	*B. D.*	*x*	*y*	Mag.	Des.	*B. D.*	*x*	*y*	Mag.
		"	"				"	"	
a	+30° 1318	+ 716	+3187	7.28	*p*	.....	+ 4	+368	11.19
b	+30° 1300	.....	.....	7.76	*q*	.....	+308	−132	11.41
c	+30° 1314	+ 274	+1949	7.98	*r*	.....	−404	−278	11.63
d	+30° 1320	+1021	+ 878	8.50	*s*	.....	+359	+345	11.73
e	+29° 1342	+1843	− 347	8.82	*t*	.....	−224	+176	11.78
f	+30° 1316	+ 643	+ 552	9.14	*u*	.....	+467	+145	12.08
g	+30° 1306	−1006	+ 142	9.36	*w*	.....	−254	+242	12.23
h	... .	+ 504	+ 289	9.71	*x*	.....	−268	+ 92	12.45
	... ...	+ 504	+ 297		*y*	.....	+270	+ 16	12.70
k	+30° 1302	−1923	+ 231	9.93	*z*	.....	+186	− 64	12.95
l	+30° 1309	− 456	+ 53	10.13	*α*	.....	+ 31	+ 97	13.05
m	+30° 1317	+ 638	− 327	10.41	*β*	.....	− 24	− 70	13.35
n	... .	− 311	− 47	10.66	*γ*	.....	+ 31	− 8	13.53
o	... .	+ 544	+ 197	11.01					

From this it appears that the spectrum resembles that of *Nova Sagittarii* on April 19, 1898. No dark lines are visible, but this is perhaps owing to the small dispersion.

The same lines, and having nearly the same intensities, appeared on similar photographs taken on March 29, 31, and April 1. They also showed the additional nebula line, λ5003, which has the intensity *2* or *3*, and is certainly brighter than *Hζ*. This line does not appear on the plate taken March 25, and indicates the first step in the change into a gaseous nebula. Three additional bright lines were detected in the later photographs, whose estimated wave-lengths are about λ4176, 4240, and 4462.

In the other new stars the appearance of line 5003 was followed by the diminution in intensity of the line *Hβ*, and the appearance and rapid increase in the nebula line, near *Hζ*, which finally became the strongest line in the spectrum.

A most important question in connection with the appearance of new stars is whether such objects can come and go without detection by astronomers. Since the Henry Draper Memorial was established, nine new stars have been discovered. Six of them, *Nova Persei* No. 1, *Nova Normae*, *Nova Carinae* No. 2, *Nova Centauri*, *Nova Sagittarii*, and *Nova Aquilae*, were found in the regular examination of the Draper Memorial photographs, and probably all of them would otherwise have escaped detection. Two, *Nova Aurigae* and *Nova Persei* No. 2, were bright, and were found visually by Dr. Anderson. The first of these might have escaped detection here, although numerous early charts were obtained which showed that it was visible to the naked eye during seven weeks before its discovery. The spectrum of Turner's *Nova* is so conspicuous on the plate taken on March 25 that when this plate was developed and examined it would doubtless have been found on it here, but for the prompt discovery and announcement by Professor Turner.

EDWARD C. PICKERING.

APRIL 3, 1903.

A LIST OF FOUR STARS WHOSE VELOCITIES IN THE LINE OF SIGHT ARE VARIABLE.[1]

THE following four spectroscopic binaries, discovered with the Mills Spectrograph, are additional to the thirty-eight binaries already announced:

[1] *Lick Observatory Bulletin* No. 31.

v Andromedae ($\alpha = 0^h\,44^m$; $\delta = +40^\circ\,32'$).

The variable velocity of this star was discovered from the second plate. Three observations have been secured, as follows:

Date	Velocity	Measured by
1902, October 8	− 17 km	Curtis
November 5	− 76	Curtis
1903, January 14	+ 49	Curtis

The spectrum of this star shows very few lines. The $H\gamma$ line is very broad, but the lines due to helium are much better and admit of fairly accurate measurements.

π^4 *Orionis* ($\alpha = 4^h\,46^m$; $\delta = +5^\circ\,26'$).

The variation in velocity was discovered from the second plate, the observations being as follows:

Date	Velocity	Measured by
1902, October 6	+ 43 km	Curtis
1903, January 4	± 0	Curtis
January 12	+ 6	Curtis

The spectrum is very similar to that of *v Andromedae*.

σ Geminorum ($\alpha = 7^h\,37^m$; $\delta = +29^\circ\,7'$).

The change in velocity was shown in the second plate of this star. The observations are as follows:

Date	Velocity	Measured by
1902, March 16	+ 74 km	Reese
1903, January 12	+ 12	Reese
January 13	+ 9	Reese
February 15	+ 69	Curtis

The lines in the spectrum of this star, though numerous, are rather hazy. Still they admit of trustworthy measurements.

ι Argus ($\alpha = 8^h\,3^m.3$; $\delta = -24^\circ\,1'$).

Director Campbell requests me to announce also the variable

velocity of ι *Argus*, discovered by him in 1898 from the second plate.

Date	Velocity	Measured by
1897, February 23	+41.9 km	Reese
1898, February 21	+49.5	Reese
1899, April 12	+49.8	Reese
April 13	+50.3	Reese
1901, April 9	+42.3	Reese
1902, February 18	+46.3	Reese

H. M. REESE.

FEBRUARY 21, 1903.

A STAR WITH A GREAT RADIAL VELOCITY.[1]

THE star ϕ^2 *Orionis* ($\alpha = 5^h\ 31^m$; $\delta = +9^\circ\ 15'$) is interesting on account of its great velocity in the line of sight. The observations are as follows:

Date	Velocity	Measured by
1902, October 28	+ 94 km	Curtis
November 24	+102	Curtis
December 30	+ 96	Curtis

Although the spectrum shows reasonably good lines, it is not likely that the range of eight kilometers in these measurements indicates a real variability in the star's motion, for the second plate was much underexposed.

H. M. REESE.

FEBRUARY 19, 1903.

THE SPECTRUM OF THE NEBULOSITY SURROUNDING *NOVA PERSEI*.[2]

THE remarkable changes of position and brightness observed in the nebulosity around *Nova Persei* have made it desirable to accumulate as much evidence as possible bearing upon the nature of these phenomena.

Unfortunately, physical observations were not attempted during the earlier stages, when the nebulosity was brighter. At the time of the discovery of these changes, it seemed unlikely that any satisfactory

[1] *Lick Observatory Bulletin* No. 31. [2] *Lick Observatory Bulletin* No. 33.

spectroscopic observations could be obtained, owing to the faintness of the nebulosity. The great uncertainty as to the real nature of the changes and the almost entire lack of positive evidence seemed to warrant the attempt, notwithstanding the difficulties.

A small slit-spectrograph, to be used with the Crossley Reflector, was designed especially for the problem. Both collimator and camera lenses are single and of quartz; the prism is also of quartz, the refracting angle being 50°. Following are the constants of the spectrograph, including those of the telescope:

Aperture of lenses and prism	1⅛ inches
Focal length of collimator and camera lenses	6 inches
Refracting angle of prism	50° 14′
Aperture of Crossley Reflector	36¼ inches
Focal length of Crossley Reflector	210 inches

The prism and lenses were mounted directly in frames of brass and attached to the body of the spectrograph, which was constructed of Spanish cedar. A reflecting slit was used, and guiding was accomplished by means of the reflected image of the *Nova* viewed in a small telescope. The nebular spectrum occupied the central 0.2 inch of the slit, on either side of which was photographed a comparison spectrum of hydrogen. The slit-width was reduced to 0.002 inch for the comparison spectrum, which was inserted for from 2^s to 5^s each night at about the middle of the exposure.

The photographs of the *Nova* in August and September 1902, with exposures of 7 to 11 hours, showed only condensation *D* with any certainty. No attempt was made, therefore, to study any other portion of the nebula. The slit was placed approximately parallel to a circle of declination, and by estimation across the densest part of condensation *D*. This region was free from small stars. The region covered was 1½′ south and from 1′ west to 4′ west of the *Nova*.

Under the above conditions and with a slit-width of 0.006 inch, a negative was obtained with the following exposures:

Duration		Remarks
1902, October 31,	$7^h\ 40^m$	Seeing 2; sky good.
November 1,	9 4	Seeing 3; haze in the beginning.
November 2,	8 0	Seeing 3; some haze in the beginning; stopped by clouds.
November 4,	9 25	Seeing 3; sky good.
Total,	$34^h\ 9^m$	

The resulting negative shows a very faint spectrum extending from $H\beta$ to about λ 360. The length of the spectrum is 0.11 inch. There is no evidence of flexure in the spectrograph, nor of any change in the position of the plate during the exposure. As the exposure on October 31 had been carried through the first traces of dawn, a negative was made, later, under as nearly the same conditions as possible, to make sure that the spectrum obtained was not that of the sky. No trace of any sky spectrum was found on this plate. Another plate which had been exposed in the brighter dawn to obtain a faint spectrum of the sky for comparison, showed that the sky spectrum was unlike that of the nebula which had been obtained. With such low dispersion the sky spectrum between $H\beta$ and K is divided into two bands of almost equal extent and intensity by the weaker region (full of absorption lines) at G. These and other considerations seem to leave no doubt that the spectrum obtained is that of the nebulosity.

In the spectrum of the *Nova* nebula fully three-fourths of the light is condensed in the region extending from $H\beta$ to $H\gamma$; above $H\gamma$ the spectrum is extremely faint, and is lacking entirely from about λ 380 to λ 390. In the upper part of the spectrum there appear to be traces of two lines, one almost coincident with $H\delta$ and the other at about λ 370. They are so extremely faint, however, that their existence cannot be affirmed with certainty.

If the above interpretation is correct, the spectrum of condensation *D* resembles that of the *Nova* during the first few days of its great brightness much more than it does that observed since July 1901. So far as can be told from the low dispersion used and the weak spectrum recorded, the general appearance of the spectrum of the nebula below λ 380 agrees very well with the spectrum of the *Nova* obtained by Campbell and Wright in February 1901. With the conditions under which their observations were made most of the light of the *Nova* would be included in the region between $H\beta$ and $H\gamma$, and the region between λ 380 and λ 390 would be too weak to show in a faint spectrum. This is also true of the region below $H\beta$. As no early observations of the spectrum of the *Nova* high up in the violet have been published, it is impossible to make any comparisons between the two spectra in this region, where some lines are suspected in the nebulosity.

The spectrum of the nebulosity does not agree with that of the *Nova* observed since July 1901. The strongest region in the spectrum of the nebulosity may possibly be made up of the lines between $H\beta$ and $H\gamma$ which the low dispersion failed to separate in a spectrum so

weak, although this seems improbable. The positions of the two suspected lines at λ 410 and λ 370 do not agree at all, however, with the strongest lines in the later spectrum of the *Nova*. There are certainly no traces in the nebulosity of any lines in the positions of those observed in the *Nova* at λ 387 and λ 397 by Campbell and Wright,[2] nor at λ 346, where a very strong line was photographed by Mr. Stebbins,[1] with the Crossley Reflector. These lines are the strongest in the photographic spectrum of the *Nova* since July 1901.

The spectrum obtained is certainly not the ordinary bright-line spectrum of the nebulæ. The suspected lines at λ 410 and λ 370 may be nebular lines which occupy approximately these positions. The region between *Hβ* and *Hγ*, however, is not at all like the characteristic nebular spectrum, where *Hβ* and *Hγ* (and occasionally λ 469) are the only strong lines. If the ordinary nebular spectrum is present at all, it is in conjunction with another spectrum, probably continuous, extending from λ 434 to λ 487.

To still further test the identity of the spectrum secured, three negatives of the spectrum of the *Nova* were obtained on February 17, 1903, with the same spectrograph. Exposures of 10, 30, and 60 minutes were given. Ten minutes barely sufficed to give a distinguishable spectrum. One hour gave a spectrum showing the five principal lines between λ 387 and λ 471. There are traces of the chief nebular line, and a very faint spectrum up in the ultra-violet. The relative intensities of the lines are not very different from those obtained in September and October 1901, by Mr. Stebbins with the small slitless spectrograph attached to the Crossley Reflector.

A comparison of the spectrum of the *Nova* on February 17, 1903, and that of the nebulosity, shows that they are not identical. The line at λ 387, which is one of the strongest in the spectrum of the *Nova*, has no counterpart whatever in the nebula. It cannot be made to coincide with the slight strengthening in the nebula at λ 370.

The conclusions are, therefore, that the spectrum of condensation *D* observed in November 1902, is not the spectrum of a gaseous nebula; that it is not the spectrum of the *Nova* since the latter has become nebular; and that it more nearly resembles the spectrum of the *Nova* during the first few days of its outburst. The question as to what epoch in the star's history the spectrum of the nebulosity should conform, if the light were reflected, is a difficult one to decide with the data in our possession. This is particularly true in the case of con-

[1] *Lick Observatory Bulletin* No. 8.

densation *D*. Some knowledge of the real distance and direction of this mass from the star, or spectroscopic observations at considerable intervals, would aid materially in the solution.

The preceding spectroscopic evidence is not inconsistent with the theory that the light of condensation *D*, at least, is reflected light emitted by the *Nova at the time of its greatest brightness;* but is not strong enough to establish this theory in the face of conflicting indications already pointed out.

The spectrum of the thirteenth-magnitude star at approximately the position-angle of 245° and 4½′ distant from the *Nova* is also shown on the negative, this star having occasionally crossed the slit. Its spectrum appears to be continuous, and extends from $H\beta$ to $H\gamma$.

I am indebted to Dr. Campbell for suggestions as to the construction of the spectrograph, and to Dr. H. D. Curtis and Fellows R. H. Curtiss and C. A. G. Weymouth for assistance in making the observations.

C. D. PERRINE.

MARCH 2, 1903.

THE SNOW HORIZONTAL TELESCOPE.

I AM glad to be permitted to announce that Miss Helen E. Snow, of Chicago, has provided for the reconstruction of the coelostat reflecting telescope of the Yerkes Observatory as a memorial to her father, the late George W. Snow. Through Miss Snow's generosity it will be possible to build the telescope in permanent and substantial form, and to provide it with solar and stellar spectrographs, spectroheliographs, and other important accessories. The great superiority of the new instrument will amply compensate for the delay occasioned by the fire of last December.

GEORGE E. HALE.

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THE PRESSURE DUE TO RADIATION.[1]

By E. F. NICHOLS and G. F. HULL.

As EARLY as 1619 Kepler[2] announced his belief that the solar repulsion of the finely divided matter of comets' tails was due to the outward pressure of light. On the corpuscular theory of light Newton[3] considered Kepler's idea as plausible enough, but he was of the opinion that the phenomenon was analogous to the rising of smoke in our own atmosphere. In the first half of the eighteenth century DeMairan and DuFay[4] contrived elaborate experiments to test this pressure of light theory in the laboratory, but, because of the disturbing action of the gases surrounding the illuminated bodies employed in the measurements, they obtained wholly confusing and contradictory results. Later in the same century Rev. A. Bennet[5] performed further

[1] Presented to *the American Academy of Arts and Sciences*, April, 1903.

[2] DEMAIRAN, *Traité physique et historique de l'Aurore boréale* (2d ed.), pp. 357, 358. Paris, 1754.

[3] *Isaaci Newtoni Opera quae Existant Omnia.* Samuel Horsley, LL.D., R.S.S., Tom. III, pag. 156. Londinium, 1782.

[4] DEMAIRAN, *loc. cit.*, p. 371. This treatise contains also the accounts of still earlier experiments by Hartsoeker, p. 368, and Homberg, p. 369. The later experiments are of more historic than intrinsic interest.

[5] A. BENNET, *Phil. Trans.*, p. 81, 1792.

experiments, but could find no repulsive force not traceable to convection currents in the gas surrounding the body upon which the light was projected, due in his opinion to the heating effect of the rays. Finding no pressure due to radiation, he made the following unique suggestion in support of the wave theory of light:

> Perhaps sensible heat and light may not be caused by the influx or rectilinear projection of fine particles, but by the vibrations made in the universally diffused caloric or matter of heat or fluid of light. I think modern discoveries, especially those of electricity, favor the latter hypothesis.

In the meantime Euler,[1] accepting Kepler's theory attributing the phenomenon of comets' tails to light pressure, had hastened to the support of the wave theory by showing theoretically that a longitudinal wave motion might produce a pressure in the direction of its propagation upon a body which checked its progress. In 1825 Fresnel[2] made a series of experiments, but arrived at no more definite conclusion than that the repulsive and attractive forces observed were not of magnetic nor electric origin.

Crookes[3] believed in 1873 that he had found the true radiation pressure in his newly invented radiometer, and cautiously suggested that his experiments might have some bearing on the prevailing theory of the nature of light. Crookes's later experiments and Zöllner's[4] measurements of radiometric repulsions showed that the radiometric forces were in some cases 100,000 times greater than the light pressure forces with which they had been temporarily confused. Zöllner's experiments are among the most ingenious ever tried in this field of work, and he missed the discovery of the true radiation pressure by only the narrowest margin. An excellent bibliography of the whole radiometric literature is given by Graetz,[5] and an account of some of the older experiments not mentioned above is given by Crookes.[6]

[1] L. EULER, *Histoire de l'Académie royale de Berlin* (2), p. 121, 1746.

[2] A. FRESNEL, *Ann. Chem. et Phys.*, **29**, 57, 107, 1825.

[3] W. CROOKES, *Phil. Trans.*, p. 501, 1873.

[4] F. ZÖLLNER, *Pogg. Ann.*, **160**, 156, 206, 450, 1877.

[5] L. GRAETZ, *Winkelmann's Handbuch der Physik*, 2*b*, p. 262. Breslau, 1896.

[6] W. CROOKES, *loc. cit.*, p. 501.

In 1873 Maxwell,[1] on the basis of the electromagnetic theory, showed that if light were an electromagnetic phenomenon, pressure should result from the absorption or reflection of a beam of light. After a discussion of the equations involved, he says: "Hence in a medium in which waves are propagated there is a pressure in the direction normal to the waves and numerically equal to the energy in unit volume." Maxwell computed the pressure exerted by the Sun on the illuminated surface of the Earth, and added:

> It is probable that a much greater energy of radiation might be obtained by means of the concentrated rays from an electric lamp. Such rays falling on a thin metallic disk, delicately suspended in a vacuum, might perhaps produce an observable mechanical effect.

Apparently independent of Maxwell, Bartoli[2] announced in 1876 that the Second Law of Thermodynamics required the existence of a pressure due to radiation numerically equal in amount to that derived by Maxwell. Bartoli's reasoning holds for all forms of energy streams in space, and is of more general application than Maxwell's equations. Bartoli contrived elaborate experiments to verify this theory, but was balked in the search, as all before him had been, by the complicated character of the gas action, which he found no way of eliminating from his experiments.

After Bartoli's work, the subject was dealt with theoretically by Boltzmann,[3] Galitzine,[4] Guillaume,[5] Heaviside,[6] and more recently by Goldhammer.[7] Fitzgerald,[8] Lebedew,[9] and Hull[10]

[1] J. C. MAXWELL, *A Treatise on Electricity and Magnetism* (1st ed.), **2**, 391. Oxford, 1873.

[2] A. BARTOLI, *Sopra i movementi prodotti della luce et dal calorie*, Florence, Le Monnier, 1876; also *Nuovo Cimento*, **15**, 193, 1884.

[3] L. BOLTZMANN, *Wied. Ann.* **22**, 31, 291, 1884.

[4] B. GALITZINE, *Wied. Ann.*, **47**, 479, 1892.

[5] CH. ED. GUILLAUME, *Arch. de Gen.* (3), **31**, 121, 1894.

[6] O. HEAVISIDE, *Electromagnetic Theory*, 1, 334. London, 1893.

[7] D. A. GOLDHAMMER, *Ann. der Phys.*, **4**, 834, 1901.

[8] G. F. FITZGERALD, *Proc. Roy. Soc. Dub.*, 1884.

[9] P. LEBEDEW, *Wied. Ann.*, **45**, 292, 1892; ASTROPHYSICAL JOURNAL, **14**, 155, 1902.

[10] G. F. HULL, *Trans. Astron. Soc. Toronto*, p. 123, 1901.

have discussed the bearing of radiation pressure upon the Newtonian law of gravitation, with special reference to the repulsion of comets' tails by the Sun. The theory of radiation pressure, combined with the known properties in negative electrons, has recently been more or less speculatively applied by Arrhenius[1] to the explanation of many cosmical and terrestrial phenomena, among which the following may be mentioned: the solar corona, zodiacal light, gegenschein, comets, origin of cometary and meteoric material in space, the emission of gaseous nebulæ, the peculiar changes observed in the nebula surrounding *Nova Persei*, the northern lights, the variations in atmospheric electricity and terrestrial magnetism and in the barometric pressure. Schwarzschild[2] computed from radiation pressure on small spherical conductors the size of bodies of unit density for which the ratio of radiation pressure to gravitational attraction would be a maximum.

Before the Congrès international de Physique in 1900, Professor Lebedew,[3] of the University of Moscow, described an arrangement of apparatus which he was using at that time for the measurement of light pressure. He summarizes the results already obtained as follows:

> Les résultats des mesures que j'ai faites jusqu'ici peuvent se résumer ainsi: L'expérience montre qu'un faisceau lumineux incident exerce sur les surfaces planes absorbantes et réfléchissantes des pressions qui, aux erreurs près d'observation, sont égales aux valeurs calculées par Maxwell et Bartoli.

No estimate of the "errors of observation" was given in the paper, nor other numerical data. Unfortunately the proceedings of the Paris Congress did not reach the writers, nor any intimation of the methods or results of Professor Lebedew's work, until after the publication of their own preliminary experiments.

The writers[4] presented the results they had obtained by

[1] S. A. ARRHENIUS, *Lehrbuch der kosmischen Physik*, Leipzig, 1903, pp. 149–58, 200–208, 226, 920–5.

[2] K. SCHWARZSCHILD, *Kgl. bayer. Akademie d. Wissenschaften*, **31**, 293, 1901.

[3] P. LEBEDEW, *Rapports présentés au Congrès international de Physique* (2), p. 133. Paris, 1900.

[4] E. F. NICHOLS and G. F. HULL, *Science*, **14**, 588 (October 18, 1901); *Phys. Rev.*, **13**, 293 (November, 1901); ASTROPHYSICAL JOURNAL, **15**, p. 62 (January, 1902).

measurements of radiation pressure at eight different gas pressures, in a preliminary communication to the American Physical Society, meeting with Section B of the American Association at Denver, August 29, 1901. The main arguments underlying the method of measurement of the radiation pressure may here be given.

In the experiments of earlier investigators every approach to the experimental solution of the problem of radiation pressure had been balked by the disturbing action of the gases which it is impossible to remove entirely from the space surrounding the body upon which the radiation falls. The forces of attraction or repulsion, due to the action of gas molecules, are functions, first, of the temperature difference between the body and its surroundings, caused by the absorption by the body of a portion of the rays which fall upon it; and, second, of the pressure of the gas surrounding the illuminated body. In the particular form of apparatus used in the present study, the latter function appears very complicated, and certain peculiarities of the gas action remain inexplicable upon the basis of any simple group of assumptions which the writers have so far been able to make.

Since we can neither do away entirely with the gas nor calculate its effect under varying conditions, the only hopeful approach which remains is to devise apparatus and methods of observation which will reduce the errors due to gas action to a minimum. The following considerations led to a method by which the elimination of the gas action was practically accomplished in the present experiments:

1. The surfaces which receive the radiation, the pressure of which is to be measured, should be as perfect reflectors as possible. This will reduce the gas action by making the rise of temperature due to absorption small, while the radiation pressure will be increased; the theory requiring that a beam, totally reflected, shall exert twice the pressure of an equal beam, completely absorbed.

2. By studying the action of a beam of constant intensity upon the same surface surrounded by air at different pressures,

certain pressures may be found where the gas action is less than at others.

3. The apparatus—some sort of torsion balance—should carry two surfaces symmetrically placed with reference to the rotation axis, and the surfaces of the two arms should be as nearly equal as possible in every respect. The surfaces or vanes should be so constructed that if the forces due to gas action (whether suction or pressure on the warmer surface) and radiation pressure have the same sign in one case, a reversal of the suspension should reverse the gas action and bring the two forces into opposition. In this way a mean of the forces on the two faces of the suspension should be, in part at least, free from gas action.

4. Radiation pressure, from its nature, must reach its maximum value instantly, while observation has shown that gas action begins at zero and increases with length of exposure, rising rapidly at first, then more slowly to its maximum effect, which, in many of the cases observed, was not reached until the exposure had lasted from two and a half to three minutes. For large gas pressures an even longer exposure was necessary to reach stationary conditions. The gas action may be thus still further reduced by a ballistic or semi-ballistic method of measurement.

Ballistic observations of radiation pressure were made at air pressures ranging from 96.3 mm to 0.06 mm of mercury. The average radiation pressure of the standard beam was found to be 1.05×10^{-4} dyne, with a probable error of 6 per cent. To compare this value of the pressure with the theoretical value as given by the Maxwell-Bartoli formula, $p = \frac{E(1+\rho)}{V}$, it was necessary to measure E, the energy of the beam. This was done by a bolometric method in which the measurement of the energy was made to depend upon the resistance of the bolometer strip. Using the value of the energy thus obtained and 0.92[1] as the reflection coefficient of the silver surfaces, it was found that the

[1] This value was obtained from the measurements of Langley, Rubens, Nichols, and Paschen for the assumed mean wave-length of the beam.

pressure directly observed was about 20 per cent. lower than that computed from the Maxwell-Bartoli formula. After the publication of the paper it was found that an error had been made in the measurement of the resistance of the bolometer strip, owing to a fault in its construction. A new estimation of the resistance gave an energy value which brought the theoretical value of the pressure very close to that found by direct measurement. But another method of measuring the energy was adopted in the later experiments described in this paper.

In the number of the *Annalen der Physik* for November, 1901, Professor Lebedew[1] published the results of a more varied series of measurements of radiation pressure than the early measurements of the present writers. The principal difference between the methods employed by him and by the writers for determining the pressure was that he used very thin metallic vanes surrounded by gas at extremely low pressures, thus following Maxwell's suggestion literally, while the writers used silvered glass vanes and worked at large gas pressures for which the gas action had been carefully and exhaustively studied and found to be negligibly small for short exposures. From our knowledge of the variation of gas action in different vacua, we feel sure that our method would not have been successful in high vacua because of the relatively large gas action. Professor Lebedew's own results, with blackened vanes of lower heat conductivity, show that his success in eliminating gas disturbance was due to the high heat conductivity of thin vanes rather than to the high vacua employed.

Professor Lebedew's[2] estimate of the accuracy of his work is such as to admit of possible errors of 20 per cent. in his final results. An analysis of Professor Lebedew's paper and comparison with our preliminary experiments seem to show that his accidental errors were larger than ours, but through the undiscovered false resistance in the bolometer our final results were somewhat farther from the theory than his. Either of the above researches would have been sufficient to establish the *existence*

[1] P. LEBEDEW, *Ann. der Phys.*, 6, 433, 1901.

[2] P. LEBEDEW, *Ann. der Phys.*, 6, 457, 1901.

of a pressure due to radiation, but neither research offered, in our judgment, a satisfactory *quantitative* confirmation of the Maxwell-Bartoli theory.

LATER PRESSURE MEASUREMENTS.

Description of apparatus: the torsion balance.—The form of suspension of the torsion balance, used to measure radiation pressure in the present study, is seen in Fig. 1. The rotation axis ab was a fine rod of drawn glass. A drawn-glass cross-arm c, bent down at either end into a small hook, was attached to the axis. The surfaces C and D, which received the light beam, were circular microscope cover-glasses, 12.8 mm in diameter and 0.17 mm thick, weighing approximately 51 mg each. To distinguish the two vanes from each other, in case individual differences should appear in the measurements, and also to mark the two faces of each vane for subsequent recognition, a letter C was marked on one, and D on the other, by diamond scratches. Through each glass a hole 0.5 mm or less in diameter was drilled near the edge, by means of which the glasses could be hung on the hooks on the cross-arm c. On opposite sides of the rotation axis at d two other drawn-glass cross-arms were attached. The cover-glasses slipped easily between these, and were thus held securely in one plane. Farther down on ab, a small silvered plane mirror m_1 was made fast at right angles to the plane of C and D. This mirror was polished bright on the silver side, so that the scale at S_5 (Fig. 2) could be read in either face. A small brass weight m_1 (Fig. 1), of 452 mg mass and of known dimensions, was attached at the lower end of ab. The cover-glasses which served as vanes were silvered and brilliantly polished on the silvered sides, and so hung on the small hooks that both silver faces or both glass faces were presented to the light. A quartz fiber f_2, 3 cm long, was made fast to the upper end of ab, and to the lower end of a fine glass rod d_1 which carried a horizontal magnet m_2. The rod d_1 was in turn suspended by a short fiber to a steel pin e, which could be raised or lowered in the bearing h. The whole was carried by a bent glass tube t, firmly fastened to a solid brass foot F, resting on a plane

ground-glass plate *P*, cemented to a brass platform mounted on three leveling screws not shown. A bell-jar *B*, 25 cm high and 11 cm in diameter, covered the balance. The flange of the bell-jar was ground to fit the plate *P*. A ground-in hollow glass stopper fitted the neck of the bell-jar, which could thus be put in

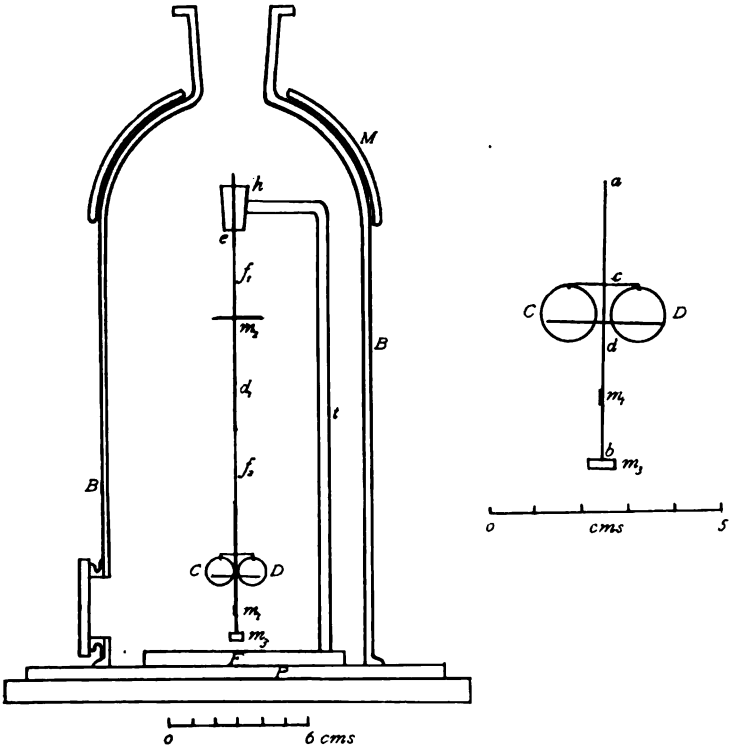


FIG. 1.

connection with a system of glass tubes leading to a Geissler mercury pump, a MacLeod pressure gauge, and a vertical glass tube dipping into a mercury cup and serving as a rough manometer for measuring the larger gas pressures employed during the observations. The low pressures were measured on the MacLeod gauge in the usual way. A semicircular magnet *M*, fitted to the vertical curvature of the bell-jar, was used to direct the suspended magnet m_2 and thus to control the zero

position of the torsion balance. By turning M through 180°, the opposite faces of the vanes C and D could be presented to the light.

THE ARRANGEMENT OF APPARATUS.

A horizontal section of the apparatus through the axis of the light beam is shown in Fig. 2. The white-hot end of the horizontal carbon S_1, of an A. T. Thompson 90° arc-lamp, fed by alternating current, served as a source. The arc played against the end of the horizontal carbon from the vertical carbon which was screened from the lenses L_1 and L_2 by an asbestos diaphragm d_2. A lens, not shown, projected an enlarged image of the arc and carbons on an adjacent wall, so that the position of the carbons and the condition of the arc could be seen at all times by both observers.

The cone of rays passing through the small diaphragm d_2 fell upon the glass condensing lenses L_1, L_2. At d_3 a diaphragm, 11.25 mm in diameter, was interposed, which permitted only the central portion of the cone of rays to pass. Just beyond d_3, the beam passed to a shutter at S_2. This shutter was worked by a magnetic escapement, operated by the seconds contact of a standard clock. The observer at T_1 might choose the second for opening or closing the shutter, but the shutter's motion always took place at the time of the second's contact in the clock. Any exposure was thus of some whole number of seconds' duration. The opening in the shutter was such as to let through, at the time of exposure, all of the direct beam which passed through d_3, but to shut out stray light. Just beyond the shutter and attached to the diaphragm d_5 was a 45° glass plate which reflected a part of the beam to the lens L_5, by means of which an image of d_4 was projected upon one arm of a bolometer at R. The glass lens L_3 focused a sharp image of the aperture d_4 in the plane of the vanes of the torsion balance B_1, under the bell-jar. The bell-jar was provided with three plate-glass windows W_1, W_2, W_3. The first two gave a circular opening 42 mm in diameter, and through the third, deflections of the balance were read by a telescope and scale. The lens L_3 was arranged to move horizontally between the stops S_3 and S_4. These were

so adjusted that when the lens was against S_3 the sharp image of the aperture d_3 fell centrally upon one vane; and when against S_4 the image fell centrally upon the other. This adjustment, which was a very important one, was made by the aid of a telescope T_2, mounted on the carriage of a dividing engine. This was used to observe and measure the position of the rotation axis, as well as the positions of the images of d_3, when the lens L_3 was against the stops. For the latter measurements the vanes could be moved out of the way by turning the suspension through 90° by the control magnet M (Fig. 1).

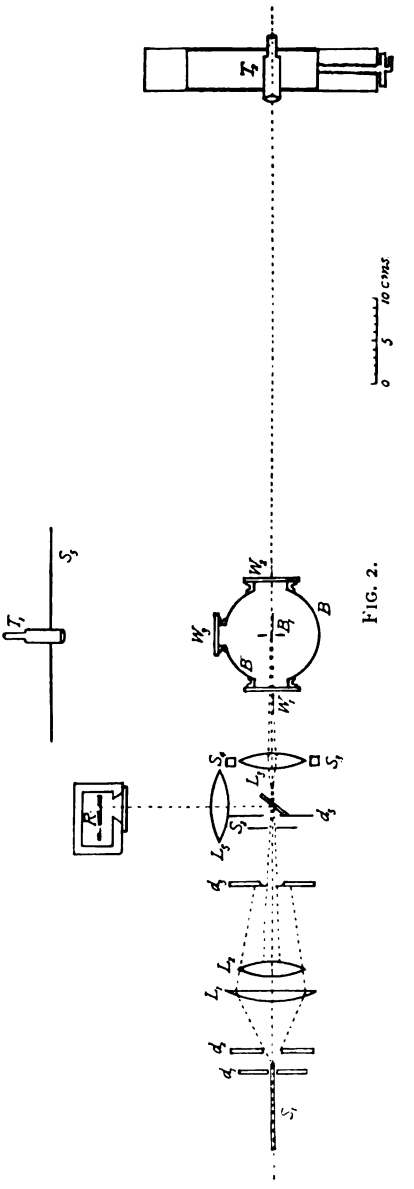


FIG. 2.

To make sure that the balance as used was entirely free from any magnetic moment or disturbance, the small magnet m_2 was clamped in one position to maintain a constant zero, and the period of the balance was accurately measured with the axis of the large magnet M in the vertical plane of the vanes and again when the axis was at right angles to the plane of the vanes. Several series of this sort failed to show a difference of 0.1 second in the period of the balance for the two positions of the magnet.

The bolometer at R (Fig. 2) was of sheet platinum 0.001 mm thick, rolled in silver. The strip was cut out in the form shown

in Fig. 3, and mounted on a thin sheet of slate *S*. Two windows had been cut in the slate behind the strips at *ABCD* where the silver had been removed leaving the thin platinum. The platinum surfaces were blackened by Kurlbaum's process. The image from L_5 (Fig. 2), fell at *D*. The silver ends between *A*

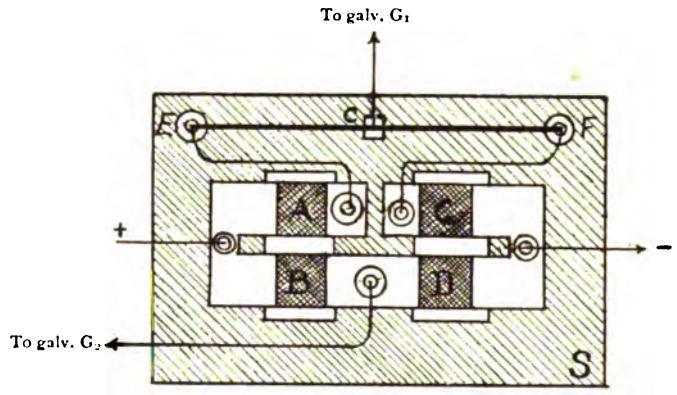


FIG. 3.

and *C* were connected with *E* and *F* respectively. On the heavy wire *EF* a sliding contact *c* served to balance the bridge, all four arms of which are shown in the figure.

METHODS OF OBSERVATION.

The observations leading to the results given later were of three different kinds: (1) the calibration of the torsion balance; (2) the measurement of the pressure of radiation in terms of the constant of the balance; and (3) the measurement of the energy of the same beam in erg-seconds by the rate of temperature rise of a blackened silver disk, of known mass and specific heat.

1. The determination of the constant of the torsion balance was made by removing the vanes *C* and *D* and accurately measuring the period of vibration. Its moment of inertia was easily computed from the masses and distribution of the various parts about the axis of rotation. The moment of torsion for 1 mm deflection on a scale 105 cm distant was 0.363×10^{-5} dyne×cm. This value divided by one-half the distance between the centers

of the light spots on the two vanes gave the force in dynes per scale division deflection. As the light spots were circles 11.25 mm in diameter the area of the image was very nearly 1 sq. cm, hence the above procedure gave roughly the pressure in dynes per square centimeter.

2. In the measurements of radiation pressure, it was easier to refer the intensity of the beam at each exposure to some arbitrary standard which could be kept constant than to try to hold the lamp as steady as would otherwise have been necessary. For this purpose, the bolometer at R (Fig. 2) was introduced, and simultaneous observations were made of the relative intensity of the reflected beam by the deflection of the galvanometer G_2, and the pressure due to the transmitted beam by the deflection of the torsion balance. The actual deflection of the balance was then reduced to a deflection corresponding to a galvanometer deflection of 100 scale divisions. The galvanometer sensitiveness was carefully tested at the beginning and end of each evening's work. All observations of pressure were thus reduced to the pressure due to a beam of fixed intensity.

At each series of radiation pressure measurements, two sets of observations were made. In one of these sets, static conditions were observed, and in the other, the deflections of the balance due to short exposures were measured. In the static observations, each vane of the balance was exposed in turn to the beam from the lamp, the exposures lasting until the turning points of the swings showed that stationary conditions had been reached. The moment of pressure of radiation and gas action combined would thus be equal to the product of the static deflection and the constant of the balance. The torsion system was then turned through 180° by rotating the outside magnet, and similar observations were made on the reverse side of the vanes. All turning-points of the swinging balance in these observations were recorded. From the data thus obtained the resultant of the combined radiation and gas forces could be determined for the time of every turning-point. Every value was divided by the deflection at standard sensitiveness of the galvanometer G_2 read at the same time and was thus reduced to standard lamp.

Results thus obtained, together with the ballistic measurements, showed the direction and extent of the gas action as well as its variation with length of exposure.

The reasons for reversing the suspension follow: The beam from the lamp, before reaching the balance, passed through three thick glass lenses and two glass plates. All wave-lengths destructively absorbed by the glass were thus sifted out of the beam by the time it reached the balance vanes. The silver coatings on the vanes absorbed therefore more than the glass. The radiation pressure was always away from the source, irrespective of the way the vanes were turned, while the gas action would be exerted mainly on the silvered sides of the vanes.

At the close of the pressure and energy measurements, when the reflecting power of the silver faces of the vanes was compared with that of the glass-silver faces, the reflection from the silver faces was found very much higher than that for the glass faces backed by silver. This result was the more surprising because the absorption of the unsilvered vanes was found by measurement to be negligibly small.[1] This unexpected difference in reflecting power of the two faces of the mirrors prevented the elimination of the gas action, by the method described, from being as complete as had been hoped for. But by choosing a gas pressure where the gas action after long exposure is small, the whole gas effect during the time of a ballistic exposure may be so reduced as to be of little consequence in any case.

By exposing each of the vanes in turn and by reversing the suspension and averaging results, nearly all errors due to lack of symmetry in the balance or in the position of the light images with reference to the rotation axis, or errors due to lack of uniformity in the distribution of intensity in different parts of the image, could be eliminated.

The changing character of the gas action, both with time of exposure and gas pressure surrounding the balance vanes, is well illustrated in eight series of static observations in which the

[1] LORD RAYLEIGH records a similar difference between the reflection from air-silver and glass-silver surfaces. *Scientific Papers*, Cambridge, **2**, 538–539, 1900.

glass faces of both vanes were exposed.[1] The results obtained

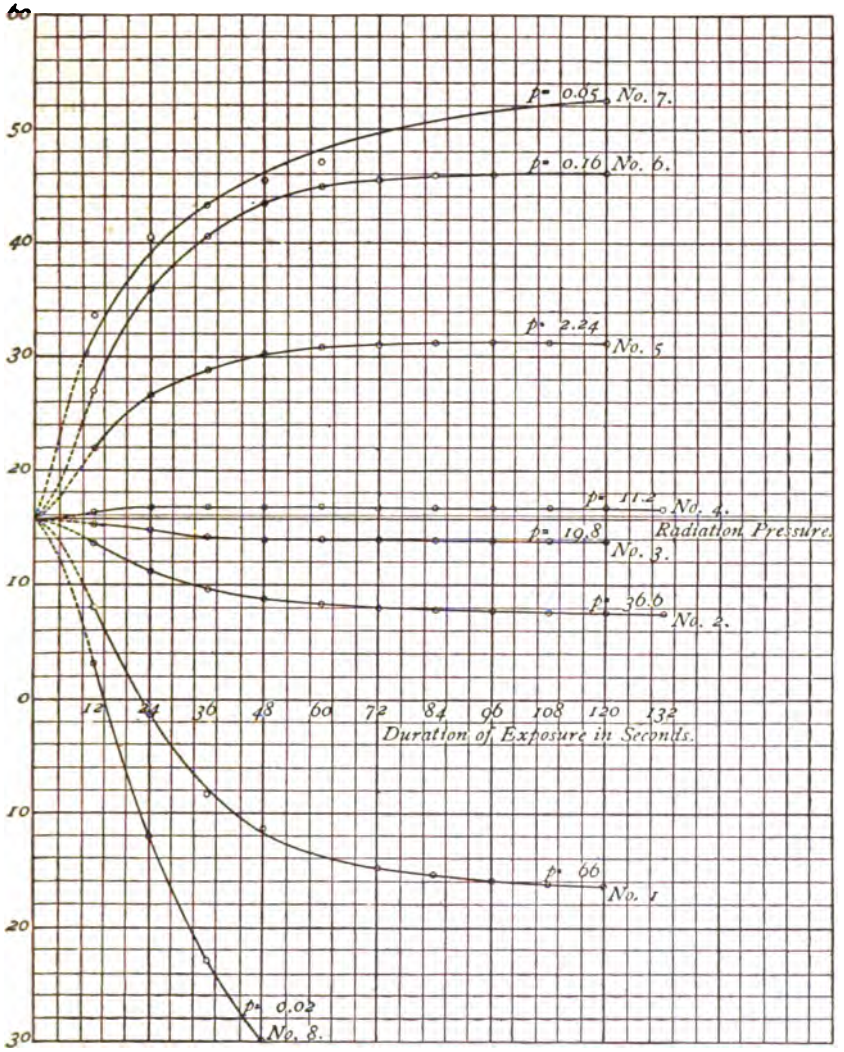


FIG. 4.

on the two vanes were averaged and plotted as curves in Fig. 5,

[1] Observations were also made on the silver faces, but the gas action when the glass faces were exposed was nearly double that for the silver faces, so the least favorable case is shown.

where static deflections due to combined radiation pressure and gas action are shown as ordinates and duration of exposure, in seconds, as abscissæ.[1] A horizontal line through the diagram gives the mean value of the moment of radiation pressure computed from the data in Table I. Decrease of the deflection with time indicates gas repulsion on the warmed silver faces and increase in deflection, gas suction. It will be seen from the curves that beginning at a gas pressure of 66 mm of mercury, the gas action was repulsion changing to suction in passing from 19.8 to 11.2 mm. In the last two cases the total gas action is small. For lower pressures the suction increases to 0.05 mm. At a gas pressure of 0.02 mm the gas action is again a strong repulsion.

The curves indicate the existence of two gas pressures, at which the gas action in our arrangement of apparatus should be zero, one between 19.8 and 11.2 mm and the other between 0.05 and 0.02 mm.[2] The former region was chosen for the ballistic measurements and nearly all of the observations were made at a gas pressure of approximately 16 mm. Even for the two pressures where the decrease in the static deflection was most rapid, *i. e.*, at gas pressures of 66 and 0.02 mm, the first throw was always in the direction of radiation pressure. The gas action is strongly influenced by very slight changes in the inclination of the plane of the vanes to the vertical and also by any object introduced under the bell-jar anywhere near the vanes. For instance, a very considerable effect was observed when a small vessel of phosphoric anhydride was placed under the jar behind the vanes, though the nearest wall of the vessel was separated from the vanes by a distance of at least 3 cm.

During the observations, the polished silver coatings on the vanes deteriorated rapidly; new coatings rarely lasted for more than two evenings' work. As the balance had to be removed

[1] Ordinates of the curves are proportional to moments.

[2] Crookes in his work with the radiometer discovered certain gas pressures for which the combined gas and radiation forces neutralized, but as he did not discriminate between forces due to radiation and gas forces his results were apparently capricious and his reasoning somewhat confused. See *Phil. Trans.*, p. 519, 1875.

and the mirrors taken from the hooks, silvered, polished, and replaced a great number of times during the entire series of measurements, although great care was taken in setting the plane of the vanes vertical, it is not likely that precisely the same conditions for gas action were ever repeated. The principal value of the static results was in indicating favorable gas pressures for work, rather than affording quantitative estimates of the gas action in short exposures. The dotted parts of the curves are not based on results of observation and might perhaps have been omitted without loss.

It was plain, therefore, that further elimination of the gas action must be sought in exposures so short that the gas action would not have time to reach more than a small fraction of its stationary value. This led to the method of ballistic observations.

THE BALLISTIC OBSERVATIONS.

In passing from the static to the ballistic observations it must always be possible to compute the static equivalent of the ballistic swings. Furthermore the exposures should be made as short as possible without reducing the size of the swing below a value which can be accurately measured.

If the exposure lasts for one-half the period of the balance, the deflection, if the gas action be small, and the damping zero, is equal to 2θ where θ is the angle at which the torsion of the fiber will balance the moment produced by the radiation pressure. If the duration of the exposure be one-quarter of the period of the balance, the angle of deflection is $\theta \mid \overline{2}$. The deflection is thus reduced by 30 per cent., but the effect of the gas action is reduced in greater proportion. It was decided, therefore, to expose for six seconds, one-quarter of the balance period. Neglecting the gas action, the equation[1] of motion of the balance is given by

$$\kappa \frac{\partial^2 \theta}{\partial t^2} + 2\epsilon \frac{\partial \theta}{\partial t} = -G\theta + L ,$$

[1] We are justified in using quantitatively this equation, containing a damping term proportional to the velocity, because the amplitudes of the successive swings of the torsion balance, when no energy fell upon the vanes, were found experimentally to follow accurately the exponential law.

where κ = the moment of inertia of the torsion balance,
ϵ = the damping constant,
G = the moment of torsion of the fiber for $\theta = 1$ radian,
and L = the moment of the radiation force.

The solution of this equation is

$$\theta = \frac{L}{G}\left\{1 - e^{-\frac{\epsilon}{\kappa}t}\cos\sqrt{\frac{G}{\kappa} - \frac{\epsilon^2}{\kappa^2}}\,t\right\},$$
$$= \frac{L}{G}\left\{1 - e^{-\frac{\epsilon}{\kappa}t}\cos 2\pi\frac{t}{T}\right\}, \qquad (1)$$

the constants of integration having been determined from the condition that

$$\theta = \frac{\partial\theta}{\partial t} = 0 \text{ when } t = 0 .$$

When

$$t = \frac{T}{4},\ \theta = \frac{L}{G} \text{ and } \frac{\partial\theta}{\partial t} = \frac{L}{G}\left(\frac{\epsilon}{\kappa}e^{-\frac{\epsilon}{\kappa}t}\cos 2\pi\frac{t}{T} + e^{-\frac{\epsilon}{\kappa}t}\frac{2\pi}{T}\sin 2\pi\frac{t}{T}\right). \qquad (2)$$

The light being cut off when $t = \frac{T}{4}$, the equation of motion becomes

$$\kappa\frac{\partial^2\theta}{\partial t^2} + 2\epsilon\frac{\partial\theta}{\partial t} = -G\theta , \qquad (3)$$

the solution of which is $\theta = Ae^{-\frac{\epsilon}{\kappa}t}\cos\left(2\pi\frac{t}{T} + \alpha\right)$, where A and α can be determined by the conditions imposed by equation (2). Neglecting very small quantities, the value of the amplitude A is expressed by the equation

$$A = \frac{L}{G}\left\{1 + r + \frac{2}{\pi}r^{\frac{1}{2}}\log\left(\frac{1}{r}\right)\right\}^{\frac{1}{2}}, \qquad (4)$$

where r is the ratio of successive amplitudes of the damped vibrations. If $r = 1$, that is, if the motion is undamped, $A = \frac{L}{G}1^{\frac{1}{2}}$. In the partial vacuum used in the experiments (16mm of mercury, a value chosen from the curves in Fig. 4), r was found to be equal to 0.783; consequently $A = 1.357\frac{L}{G}$. (5)

From this it is seen that the total angle of deflection of the torsion balance in the ballistic measurements is equal to 1.357 times the angle at which the moment of the torsion of the fiber balances the moment of the radiation pressure.

The duration of exposure was always six seconds without appreciable error, but the period of the balance on account of slight accidental shifting of small additional masses upon the counterpoise weight m_3 (Fig. 1), differed from twenty-four seconds sometimes by 1 per cent. It is necessary, therefore, to find the error in the deflection due to this variation in the period. This is done by making $t = \frac{T}{4} + \delta$ in equation (2) and in introducing the new conditions in equation (3). But it is simpler and sufficiently accurate to assume the motion as undamped. For this condition the amplitude

$$A = \frac{L}{G}\left\{2 + 2 \sin 2\pi \frac{\delta}{T}\right\}^{\frac{1}{2}} = \sqrt{2}\frac{L}{G}\left(1 + \pi \frac{\delta}{T}\right) \text{ nearly.}$$

For $T = 23.75$ seconds, $\frac{T}{4} = 5.94$ and $\delta = 0.06$. Hence

$$A = \sqrt{2}\frac{L}{G}(1.008).$$

If $\delta = 0$, $A = \sqrt{2}\frac{L}{G}$, consequently an error of 1 per cent. in T causes an error of 0.8 per cent. in A.

To make sure that the observed radiation pressures depended only on the intensity of the beam, and were uninfluenced by the wave-length of the incident energy, the ballistic observations of pressure, the thermal measurements of intensity, and the determination of the reflection coefficients, were carried out for three entirely different wave-groups of the incident radiation. In the measurements designated "through air," no absorbing medium was introduced in the path of the beam between the lamp and the balance except the glass lenses and plates already mentioned. In the measurements "through red glass," a plate of ruby glass was put in the path of the beam between L_2 and d_1 (Fig. 2). For the observations "through water cell," a 9 mm layer of distilled water in a glass cell was placed in the path of the beam at the same point.

The separate observations entering into a single series of ballistic measurements and their treatment will appear from Table II, which is copied direct from the laboratory notebook and represents an average ballistic series. The designations

EVC_s, WVD_s, EVD_g, WVC_g mean that the vane C in the first case was on the east side of the rotation axis with its silver face toward the light. The subscript g signifies that the glass face of the vane was toward the light. The second column of the table gives the zero reading of the balance before opening the shutter; the third, the end of the swing produced by a six-second exposure; the fourth, the deflection of the balance; the fifth, the ballistic deflection of the lamp galvanometer G_2. Columns six and seven give the balance deflection reduced to standard lamp.

The results of all the ballistic pressure measurements "through air" are collected in Table II. In the fourth and fifth columns two values are given for the constant of the lamp galvanometer G_2, since reversing the magnet on the balance bell-jar to reverse the suspension within affected the constant of the galvanometer slightly, the values for the silver and glass faces forward were never the same. The subscripts show to which series, silver or glass, the constant belongs. The values of the lever-arm l of the balance, in the sixth column, are obtained by measuring the distance between the centers of the images when on the east and west vanes (by the dividing engine T_2, Fig. 2) and dividing by two. The columns headed $\frac{C_s + D_s}{2} = P_s$ and $\frac{C_g + D_g}{2} = P_g$ are the average moments due to pressures for the silver and glass sides of the vanes respectively toward the light. The next two columns contain these moments corrected for a period of 24 seconds of the torsion balance. The columns headed $\frac{P_s \times G_s}{l}$ and $\frac{P_g \times G_g}{l}$ are the corresponding forces reduced to standard sensitiveness, $G = 1000$. The final column contains the averages of the two columns which precede it. Table III exhibits corresponding data for "red glass" and "water cell." The air pressure, period of the balance, lever arm and galvanometer constants are those given in Table II for the same date.

In these ballistic measurements the lamp reading was the throw due to an exposure of the light upon the bolometer for six seconds, but in the energy measurements the lamp reading was a stationary deflection due to prolonged exposure. To bring the pressure values into comparison with the energy meas-

TABLE I.

AUGUST 28. LIGHT PRESSURE. BALLISTIC MEASUREMENTS. AIR.

Surface	Zero	Throw	Deflection	Lamp	Deflection (Lamp 100)	
					E. V.	W. V.
			mm	mm	mm	mm
E V C_s....	281.4	248.5	32.9	164.3	20.0	
W V D_s...	281.5	313.9	32.4	164.5		19.7
E V C_s....	281.4	249.8	31.6	157.9	20.0	
W V D_s...	281.5	310.5	29.0	147.0		19.8
E V C_s....	281.5	252.6	28.9	144.8	20.0	
W V D_s...	281.5	309.6	28.1	141.8		19.8
E V C_s....	281.5	252.9	28.6	143.5	19.9	
W V D_s...	281.5	309.3	27.8	140.4		19.8
				Average.....19.97		19.77

Average, $\frac{C_s + D_s}{2} = 19.87$

MAGNET REVERSED.

E V D_g....	280.1	246.0	34.1	180.4	18.92	
W V C_g....	280.0	317.8	37.8	187.3		20.20
E V D_g....	279.8	247.2	32.6	170.8	19.20	
W V C_g....	279.4	313.7	34.3	169.4		20.25
E V D_g....	279.1	248.9	30.2	161.1	18.80	
W V C_g....	279.0	311.9	32.9	161.6		20.35
E V D_g....	279.0	249.1	30.0	158.9	18.90	
W V C_g....	278.9	311.2	33.2	164.4		20.20
				Average.....18.97		20.25

Average, $\frac{C_g + D_g}{2} = 19.61$

TABLE II.

RADIATION PRESSURE. BALLISTIC MEASUREMENTS, THROUGH AIR.

Date	Air Pressure in mm of Hg.	Balance Period T	Sens. of Galvanometer G_s (Silver)	Sens. of Galvanometer G_g (Glass)	l — Lever Arm cm	$\frac{C_s + D_s}{2} = P_s$	$\frac{C_g + D_g}{2} = P_g$	P_s corrected for $T = 24$	P_g corrected for $T = 24$	$\frac{P_s \cdot G_s}{l}$	$\frac{P_g \cdot G_g}{l}$	Average
June 19.....	32.5	23.75	[734]		0.814	Aver.	18.88	Aver.	18.73	Aver.	16.80	16.89
20.....	32.5	23.75	756.4	768	0.814	19.67	19.94	19.51	19.81	18.12	15.86	17.00
23.....	37.0	23.75	700	716	0.814							
July 23.....	16.0	23.75	682	707	0.831	21.16	20.42	21.00	20.26	17.25	17.25	17.25
25.....	16.6	23.75	684	684	0.815							
26.....	16.6	23.75	720	710	0.815	19.34	19.98	19.18	19.82	16.93	17.28	17.10
August 27.....	16.8	23.82	724	710	0.823	20.10	19.40	20.00	19.25	17.66	16.91	17.10
28.....	15.7	23.82	721	716	0.824	19.87	19.61	19.73	19.48	17.26	16.97	17.10
29.....	13.7	23.82	712	701	0.824	19.68	20.07	19.53	19.92	16.99	16.97	16.94
31.....	14.0	24.00	718	713	0.810	18.55	18.94	18.55	18.94	16.44	16.60	16.52
September 1.....	16.6	24.00	692	672	0.808	19.14	20.17	19.14	20.17	16.40	16.78	16.59
20.....	16.4	23.78	670	670	0.812	20.96	22.02	20.81	19.87	17.17	16.54	16.86
23.....	16.4	23.78	666	684	0.810	21.32	20.27	21.16	20.12	17.27	16.87	17.07
24.....	16.2	23.78	667	669	0.816	20.76	19.80	20.60	19.65	16.84	16.11	16.47
								Average.....		17.11	16.71	16.91 ± 0.053

urements it is necessary to reduce the average of the quantities in the last column to pressures in dynes by multiplying by 0.363×10^{-5}, the torsion coefficient of the quartz fiber, and to reduce not only to a static deflection of the torsion balance but also to a static deflection of the lamp galvanometer G_2. The ratio of a ballistic to a static deflection of the galvanometer G_2 was obtained from a long series of lamp exposures. This ratio was found "through air" to be $= 1.55$; "through red glass" $= 1.535$; "through water cell" $= 1.502$. These differences are probably due not solely to the damping constant of the galvanometer but to the peculiar manner in which the bolometer was warmed up to its stationary conditions by the beam from the lamp. Applying these reduction factors to the averages in Tables II and III, we obtain the following results. The pressure of the standard light beam which has passed

(*a*) through air $= 16.91 \times \frac{1.55}{1.357} \times 0.363 \times 10^{-5} =$
$(7.01 \pm 0.023) \times 10^{-5}$ dynes ;

(*b*) through red glass $= 16.91 \times \frac{1.535}{1.357} \times 0.363 \times 10^{-5} =$
$(6.94 \pm 0.024) \times 10^{-5}$ dynes ;

(*c*) through water cell $= 16.20 \times \frac{1.502}{1.357} \times 0.363 \times 10^{-5} =$
$(6.52 \pm 0.028) \times 10^{-5}$ dynes .

TABLE III.

RADIATION PRESSURE. BALLISTIC MEASUREMENTS.

	Through Water Cell.						
Date	$\frac{C_s + D_s}{2}$	$\frac{C_g + D_g}{2}$	P_s corrected for T 24°	P_g corrected for $T=24°$	$\frac{P_s \times G_s}{l}$	$\frac{P_g \times G_g}{l}$	P Average
June 20	18.62	17.10	18.46	16.96	17.14	16.00	16.57
July 25	19.00	20.10	18.85	19.94	15.82	16.74	16.28
26	18.03	19.39	17.89	19.33	15.80	16.84	16.32
August 27	18.63	18.66	18.50	18.53	16.29	15.99	16.14
29	18.25	19.02	18.10	18.87	15.68	16.06	15.87
September 20	20.39	19.14	20.23	19.00	16.69	15.82	16.25
23	20.21	19.51	20.05	19.36	16.37	16.23	16.30
24	19.84	18.91	19.69	18.77	16.10	15.40	15.70
Average	.....	.....	.....	.....	16.24	16.15	16.20 ±0.066

TABLE III—*Continued.*

THROUGH RED GLASS.

June 23	19.99	18.40	19.83	18.26	17.05	16.06	16.56
July 25	20.70	20.94	20.54	20.77	17.24	17.43	17.33
August 27.....	19.97	19.25	19.82	19.10	17.46	16.46	16.96
28.....	19.99	19.42	19.84	19.28	17.36	16.75	17.05
29.....	19.99	19.92	19.84	19.77	17.14	16.82	16.98
31.....	18.98	19.14	18.98	19.14	16.82	16.84	16.83
September 20.....	21.00	19.97	20.83	19.82	17.19	16.50	16.84
23.....	21.48	20.34	21.31	20.18	17.39	16.92	17.15
24.....	21.00	19.68	20.83	19.53	17.03	16.03	16.53
Average	.....	.....	.....	.....	17.18	16.65	16.91 ±0.051

THE ENERGY MEASUREMENTS.

Before rejecting the bolometer method used in the preliminary measurement of energy, a second bolometer of slightly different construction was tried; but the lack of uniformity of resistance, already mentioned, made its indications too uncertain for the present work. The radiant intensity of the beam used in the later experiments was determined by directing it upon the blackened face of a silver disk, weighing 4.80 grams of 13.3 mm diameter and of 3.58 mm thickness, and by measuring its rate of temperature rise as it passed through the temperature of its surroundings. The disk was obtained from Messrs. Tiffany & Co., and was said by them to be 99.8 per cent. fine silver. Two holes were bored through parallel diameters of the disk, one-fourth of the thickness of the disk from either face. Two iron-constantan thermo-junctions, made by soldering 0.1 mm wires of the two metals, were drawn through the holes into the center of the disk. To insulate the wires from the disk, fine drawn glass tubes were slipped over them and thrust into the holes, leaving less than 2 mm bare wire on either side of the junctions. The wires were sealed into the tubes, and the tubes into the disk by solid shellac. The tubes projected 15 mm or more from the disk and were bent upward in planes parallel to the faces of the disk. The general arrangement will be seen in Fig. 5. The disk was suspended by the four wires some distance below a small flat wooden box. On the box was fastened a calorimeter can swathed in cotton and filled with kerosene in which the con-

stant thermo-junctions were immersed. Copper wires soldered to the two ends of the thermo-electric series were brought out of the calorimeter, and the circuit was closed through 1,000 ohms in series with the 500 ohms resistance of galvanometer G_1. The thermo-junctions in the disk were in series, and as each junction was midway between the central plane of the disk and either face, it was assumed that when the disk was slowly warmed by heating one face the electro-motive forces obtained corresponded to the mean temperature of the disk. One face of the disk was blackened by spraying it with powdered lampblack in alcohol containing a trace of shellac. This method was suggested by Professor G. E. Hale and gives very fine and uniform dead black coatings not inferior to good smoke deposits.

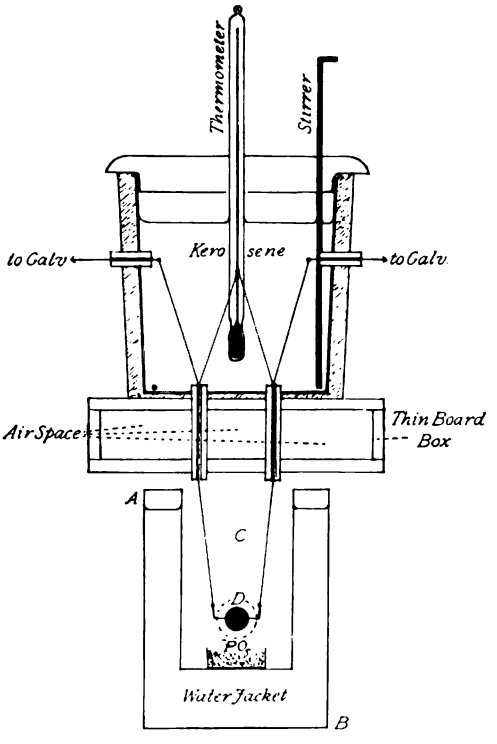


FIG. 5.

For the energy measurements the bell-jar and the torsion balance were removed from the platform P (Fig. 1) and a double-walled copper vessel, AB (Fig. 5), which served as a water jacket surrounding a small air chamber C, was mounted in the same place. A tube 2 cm in diameter was soldered into the front face of the jacket to admit the light beam into the chamber C. This opening was covered by a piece of plate glass similar to the plates forming the larger windows in the bell-jar.

The needle system in G_1, a four-coil du Bois-Rubens galvanometer, was suspended in a strong magnetic field so that its period was about four seconds. The system was heavily damped by a mica air-fan of large surface. The disk junctions and

galvanometer responded quickly to the radiation, as was shown by the reversal of motion of the magnet system 1.2 seconds after the light was cut off from the disk, when the latter was a few degrees above the temperature of the room.

The disk was calibrated for temperature in terms of the deflection for a definite sensitiveness of the galvanometer G_1. For this purpose the disk was immersed in a kerosene bath and the galvanometer deflection measured for two different temperatures of the disk. One of these was about 18° C. above the comparatively steady temperature of the room, or calorimeter containing the standard temperature junctions (see Fig. 5), and the other about the same number of degrees below the room temperature. These two temperatures were measured by a Fuess Standard Thermometer divided into tenths of a degree and calibrated at the *Reichsanstalt.* Two calibrations of the silver disk were made some days apart. One of these series appears in full in Table IV. The first three columns of the table give the zero,

TABLE IV.

CALIBRATION OF SILVER DISK.

Cold Bath.								
G_1 Readings			Disk T_1°	Room cal. T_2°	Deflection of G_1	Means of Alternate Deflections	G_1 Means	$T_2 - T_1$
Reversed	Zero	Direct						
	221.2							
	221.0	35.2	1°.58	20°.05	185.8			
	220.9							
402.0	221.0		1.60	20.07	181.0	185.7	183.4	18.47
	221.2							
	221.2	35.7	1.57	20.10	185.5	181.3	183.4	18.53
	221.2							
403.1	221.5		1.56	20.14	181.6	185.8	183.7	18.58
	221.9							
	222.0	35.9	1.54	20.16	186.1	182.2	184.1	18.62
	222.1							
405.0	222.2		1.52	20.20	182.8	186.3	184.5	18.68
	222.4							
	222.7	36.2	1.57	20.22	186.5	182.7	184.6	18.65
	223.0							
405.8	223.1		1.60	20.26	182.7	186.7	184.7	18.66
	223.2							
	223.3	36.3	1.59	20.30	187.0			
	223.5						184.0	18.60
	Correction to $T_1 = 0°.00$.							

TABLE IV—*Continued.*

				WARM BATH.				
	217.3							
	218.6	434.2	41°45	20°40	215.6			
	219.9							
2.0	*220.5*		41.35	20.42	218.5	213.7	216.1	20.93
	221.1							
	222.4	434.2	41.25	20.44	211.8	216.4	214.1	20.81
	223.7							
10.0	*224.4*		41.08	20.50	214.4	209.1	211.7	20.58
	225.1							
	225.7	432.1	40.90	20.55	206.4	212.4	209.4	20.35
	226.3							
16.3	*226.7*		40.80	20.60	210 4	204.8	207.6	20.20
	227.2							
	227.8	431.0	40.67	20.61	203.2	208.5	205.8	20.07
	228.4							
21.8	*228.5*		40.55	20.63	206.7	201.8	204.2	19.92
	228.7							
	229.0	429.4	40.43	20.65	200.4			
	229.3						209.8	20°41
	Correction to $T_1 = 0°10$.						Corrected, 20.51	

direct, and reversed reading of the galvanometer G_1. The fourth column gives the temperature of the bath in which the disk was immersed, and the fifth, that of the constant temperature calorimeter. The sixth column gives the deflections of G_1. The seventh column the means of the alternate deflections. The eighth, the mean of the two columns preceding it. The last column gives the difference in temperature between the two calorimeters in degrees C. For the total temperature range in the table, 39°11, the deflection of G_1 was 393.8 scale divisions for a sensitiveness of $G_1 = 996$. A range of one degree would thus give a deflection of 10.03 divisions for a sensitiveness of $G_1 = 1000$. The mean of two separate calibrations was 9.96 scale divisions for one degree temperature difference.

Before beginning a series of intensity measurements the disk was suspended in an air-chamber containing phosphoric anhydride and surrounded by a jacket of ice and salt. The disk was thus lowered to a temperature of about zero degrees and was then quickly transferred to the chamber C (Fig. 5), and the beam was directed upon it. When its temperature had risen to

within five or six degrees of that of the chamber C, galvanometer readings were made at intervals of five seconds until the disk was heated to a temperature several degrees above its surroundings. The temperature of the chamber C was determined by removing the disk and cooling it to a point near the room temperature, then replacing it and observing its rate of temperature change for several minutes.

The notebook record of one series of observations showing the heating of the disk by the light beam is given in full in Table V. It will be seen from the table that the temperature of the disk passed that of the chamber thirty seconds after the beginning of the series. The readings of G_1 at equal time intervals on either side of the zero are on horizontal lines. The last column of the table contains the rate at which the galvanometer deflection was changing when the disk and its surroundings were at the same temperature.

Energy series were made "through air," "through red glass," and "through water cell," as in the pressure measurements. During the experiment the black coatings were frequently cleaned off from the disk and new ones deposited. The final result therefore does not correspond to an individual, but to an average coating.

To correct for any inequality between the two disk thermojunctions or any lack of symmetry in their positions, referred to the central plane of the disk, which might prevent the mean temperature of the two junctions from representing the mean temperature of the mass, series of observations were made on each face of the disks. The black coating was always cleaned off from the face of the disk away from the light. All of the series of energy measurements are gathered together in Tables VI and VII. In the tables, under the head of "through air," the first column contains the observed rate of increase in the galvanometer deflection G_1, when the disk and its surroundings were at the same temperature; the second column, the corresponding mean lamp deflections of galvanometer G_2. The third and fourth columns contain the sensitiveness of galvanometers G_1 and G_2 respectively, and the last column the values of the

TABLE V.

AUGUST 16. ENERGY MEASUREMENTS. THROUGH AIR. SERIES 4.

Zero of galvanometer G_1 (closed circuit) determined by method of cooling = 216.8 = reading at room temperature.

Time	G_1	Time	G_1	ΔG_1	Δt	$\frac{\Delta G_1}{\Delta t}$ (in mm per sec.)
0 secs.	174.5	60 secs.	253.2	78.7	60 secs.	1.312
5 "	182.0	55 "	247.3	65.3	50 "	1.306
10 "	189.0	50 "	241.3	52.3	40 "	1.308
15 "	196.2	45 "	235.2	39.0	30 "	1.300
20 "	203.0	40 "	229.1	26.1	20 "	1.305
25 "	209.7	35 "	222.8	13.1	10 "	1.310
30 "	216.4			Average.....		1.307

The lamp reading (G_2) was 924.

The sensitiveness of G_2 was 667, and of G_1 was 996.

$\frac{\Delta G_1}{\Delta t}$ reduced to standard conditions becomes

$1.307 \times 667 \times 996 \div (924 \times 1000) = 0.943$ mm per sec.

first column reduced to standard lamp and standard sensitiveness of both instruments. The series on the two faces of the disk are recorded and averaged separately, then combined with their probable errors in the general average at the end of Table VII.

Tables VI and VII give the following results. The average increase in the reading of G_1 for standard conditions is 0.966 mm per second. From the thermal calibration, a deflection of 9.96 divisions corresponds to a temperature difference of 1° C. Consequently the rise in temperature of the silver disk per second when the light passed:

(*a*) through air $= 0.966 \div 9.96 \quad (0.^{\circ}0970 \pm 0.^{\circ}00034)$ C.;

(*b*) through red glass $= 0.942 \div 9.96 = (0.^{\circ}0946 \pm 0.^{\circ}00036)$ C.;

(*c*) through water cell $= 0.880 \div 9.96 = (0.^{\circ}0884 \pm 0.^{\circ}00064)$ C.

The mass of the silver disk was 4.80 grams, its specific heat[1] at 18° C. = 0.0556; the mechanical equivalent of heat at 18° C.

[1] U. BEHN, *Ann. der Phys.*, **4**, 266, 1900.

TABLE VI.

FRONT FACE.

Date	Through Air					Through Red Glass			Through Water Cell		
	$\frac{\delta G_1}{\delta t}$	G_2 (Lamp)	S_1	S_2	$\frac{\delta G_1}{\delta t}$ Reduced to Stan.	$\frac{\delta G_1}{\delta t}$	G_2	$\frac{\delta G_1}{\delta t}$ Reduced to Stan.	$\frac{\delta G_1}{\delta t}$	G_2	$\frac{\delta G_1}{\delta t}$ Reduced
August 10..	1.387	980	990	689	0.965				0.437	345	0.864
10..	1.263	920	990	689	0.936				0.400	311	0.877
10..									0.369	279	0.902
11..	1.244	866	986	701	0.992	0.750	546	0.950	0.412	315	0.905
11..	1.455	1010	986	701	0.995	0.750	546	0.950	0.510	382	0.922
11..	1.505	1047	986	701	0.994				0.516	381	0.935
16..	1.447	1022	996	669	0.942	0.736	529	0.927	0.416	327	0.873
16..	1.284	886	996	669	0.966	0.740	527	0.936	0.451	352	0.853
16..	1.316	925	996	669	0.948	0.797	550	0.965	0.502	382	0.875
16..	1.307	924	996	669	0.943						
18..	1.598	1110	995	667	0.955	0.738	515	0.952	0.449	333	0.895
18..	1.550	1047	995	667	0.984	0.732	518	0.940	0.445	342	0.865
18..	1.548	1031	995	667	0.995	0.730	518	0.938	0.451	346	0.867
18..	1.410	957	995	667	0.977						
18..	1.330	898	995	667	0.983						
19..	1.241	862	1001	675	0.975	0.760	532	0.965	0.451	343	0.892
19..	1.360	934	1001	675	0.985	0.728	512	0.960	0.452	338	0.904
19..	1.324	905	1001	675	0.990	0.738	525	0.950	0.466	351	0.898
19..	1.364	934	1001	675	0.988						
					Average..... 0.973 ±0.003			Average..... 0.948 ±0.002			Average..... 0.888 ±0.004

TABLE VII.

REAR FACE.

Date	Through Air					Through Red Glass			Through Water Cell		
	$\frac{\delta G_1}{\delta t}$	G_2 (Lamp)	S_1	S_2	$\frac{\delta G_1}{\delta t}$ Reduced to Stan.	$\frac{\delta G_1}{\delta t}$	G_2	$\frac{\delta G_1}{\delta t}$ Reduced	$\frac{\delta G_1}{\delta t}$	G_2	$\frac{\delta G_1}{\delta t}$ Reduced
August 12..	1.374	960	991	684	0.970	0.808	578	0.949	0.495	370	0.906
12..	1.331	932	991	684	0.968	0.740	536	0.935	0.434	320	0.919
12..	1.284	900	991	684	0.967	0.705	542	0.957	0.489	371	0.895
15..	1.428	992	996	670	0.960	0.703	506	0.926	0.490	368	0.890
15..	1.428	984	996	670	0.968	0.742	526	0.941	0.466	352	0.885
15..	1.531	1068	996	670	0.962	0.765	551	0.926	0.440	337	0.873
20..	1.477	1047	996	685	0.961	0.703	522	0.918	0.458	375	0.833
20..	1.520	1090	996	685	0.951	0.760	537	0.905	0.497	400	0.848
20..	1.576	1130	996	685	0.951	0.781	570	0.935	0.507	408	0.848
20..	1.568	1124	996	685	0.950						
21..	1.783	1224	995	668	0.970	0.846	604	0.932	0.503	393	0.852
21..	1.773	1232	995	668	0.957	0.790	575	0.915	0.481	377	0.850
21..	1.705	1190	995	668	0.953	0.803	575	0.930	0.484	373	0.862
21..	1.452	1019	995	668	0.948						
					Average.. .. 0.960 ±0.0014			Average..... 0.930 ±0.003			Average..... 0.872 ±0.005
Average of front and rear face, 0.966 ±0.0034								0.942 ±0.0030			0.880 ±0.0064

$= 4.272 \times 10^7$ ergs.[1] Consequently the energy of the standard radiation is

(a) through air, $0.0970 \times 4.80 \times 0.0556 \times 4.272 \times 10^7$
or $E_a = (1.108 \pm 0.004) \times 10^6$ ergs per second.

(b) through red glass, $E_r = (1.078 \pm 0.004) \times 10^6$ ergs per second.

(c) through water cell, $E_w = (1.008 \pm 0.007) \times 10^6$ ergs per second.

REFLECTING POWER OF THE SURFACES USED.

According to Maxwell and Bartoli, the pressure in dynes per square centimeter for normal incidence is equal to the energy in ergs in unit volume of the medium. The energy in unit volume is made up of both the direct and reflected beams. If E is the intensity of the incident beam and ρ the reflection coefficient, the pressure $p = \frac{E(1+\rho)}{V}$, where V is the velocity of light. The methods for measuring p and E have already been described. The determination of ρ for both sides of the vanes C and D was made as follows. The supports of the torsion balance were replaced by the divided circular plate A (Fig. 6), of a force table which could be rotated about a central, vertical axis. The rod about which the plate turned passed up through the plate and at its top the mirror holder bb was fastened. The vanes were freshly silvered and mounted on a plate-glass carrier aa, which was held by a clamp against the back face of bb. The beam was directed on the vanes by the lens L_1 (Figs. 2 and 6) exactly as it had been in the pressure observations. After reflection from the vane the beam fell on a concave mirror M which projected an image of the vane upon a simple sheet bolometer B, forming the unknown resistance of a post-office box bridge. The current was supplied from storage cells and the galvanometer was the same used in the energy determinations but fitted with low resistance coils. The bolometer was covered by the bell-jar used earlier. The mirror M, the bell-jar, and bolometer were attached to the plate of the force table. The full line diagram shows the arrangement for reflection. The dotted figure shows the position for a measurement of the direct beam. All measurements of direct reflection were made for an angle of incidence of $12^\circ.5$.

[1] Mean of Rowland's and Griffith's values. *Phil. Trans.*, 5, 184, 496, 1893.

The method of observing will be seen from the notebook record of a single series of measurements given in Table VIII. In the table, D and R indicate direct and reflected beams, respectively. The first and second columns contain the zero points and end of swings of the galvanometer G_1, and the third column, the deflection. The remaining columns, in order, contain the lamp galvanometer deflection; the deflection of G_1 reduced to constant lamp; the means of each pair of D and R values; the means of alternate readings; and the final column, the quotients of the two preceding columns which are the reflection coefficients sought. In all, three series of measurements were made on the silver, and two series on the glass-silver faces of each vane. To get average coefficients which would represent the range of condition of the mirrors during the pressure measurements, the vanes were cleaned and new silver coatings deposited between each two series on the same vane. The reflection coefficients are collected in Table IX. For each surface studied the diffused reflection for a beam which had traversed air was determined by setting the mirror holder for normal incidence. The diffuse energy reflected at an angle of 25° falling on the full aperture of the mirror M was measured, and the total diffuse energy for the hemisphere computed on the basis of the cosine law. It may be shown that of the diffuse reflection, two-thirds is effective as light pressure. This increases the air-silver reflection coefficients by 0.9 per cent. and the glass-silver

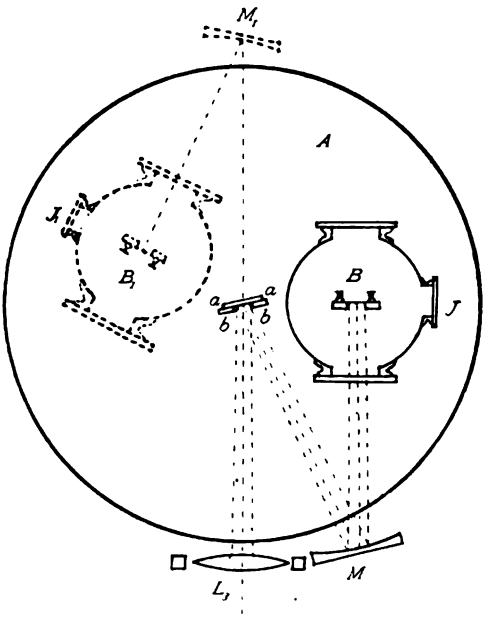


FIG. 6.

TABLE VIII.

OCTOBER 31, 1902. REFLECTION COEFFICIENT OF D_g. AIR.

	G_1 Zero	G_1 Turning Point	Deflection G_1	Lamp	G_1 Reduced to Standard	Averages	Alternate Averages	Reflection Coefficient
R	350.0.....	159.5	190.5	132.6	143.8	142.5		
	349.0.....	152.0	197.0	139.3	141.3			
D	349.5.....	100.5	249.0	136.8	182.1	182.5	142.0	0.779
	350.0.....	111.5	238.5	130.5	183.0			
R	346.0.....	177.0	169.0	119.3	141.2	141.5	182.5	.775
	347.0.....	171.0	176.0	124.4	141.7			
D	348.5.....	123.0	225.5	124.0	181.8	182.4	141.1	.773
	348.5.....	120.0	228.5	125.0	183.0			
R	345.0.....	172.0	173.0	122.6	141.0	140.6	182.7	.770
	345.0.....	171.0	174.0	124.0	140.3			
D	346.0.....	132.0	214.0	115.5?		183.0	141.2	.773
	346.0.....	124.0	221.0	120.7	183.0			
R	344.0.....	173.0	171.0	120.7	141.8	141.7	182.1	.778
	344.5.....	171.0	173.5	122.6	141.6			
D	346.0.....	119.0	227.0	125.3	181.0	181.2	141.6	.781
	346.0.....	117.5	228.5	126.0	181.3			
R	342.0.....	174.0	168.0	118.0	142.3	141.5	181.8	.780
	342.0.....	170.5	171.5	122.0	140.8			
D	347.0.....	130.0	217.0	119.0	182.3	182.5	141.4	.775
	347.0.....	134.0	213.0	116.7	182.7			
R	341.5.....	174.5	167.0	118.0	141.3	141.2		
	341.0.....	173.0	168.0	119.0	141.1			

Average 0.776

TABLE IX.

REFLECTION COEFFICIENTS IN PERCENTAGES.

	C_s				C_g			
	Through Air	Red Glass	Water	Diffuse	Through Air	Red Glass	Water	Diffuse
	92.8	94.5	88.9	0.98	77 8	75.9	80.8	
	89.8	96.8	86.0	0.92	77.6	76.6	80.0	1.6
	90.8			1.23				
Average..	91.1	92.7	87.5	1.04	77.7	76.3	80.4	1.6
	D_s				D_g			
	95.0	96.3	91.5	2.2	77.6	76.5	81.0	2.8
	92.0	94.0	90.4		76.7	75.2	79.7	2.2
	94.8	95.0	92.3	0.8				
Average..	93.9	95.1	91.4	1.5	77.2	75.9	80.4	2.5

TABLE IX—*Continued.*

AVERAGE REFLECTION.

Air-Silver				Glass-Silver			
92.5	93.9	89.5	1.3	77.5	76.1	80.4	2.0

CORRECTED REFLECTION COEFFICIENTS.

92.0	93.4	89.0	77.6	76.2	80.5	

Average Coefficients through Air, *84.8;* Red Glass, *84.8;* Water, *84.8.*

values by 1.3 per cent. The small glass rod *d* (Fig. 1), not present in the reflection measurements, decreased the reflecting area of the silvered surfaces in the pressure measurements by 1.54 per cent. The air-silver values are thus decreased by $0.92 \times 1.54 = 1.4$ per cent., and the glass-silver values by $0.78 \times 1.54 = 1.2$ per cent. The application of these two corrections gives the final corrected coefficients in Table IX. The diffuse reflection of black coatings deposited by the method used in blackening the silver disk was measured and computed in the same manner as the diffused reflection from the vanes *C* and *D*. The agreement found by Ångström[1] between the diffuse reflection of matte surfaces for normal incidence and the cosine law was abundantly close for the present purpose. Five determinations of this reflection were made under different conditions and with different coatings. The values in percentages of the incident beam were 4.4 per cent., 4.5 per cent., 4.2 per cent., 4.6 per cent., and 5.2 per cent.; average, 4.6 per cent. Thus only 95.4 per cent. of the incident beam was absorbed by the black coating on the silver disk in producing the temperature increase observed. Hence the true energy of the beam is equal to the observed energy divided by 0.954.

The silver disk, diameter 13.3 mm, used in the energy measurements, received long waves and scattered radiation which passed round and through the light pressure vanes of diameter 12.8 mm. This amount was experimentally determined for both thin and thick silver coatings in order to approximate to the

[1] K. ÅNGSTRÖM, *Wied. Ann.*, **26**, 271, 1885.

average condition of the coatings in the light pressure measurements, and it was found to average (a) through air, 1.40 per cent.; (b) through red glass, 1.44 per cent.; (c) through water, 0.46 per cent. On this account the energy E of the standard radiation must be reduced by the above percentages.[1] Applying these corrections and the corrections due to the diffused radiation from the black coating on the silver disk, the energy of the standard radiation becomes

(a) through air, $E_a \times \frac{0.986}{0.954}$;

(b) through red glass, $E_g \times \frac{0.986}{0.954}$;

(c) through water, $E_w \times \frac{0.995}{0.954}$;

Hence the pressure produced by standard radiation, calculated by Maxwell's formula, $p = \frac{E(1+\rho)}{3 \times 10^{10}}$, since $\rho = 0.848$, becomes

(a) through air $= E_a \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$

$= 1.108 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^6$ dynes

$= (7.05 \pm 0.03) \times 10^{-5}$ dynes ;

(b) through red glass $= E_g \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$

$= 1.078 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^6$ dynes

$= (6.86 \pm 0.03) \times 10^{-5}$ dynes ;

(c) through water $= E_w \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954}$

$= 1.008 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954} \times 10^6$ dynes

$= (6.48 \pm 0.04) \times 10^{-5}$ dynes.

[1] As the average pitch of the cone of the incident beam was about one part in forty, no correction need be applied for inclination. Furthermore, the inside of the bell-jar was blackened and the zero of the balance was so chosen that energy reflected from the window admitting the beam could produce no pressure effects.

A comparison of observed and computed pressures follows:

	Observed Values in 10^{-5} Dynes	Computed Values in 10^{-5} Dynes	Obs.-Comp. in Percentages
Through air.....	$p=7.01\pm0.02$ [1]	7.05 ± 0.03	-0.6
Through red glass ...	$p=6.94\pm0.02$	6.86 ± 0.03	$+1.1$
Through water	$p=6.52\pm0.03$	6.48 ± 0.04	-0.6

An estimate of the approximate magnitude of the gas action, not eliminated by the ballistic method of observation, may be reached from the following considerations.

When radiation falls upon a vane of the torsion balance, part of it is absorbed by the silver surface. From the amounts directly and diffusely reflected, as given in Table IX, the amount transmitted by the average surface (experimentally determined but not given in Table IX), the effect of the glass rod and the reflection coefficient of the glass surface, it was found that, when the silver side of the vane was toward the radiation source, the absorption coefficient for radiation through air was 6 per cent., and when the glass surface was forward, it was 18 per cent.

The total force acting on the vane is made up of two parts, that due to radiation pressure and that due to gas action. Let F_r be the force due to the first cause, assuming that all the radiation is absorbed, and F_g the effect due to the second, on the same condition. Then the total effect, when the silver side of the vane is forward and the radiation is "through air," is $1.92\,F_r+0.06\,F_g$. When the glass side is forward the total effect is $1.776\,F_r-0.18\,F_g$. Making these expressions equal to the reduced deflections (Table II, columns 11 and 12) on the silver and glass surfaces respectively, we have two equations by means

[1] The pressure and energy measurements for the three different wave groups through air, red glass, and water cell, constitute three independent experiments. In the values for pressure, 7.01, 6.94, and 6.52 in the three cases are only accidentally related. The difference arises from the different reflecting power of the 45° glass plate (Fig. 2) for the different beams and from the fact that the indications of the lamp galvanometer G_2 connected with bolometer A', were probably not strictly proportional to energy for throws differing as widely as 33, 60, and 100, which, roughly, were the relative intensities of the beams through water cell, red glass, and air. The function of the lamp bolometer and galvanometer was purely to keep a check on the small variations of the lamp, which rarely fluctuated more than 10 per cent. on either side of the mean value.

of which the values of F_r and F_g may be obtained. Hence the effect due to gas action on each face of the vane is approximately determinate, as is also the part ($0.06\,F_g$) not eliminated when we average the two columns to obtain column 13.

Applying this method to all the results of Table II (with the exception of those results taken with poor mirrors as shown by our notes), the gas action present in the ballistic deflections "through air" is 0.8 per cent. Applying the corresponding data and equations to Table III, the gas action present in the red glass values is 1.1 per cent. and in the water cell values, 0.3 per cent. The sign of F_g comes out negative, which means that the gas action was suction.

This reasoning assumes that the glass faces of the vanes during the six seconds exposure are not warmed by absorption nor by the conduction of heat through the thin glass from the silver coating. The effect of any such absorption or conduction would be to diminish the computed gas action. As estimated from the static observations, the gas action in the ballistic measurements is comparable in magnitude with the computed values obtained above, and of the same sign. Both results show that the uneliminated gas action by the most liberal estimate cannot have exceeded 1 per cent. of the radiation pressure. Because of its smallness and indefiniteness no correction for gas action has been made to the final pressure values. If corrections were applied, its effect would be to reduce slightly the observed pressures.

Aside from the measurements of pressure and energy for which the probable errors are given, the percentage accuracies in the other measurements entering into the computations, and their effects upon the final result follow:

1. Quantities which affect individual series:

(*a*) Pressure values—

Period of balance	T,	accurate to 0.2 %;	effect on result		0.0 %
Lever arm of balance	L,	" 0.1 %;	"	"	0.0 %
Constant of galv'meter	G_2,	" 0.5 %;	"	"	0.0 %
Estimate of possible error due to changing ratio of period of G_2 to length of exposure of bolometer		" 0.4 %;	"	"	0.1 %

(b) Energy values—

Constant of galv'meter G_1, accurate to 0.1 %; effect on result 0.0 %

" " G_2, " 0.5 %; " " 0.0 %

2. Quantities which affect final averages:

(a) Pressure values—

Torsion of fiber,	accurate to	0.2 %	: effect on result		0.2 %
Reducing factor, 1.357	"	0.1 %	; "	"	0.1 %
Reducing factor, 1.550 for G_2	"	0.2 %	; "	"	0.2 %
Reflection of surfaces of vanes	"	0.4 %	; "	"	0.2 %

(b) Energy values—

Mass of silver disk	"	0.1 %	; "	"	0.1 %
Thermal calibration of disk	"	0.5 %	; "	"	0.5 %
Diffuse reflection black coating	"	5.0 %	; "	"	0.1 %

From the agreement within the probable error of the air, red glass, and water values with the theory, it appears that the radiation pressure depends only upon the intensity of the radiation and is independent of the wave-length.

The Maxwell-Bartoli theory is thus quantitatively confirmed within the probable errors of observation.

WILDER LABORATORY, DARTMOUTH COLLEGE,
Hanover, N. H., February 1903.

THE APPLICATION OF RADIATION PRESSURE TO COMETARY THEORY.

By E. F. NICHOLS and G. F. HULL.

IN the experiments described in the foregoing paper the close agreement of theory with experiment warrants the rigid application of the radiation pressure theory in the explanation of cosmical phenomena.

In any balancing of radiation pressure against gravitation in comets the size of particles is the determining factor. The repulsion due to radiation pressure depends upon the intensity of the rays, the absorbing and reflecting power of the surface, and the cross-section of the body exposed. Gravitational attraction depends only upon mass, or the product of volume and density. It will be seen, therefore, that for spheres of a given substance the weight at a fixed distance from the Sun will vary with the cube of the radius, while radiation pressure will depend upon the radius squared. The ratio of pressure to weight will thus be inversely as the radius. This relation holds down to the point where the particles become so small that they begin to lose in absorbing and reflecting power through diffraction.

The intensity of the solar radiation and gravitation diminish with distance in accordance with the same law, so that the ratio of pressure to weight is a constant for the same body at all distances from the Sun.

For spheres of the same size, and the same absorbing and reflecting power, the ratio of pressure to gravitation is inversely as the density. The variation of this ratio, as it depends upon size and density, has been used by Lebedew[1] and Arrhenius[2] in the computation of the repulsion upon the finely divided matter of comets' tails, but the limiting value of the ratio for diminishing

[1] *Wied. Ann.*, **45**. 292, 1892; also ASTROPHYSICAL JOURNAL, **14**, 155, 1902.

[2] *Lehrbuch der kosmischen Physik*, p. 150, Leipzig, 1903.

spheres of the same density due to diffraction first appears in Schwarzschild's paper.[1]

Comet heads.—In the heads of comets the phenomena are most complicated and difficult of explanation, yet it seems worth while to try to gather together a few of the separate causes which may be at work in producing this intricate structure.

The heat received from the Sun by the nucleus of a comet may be spent in three ways: (1) In raising the temperature of the nucleus. As the nucleus is of relatively small mass and probably of low heat conductivity no very considerable quantity of heat is required for this purpose. (2) Heat may be, and doubtless is, used in the vaporization of volatile hydrocarbons and other substances in the nucleus. (3) Large quantities of heat are lost from the nucleus by radiation.

The porous structure of meteorites points to a similar structure in cometary nuclei. The jets from the nucleus outward to the envelope of the head may be formed by the heating of the vaporizable materials in the interior of the nucleus and the consequent shooting out under pressure of a mixture of gases and dust through holes in a loose outer crust. Lack of sufficient means of escape in this way may cause a bursting of the nucleus sometimes observed.

The general upward current of vapors from the nucleus to the envelope, aside from jets, may be due to convection away from the more strongly heated center.[2]

Because of the counter-pressure due to the radiation of the nucleus itself, the rising of even small solid particles from the nucleus to the envelope would not encounter as strong an unbalanced pressure from the solar rays as particles in the tail. For, if all the heat received from the Sun were again radiated from the nucleus on the side toward the Sun, these two counter-radia-

[1] *Sitzungsberichte der math.-phys. Classe der k. b. Akademie der Wissenschaften zu München*, 31, 293, 1901.

[2] Matter in the form of gases and vapors is not subject to radiation pressure, as solid and liquid particles are, because of the minuteness of molecular dimensions. Except in the spectrum regions of characteristic absorption, radiation can, theoretically, exert no pressure whatever upon a gas. Hence gases might rise from the nucleus toward the Sun practically unhindered by radiation pressure.

tion pressures would exactly balance at the surface of the nucleus.[1]

Small particles may also be aided in rising from the nucleus toward the Sun by gas forces. By numerous experiments on larger bodies immersed in a gas and illuminated on one side, it has been shown that they may be either repelled from the light source or drawn to it, depending upon the pressure of the surrounding gas (see curves, in the foregoing paper, p. 329.) If the gas pressure is not too low, particles after leaving the nucleus might first be drawn toward the Sun until a region of higher vacuum was reached in the ascent, and then be repelled.[2]

The brilliant envelope of the head may be regarded as forming at the height where condensation, caused by expansion and cooling, takes place. Here the repelling action of the solar radiation would reach a high value and the particles in the envelope would be driven backward to form the tail.

According to Arrhenius[3] this condensation in the envelope is assisted by the influx of negatively charged nuclei from the Sun, which serve as condensation centers for the ascending vapors. The height above the nucleus of the comet at which this condensation would occur would thus, in some measure, be governed by the supply of negative particles. These would be found in increasing numbers with diminishing distance from the Sun. This action may be responsible for the contraction of the head and envelope as comets approach the Sun.

[1] It is worth noting in this connection that the longer and invisible waves are as effective in producing pressure as the visible radiations, and that these long waves strongly preponderate in the spectra of solid bodies at temperatures low in comparison with the solar temperature.

[2] It is possible also that electrostatic forces may play a small part in the formation of the head from the nucleus. Arrhenius believes the Sun to have a positive electrical charge, due to the fact that it loses more negative electrons by condensation into nuclei and subsequent repulsion by radiation pressure than it does of *positive* electrons which do not as readily serve as centers of condensation. Streams of negatively charged particles would communicate a negative charge to the matter surrounding the comet's nucleus, which would thus be attracted by the Sun. As this attraction would oppose the formation of the tail in the same measure as it assisted that of the head, it cannot be a dominating influence.

[3] *L. c.*, p. 208.

The brilliancy of the envelope may be attributed, in large part, to the fact that bodies of sufficient size to reflect solar rays are first formed out of the vapors of the head in this region. The negative nuclei from the Sun would here experience an obstruction and lose the greater part of their motion by friction. Electrical interchanges and discharges would be more active, and the hydrocarbon spectrum be brighter in the envelope than in other parts of the head.

If the brightness of the head and its envelope depend upon the number of negatively charged nuclei which strike the comet, and if, as Arrhenius maintains, the nuclei move out from the Sun radially and in greatest numbers from regions of greater solar activity, comets crossing the surface defined by solar radii drawn through the Sun-spot belts should show a marked increase in brightness, especially in maximum Sun-spot years. The writers are not aware that any such influence has been looked for in the cases where sudden changes of brightness in comets have been observed.

Comet tails.—The maximum ratio of radiation pressure to gravitation, obtained theoretically by Schwarzschild for sunlight upon opaque reflected spheres of 0.8 density, under the most favorable conditions, was about 20 to 1, if the recent estimates (ranging from 3.5 to 4) of the solar constant were used.

In Bredichin's three types of cometary tails, the highest ratio of attraction to repulsion required is about 18 to 1.[1] The multiple tails observed in such comets as Donati's may thus be satisfactorily explained by the sifting action of radiation pressure in two ways; either by assuming, with Bredichin, that the particles in the different tails are of different densities, but of uniform size, or by assuming uniform density and particles of several different sizes.

While radiation pressure alone thus affords a satisfactory explanation of comets' tails, there is no reason to assume that it is the only cause of the repulsive action observed. There are several ways in which the gases and vapors present in the tail

[1] In comet 1893 II Hussey observed a considerably larger ratio of repulsion to gravitation.

may exert a force upon the small solid or liquid particles which are known to exist there:

1. Small particles, if warmed on one side when surrounded by gases or vapors, even under pressures so low that electrical discharges take place only under relatively high voltages, experience a strong repulsion, similar to that upon a vane of a Crookes radiometer.

2. Occluded gases or volatile materials upon the surface of the particles would be driven off by the Sun's heat on the illuminated sides, and the particles would thus receive a thrust in the direction away from the Sun.

3. If the particles were porous or loosely put together, containing cavities filled with more easily vaporizable substances, the resulting vapors would be shot out upon the hotter sides and the particles driven back by a kind of rocket action.

That these combined gas forces are still large, even in high vacua, will be seen from an actual experiment described later.

If we accept Arrhenius' theory that the solar activity produces numberless negative electrons which serve as condensation points for the vapors surrounding them in the solar atmosphere, and thus form small, negatively charged nuclei, which are driven from the Sun by radiation pressure,[1] these nuclei would exert a battering action upon the particles of the tail. In the last case a strange meeting point is found between the oldest, or Keplerian, and the latest explanation of the solar repulsion of comets' tails.

Finally Professor J. J. Thomson,[2] in investigating the action of electric waves upon charged bodies immersed in the medium, has found that a small repulsive effect may arise from this cause. This repulsive force is entirely distinct from the radiation pressure so far considered, but on the electromagnetic theory of light it may be competent to drive away electrons formed above the photosphere of the Sun, independently of the

[1] The supposed electrical discharges in the tail of a comet which give rise to its gaseous emission spectrum are attributed by Arrhenius to the electrical disturbances caused by the influx of these negative nuclei.

[2] *Phil. Mag.*, **4**, 253, 1902.

sign of the charge and of whether they have formed nuclei by condensation or not.

These last two causes of repulsion are in all probability of very minor importance when compared with radiation pressure, or even with gas action.

Experiment with a laboratory comet's tail.—Some of the above considerations led the writers to try to reproduce, as nearly as possible, in a vacuum tube some of the conditions believed to exist in comets' tails. The result of a hasty computation of the magnitude of the effect which might be expected from radiation pressure provided a suitable dust could be found was most encouraging.

At the outset it was apparent that it would be very difficult to manufacture a powder the grains of which would be sufficiently small, light, and uniform for the purpose; so the spores of a great variety of degraded vegetable forms were examined. Finally a puff ball of the genus *Lycoperdon* was discovered, the spores of which averaged two microns in diameter, and were as nearly spherical and uniform in size as a pile of apples from the same tree. These spores were light, cellular structures filled mainly with oil. They were calcined by heating to redness and all the vaporizable material driven off, leaving only sponge-like charcoal spheres behind. The density of a mass of these spheres (individuals could obviously not be dealt with) was measured and found to be about one-tenth that of water. Making liberal allowances for the spaces between spheres in the pile, the density of a single sphere could not exceed 0.15.[1]

These spores, together with a quantity of emery sand, were placed in a glass tube the form of which was suggested by the hour-glass. Smaller tubes led off from either end. One of these was fused to a good mercury pump of the Geissler type, the other bent down and joined to a small flask containing mercury.

All of the tubes were wrapped with wire gauze and heated to a temperature just below the softening point of glass and the

[1] According to Schwarzschild's formula, the ratio of radiation pressure to solar gravitation for spheres of the size and density of these spores would be about 6 to 1.

pump was worked many hours. When the pump showed no further signs of gas, the mercury in the flask was boiled and mercury vapor driven through the tubes to carry off any permanent gases which the pump alone could not reach. After this had continued for an hour or more the tube system was sealed off from the pump and the mercury flask was surrounded by solid carbon dioxide and ether, and the hour-glass still heated. In this way all of the mercury vapor which could be condensed at a temperature of −80° C. was drawn out of the tubes. After nearly an hour the mercury flask with its frozen contents was sealed off from the hour-glass.

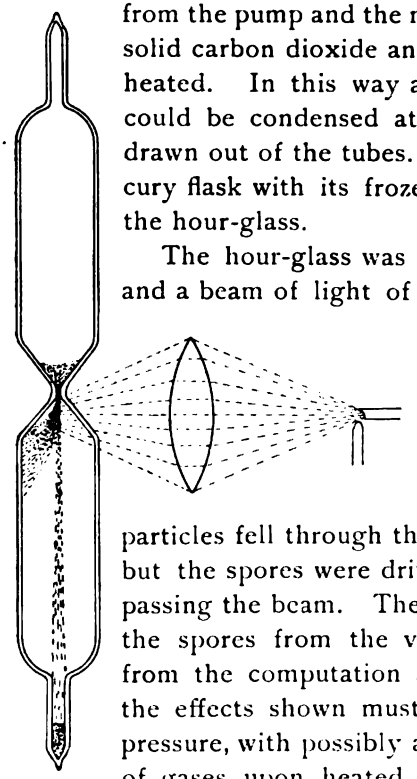


FIG. 1.

The hour-glass was then held in a vertical position and a beam of light of approximately known intensity was directed horizontally on the lower half of the tube just below the neck, Fig. 1. By tapping the tube a fine stream of sand and charcoal puff-ball spores descended. The sand particles fell through the beam, showing no deflection, but the spores were driven from the stream sidewise in passing the beam. The observed angle of deflection of the spores from the vertical was roughly that given from the computation and the observers believed that the effects shown must be due almost entirely to light pressure, with possibly a slight gas action. The action of gases upon heated bodies of this size had, so far as we know, never been studied, but one of the writers[1] had studied the gas action on larger bodies down to a pressure of permanent gases of 0.0005 mm of mercury, as shown by a McCleod gauge, and had observed that for this pressure the gas action had begun to fall off sharply. The pressure of the permanent gases in the hour-glass must have been well below

[1] A result gained in a series of unpublished experiments on gas forces by W. v. ULJANIN and E. F. NICHOLS. See also W. CROOKES, *Phil. Trans.*, p. 300, 1878.

this value and it was thought that nearly all pressure due to vapor had been frozen out.

Later, a review of the preliminary computation was made and an error discovered which had the effect of bringing out the computed light pressure on bodies of this size and density far too large. It was plain, therefore, that the force of deflection due to gas action, probably of the character of rocket action, was at least ten times as large as the effect attributable to radiation pressure. Radiation pressure alone would produce a measurable effect under the conditions of observation, but would have been far less pronounced than the effect obtained.

The experiment had unfortunately to be tried under circumstances much more unfavorable for a pronounced effect of radiation pressure than exist in comets; for the deflection produced by repulsion must be measured in terms of terrestrial gravitation, which is over 1600 times as great as solar gravitation at the distance of the Earth. To approach cometary conditions therefore it would have been necessary to use a light beam 1600 times as intense as sunlight at the Earth. In the experiment, beams from twenty to forty times as intense as sunlight were used.

Because of the meagerness of present knowledge concerning the actual conditions in comets' tails, it is impossible to say how closely the foregoing experiment fulfilled the purpose for which it was tried. It would be difficult to prove from present astronomical data that the hydrocarbon vapors known to exist in comets' tails exert no radiometric repulsion upon the small reflecting particles present. Still more difficult would it be to show that nothing which corresponds to what has been called rocket action occurs. This latter repulsion does not require the presence of any generally diffused atmosphere whatever, but simply that the particles send off gases toward the Sun under the action of the Sun's heat. Thus in passing from the era where no adequate physical causes which would meet the required conditions could be assigned for the repulsion seen in comets, we are now likely to be embarrassed in discriminating between several contributing influences.

The writers hope to repeat the comet's tail experiment using smaller spores, if they can be found, and a tube of the new silica glass which will stand stronger heating during the pumping, and thus make it possible to reach higher vacua.

THE WILDER PHYSICAL LABORATORY,
Dartmouth College, Hanover, N. H., April 1903.

PHOTOGRAPHIC REVERSALS IN SPECTRUM PHOTOGRAPHS.

By R. W. Wood.

The importance of distinguishing between photographic and true reversals of lines in spectrograms has made it seem worth while to investigate with some care the conditions under which reversals due entirely to photographic action can occur. Professor Trowbridge has advanced the theory of selective reversibility of the silver salts in the sensitive film for certain wave-lengths, and it was in part to determine whether the tendency of a line to reverse was a function of the wave-length that the present investigation was undertaken.

As I showed several years ago, the Clayden effect, or the type of reversal giving rise to the phenomenon of dark lightning, results from the action of a light-shock on the plate before its exposure to diffuse light. The effect of this light-shock, which must be of very brief duration, is to decrease the sensibility of the plate, resulting in a less energetic action during the subsequent illumination. I made no attempt at the time to determine the maximum duration of the light-shock which would still give the Clayden reversal, but expressed the opinion that it could not exceed 1-10000 of a second. This opinion was based on a single experiment, and I have since found that by a suitable adjustment of the conditions the duration may be as great as 1-1000 of a second, though only a very slight trace of reversal occurs under these conditions.

It appears to me now that there are at least four different types of photographic reversal, or perhaps five, if we allow the chemical treatment of the plate between two exposures. As any one of these four types is liable to occur in any photographic work, when the proper conditions are fulfilled, it may be well to enumerate them at the beginning.

First type.—The ordinary overexposure reversal, which occurs

when the plate is given three or four hundred times its normal exposure and then developed in the usual way.

Second type.—The reversals produced by developing the plate in full lamplight, the plate having been more or less overexposed to begin with. This type has been extensively studied by Nipher, the results of his experiments being given in the *Proceedings of the St. Louis Academy of Science.*

Third type.—This type must occur frequently, though I never remember to have seen it described. It happens when a normally or underexposed plate is developed and then exposed to light for a minute or two before the hypo bath. The fogging, which is usually of a reddish-brown color, does not occur on the portions of the plate where there is a developed image, and even if this image is very feeble it remains clean and almost transparent. I first noticed this effect in some photographs of spectra which showed strong reversals along the edges where the illumination must have been very feeble, and was unable to explain it. Further experimenting showed that it had resulted from turning up the light before the plate had been thoroughly fixed. Doubtless this effect has been described time and again in the photographic journals, but it was new to me, and may be to some others. Reversals of this type appear in the second figure of Plate XII illustrating my paper on "Screens Transparent Only to Ultra-Violet Light" (ASTROPHYSICAL JOURNAL, **17**, 136, March 1903).

Fourth type.—The Clayden effect, which is the type chiefly to be dealt with in the present paper. This occurs when an exposure of about 1-1000 of a second or less is given, and the plate subsequently fogged by exposure to diffuse light before development. If images of electric sparks are thrown on a plate and the plate then exposed to the light of a candle for a few seconds, the spark images will develop reversed, which is not the case if the exposure to candle light precedes the impression of the spark images.

Fifth type.—I have found that the condition produced in the sensitive film by light-shock can be imitated by treating the plate, after exposing portions of it to the action of a feeble light for a few seconds, with an oxidizing bath of bichromate of potash

and nitric acid. If the plate is dried, and then fogged by candle light and developed, the previously exposed portions will come out reversed.

This effect is shown in Plate XXII, Fig. 4. A series of spark images was impressed on the plate, which was then covered with a piece of black paper in which a narrow slit had been cut. A number of images of this slit were then impressed on the plate by exposure to the light of a candle. If the plate in this condition were then fogged by candle light and developed, the spark images would come out reversed and the slit images not reversed. Before fogging it, one end (the upper in the print) was immersed for a few minutes in a very dilute solution of bichromate of potash, slightly acidified with nitric acid. It was then dried, exposed to candle light, and developed. In the lower portion of the print we find the sparks black and the slit images white; in the upper portion both sets of images come out dark, the reversal of the sparks being much stronger than on the untreated portion of the plate.

In the present paper I propose to discuss the Clayden effect, not only in connection with the selective reversibility hypothesis adopted by Professor Trowbridge to explain his spectrum photographs, but also in relation to the time factor, and radiations other than light, such as the Becquerel and Roentgen rays, which are quite different in their action from light.

I shall, in dealing with the subject, speak of the initial exposure of brief duration as the light-shock. The subsequent illumination which causes the reversal of the impression of the shock I shall call the fogging exposure.

The first subject investigated was the relation of the phenomenon to the wave-length of the light. The light-shock in this case was administered by exposing the plate to the spectrum of one or more sparks between cadmium electrodes by means of a small quartz spectrograph. Even with a small diaphragm the illumination by a single spark yielded a developable image of the spectrum down to the extreme ultra-violet. Six spectra were impressed on the same plate with different sized diaphragms: the plate was then exposed to the light of a candle

for a few seconds and developed, the result being reproduced in Plate XXII, Fig. 1. It will be seen that the lines and the continuous background come out positive in the two upper spectra, showing that if the shock is too intense, no reversal takes place —a circumstance in which the Clayden effect differs essentially from ordinary reversal due to overexposure. Professor Trowbridge says that his reversals occur where bright lines fall on a continuous background, and considers the reversing action proportional to the product of the two effects. If his reversals are of this nature (Clayden effect), this cannot be the case, for by making one factor (the light-shock) large, no trace of reversal appears. As I shall show presently, this statement requires some modification, for as we increase the intensity of the shock, we can by increasing the fogging exposure still get reversal. As I said in my previous note, it appears to me probable that in the case of photographs of spectra of single sparks in narrow capillaries the shock was the almost instantaneous exposure to a bright-line spectrum of exceeding brief duration, followed by an exposure to a superimposed continuous spectrum of longer duration, which may have been due to incandescence of the inner wall of the capillary tube, or to phosphorescence of the gas. By employing a tube with a bore of about 0.25 mm I have obtained reversed lines in the blue with single discharges of a medium-sized induction coil and condenser.

The fact that the faint continuous spectrum of the spark is uniformly reversed shows that there is no selective reversibility so far as the initial light-shock is concerned.

I next endeavored to determine whether the wave-length of the fogging light had anything to do with the matter, in which case we should expect reversals in some parts of the spectrum and not in others, in the particular case where the fogging illumination was spread out into a spectrum, as in Professor Trowbridge's photographs.

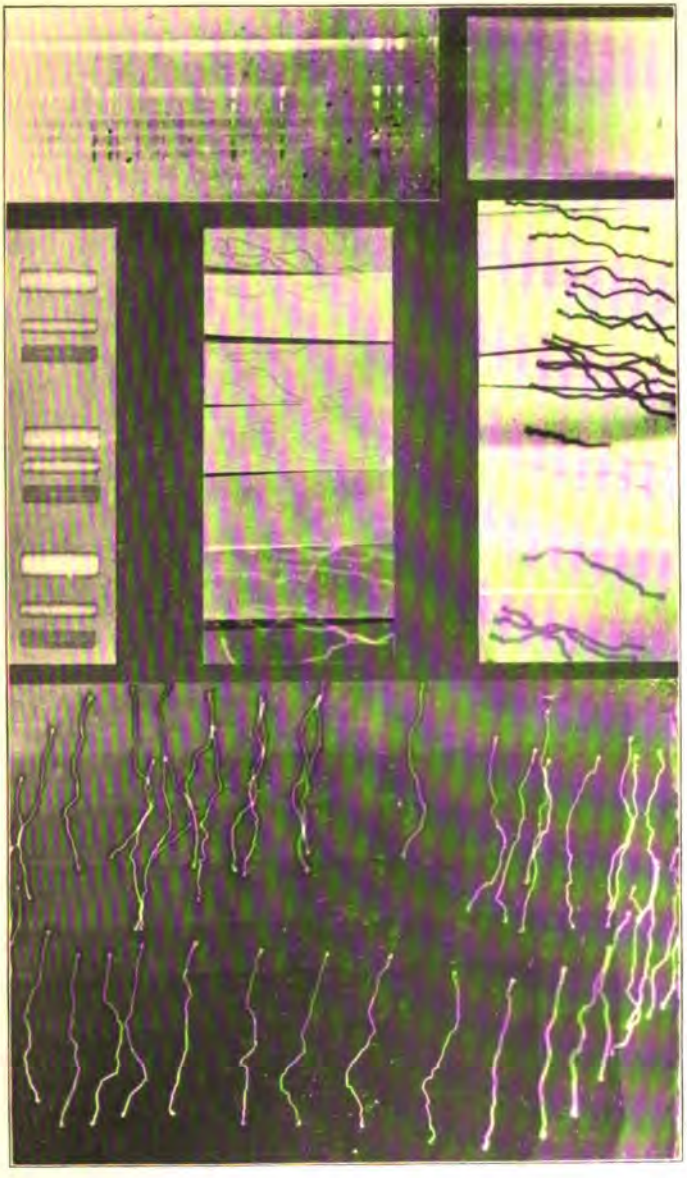
Having already found that fogging the plate with X rays never gave reversals of spark images, it occurred to me that possibly ultra-violet light might act in a similar manner. Having impressed a number of spark images on the plate, it was illumi-

PLATE XXII.

3

1

6



2

4

5

PHOTOGRAPHIC REVERSALS.

nated with light of wave-length in the neighborhood of $\lambda = 2300$, from a discharge between cadmium electrodes, a screen provided with a slit being placed in the focal plane of the quartz spectrograph, with the plate a short distance behind it. The spark images were not reversed, and it appeared at first sight as if ultra-violet light of this wave-length acted like the X rays. I was, however, not willing to accept this conclusion without further study, since the fogging illumination in this case consists in reality of a number of feeble light-shocks; that is, it is of much briefer duration than in the case of candle light. In the first experiment the fogging illumination by ultra-violet light was produced by the passage of perhaps a score of sparks before the slit of the spectrograph. To get a feebler illumination of longer duration I moved the next plate to a distance of about two meters from the screen and let the coil run for about a minute. The room was absolutely dark, the spark terminals and the front of the spectrograph being covered with heavy black cloth, so that the only light that reached the plate was of the wave-length above mentioned. On this plate the spark images which had been previously impressed were strongly reversed, showing that the time factor comes into play in the case of the fogging light as well as in that of the light-shock, and that ultra-violet light is as efficient as any other, if it is not of too brief duration. This appears to dispose of the idea of selective reversibility, at least so far as the Clayden effect is concerned.

Investigation of the time factor.—To determine the maximum duration of time which the light-shock may have and still reverse, the following method was used. A disk of cardboard 50 cm in diameter was mounted on the shaft of an electric motor, the speed of which could be determined by the tracing of a tuning-fork on a smoked metal plate mounted on the same shaft. Near the rim of the disk a number of narrow slits were cut, varying in width from 1 mm to 5 mm. An arc light was focused on the rim by means of a large condensing lens, the image of the crater being about half a millimeter in diameter. By driving the disk at a high rate of speed intermittent flashes of very brief duration were obtained as the slits passed across the arc's image. A short

distance behind the disk a rectangular metal tube, provided with a slit 1.5mm wide, was mounted immediately opposite the point where the image of the arc fell on the disk. Down this tube the plate was dropped, receiving in its passage before the slit light flashes of varying duration. The plate was subsequently exposed to candle light and developed. The images of the slit in the case of the briefest flashes were perfectly sharp; in other cases they were broadened, owing to the rapid motion of the plate. This made the interpretation of some of the records difficult, and it was found better, when working with flashes longer than 1-2000 of a second in duration, to lower the plate down the tube with a thread. A print from one of these plates is shown in Fig. 2. In this case there were two 1mm slits on the rim of the disk not very far apart, then further around a somewhat wider slit, and after this a still wider one. It will be seen that the slit images formed by the two very short flashes are completely reversed, while the others are reversed on their edges only.

If the plate moves during the exposure, as was the case in this photograph, it is obvious that the edges of the slit image will receive less exposure than the center, which accounts for the partial reversal. As the result of exposing about two dozen plates, it was found that the duration of the shock could be as long as 1-1000 of a second, and still yield reversals. It was only by carefully regulating the intensity of the fogging light and the duration of the development that these reversals could be obtained. When the duration is less than 1-2000 sec. reversals could be obtained without difficulty. Flashes varying in duration from 1-15000 sec. to 1-500 sec. were studied, and it was found that as the duration of the shock was increased the reversals became weaker, the images finally failing to appear at all on the plate, notwithstanding the longer duration of the flash. On still further increasing the duration the images came out not reversed. There may be some connection between the condition in which the light-shocks fail to develop at all, and the zero condition of the plate described by Nipher.

The experiment in which the fogging of the plate was effected

by exposure to ultra-violet light furnished by a quartz spectrograph shows that the time factor plays a rôle in the case of the fogging light as well as in the light-shock.

It was found that if the fogging light was rather intense, but of short duration, the image of the light-shock did not reverse; if the light was less intense, but of a little longer duration, no trace of the shock appeared; while if the light was still less intense, and of somewhat longer duration, the image came out reversed. With a suitable ratio of intensities and durations of time, it is possible to superpose two impressions on a photographic plate, only one of which appears on development.

This effect is shown in Fig. 3. A series of spark images of equal intensity was impressed on the plate, which was then fogged in sections, the lower strip being exposed to the light of one spark at a distance of a meter, the next to the light of four sparks at a distance of two meters, the next to nine sparks at three meters, and so on. The total amount of fogging light was thus approximately the same in each case, though it was found that considerably greater action was produced by a large number of sparks at a considerable distance than by a single spark close to the plate. It will be noticed that on the third strip from the bottom there is scarcely a trace of the spark images. The ratio of the times of duration of the shock and the fogging light was in this case about 1 : 9. On the two strips below this one the sparks appear not reversed, while on all the strips above reversal has taken place.

A more careful quantitative investigation of these effects is much to be desired, with apparatus of such design that the duration and intensity of both the light-shock and fogging light can be accurately controlled.

I am of the opinion that the result of such an investigation would be the establishment of the fact that with very brief and intense light-shocks, comparatively intense fogging light of short duration will yield reversals, while in the case of shocks of say 1-1000 second duration, the fogging light must be feeble and of long duration in order that reversals may be obtained. My plates appear to indicate this qualitatively, but quantitative

data could doubtless be obtained with suitable apparatus. As I shall show later, it is possible to administer the shock in such a manner that it comes out reversed even when the fogging light is the flash of a single spark.

I have tried to obtain some idea of the action of the light-shock, by attempting to transform its effect on the plate into an effect similar to that produced by ordinary exposure, by means of the action of various chemical agents. These experiments were all failures, but the interesting fact was ascertained that an ordinary exposure appeared to be transformed into a shock exposure by the action of a dilute bath of bichromate of potash slightly acid with HNO_3. This effect is shown in Fig. 4. A series of spark images was impressed on the plate, and then a series of images obtained by illuminating the plate with the light of a candle shining through a slit in a piece of black paper. One-half of the plate was then dipped into the bath, washed and dried, exposed to the light of a candle, and developed. A print from this plate is reproduced in Fig. 4. On the upper portion, which was treated with bichromate, both the spark images and slit images appear reversed; on the lower the latter are not reversed. This experiment merely shows that a plate which has been exposed to light in certain places and then treated with the bichromate solution is less sensitive to the action of subsequent illumination on the spots which have previously received light. The condition may appear at first sight to be similar to that produced by a light-shock, but there is in reality no connection between the two, for while light-shocks not followed by fogging can be developed as not reversed images, the "bichromatized images" do not develop at all, unless the plate is fogged before development.

The nearest approach which I have been able to make to the transformation of the effect of a light-shock into that due to ordinary exposure, is by the action of the X rays. It was found that spark images could not be reversed under any circumstances, if the plate was fogged by these rays instead of candle light. To prove that the case was not analogous to the one in which ultra-violet light failed to give reversals owing to the

comparatively brief duration of the illumination, long exposures were made with the X-ray tube at a considerable distance. Not only were reversals never obtained, but it was found that, after a brief exposure to the rays, fogging the plate by lamplight failed to reverse the spark images. This seemed very remarkable, for it was subsequently ascertained that X-ray images could be reversed even when produced by long exposure to feeble radiation, by subsequent exposure of the plate to lamplight. This effect is shown in Plate XXII, Fig. 5.

The spark images were impressed first. The plate was then exposed in vertical strips to the action of the X rays for varying lengths of time, the left-hand strip receiving the longest exposure, while the right-hand strip was not exposed at all. Following this, came an exposure in horizontal strips to lamplight, the lower strip having the longest exposure, and the upper none at all. It will be seen that there is no trace of reversal in the upper left-hand corner, where the fogging is due almost wholly to X rays, while reversed edges appear on all of the sparks in the lower right-hand corner, where the fogging was due to light; moreover, in the lower left-hand corner, where the X-ray fog preceded the light fog, the images are not reversed.

On investigating the matter further I found that shocks administered by single powerful flashes of X rays were reversed by subsequent exposure to lamplight. In this case, however, the time element appears to be without much influence, for images formed by long exposure to very feeble X radiation reverse in the same manner. This seems very remarkable, when we consider the fact that exposure to these rays changes the condition produced by light-shock in some manner, so that it is impossible to reverse it by further fogging.

The reversal by X rays is illustrated in Fig. 6. A plate was wrapped up in black paper and exposed to the radiation for several minutes, a vertical iron rod shielding the center strip of the plate. The plate was then exposed to lamplight for different lengths of time in strips perpendicular to the shadow of the iron rod. On development it was found that on the end of the plate which had received the shorter exposures to light, the central

strip came out lighter than the background, while on the opposite end of the plate it was darker. At a certain point near the center of the plate all trace of the shadow of the rod had disappeared, showing that exposure to X rays for some time and then to light for a certain time produces an image no blacker than the light alone would have produced.

I next ascertained that if the plate be exposed simultaneously to light and X rays, the latter inhibit the action of the former. A candle and an X-ray tube were set up at some little distance apart in front of a plate, the latter being much nearer the plate, however, owing to its less energetic action. An iron rod mounted in front of the plate cast two shadows upon the sensitive film, one projected by light, the other by X rays. After exposure the plate was developed, with the curious result that one shadow was darker than the background, the other lighter, showing that the light was more energetic in its action on the area screened from the X radiation. It may be worthy of mention that both this result and the preceding one were predicted before the actual experiments were tried. The prediction was the result of an attempt to apply Bose's strain theory of photographic action[1] to the phenomena in question. This theory seemed rather promising at first, especially as it enabled me to predict new phenomena, but it failed to account for so many things that I was finally forced to abandon it.

The action of other stimuli was next investigated. It has long been known that pressure marks on the film can be developed. I found that if the plate was fogged by lamplight before developing, the pressure marks came out reversed. It then occurred to me to try the effect of light-shocks on pressure marks, and I found to my surprise that the flash of a single spark was as effective in reversing the pressure mark as the exposure to the lamp. The pressure marks can also be reversed by exposure to X rays.

As a result of numerous other experiments, I finally found that if we arrange the stimuli in the following order, pressure marks, X rays, light-shock, and lamplight, an impression of any

[1] J. C. Bose, *Proc. Roy. Soc.*, **70**, 185, 1902.

one of them can be reversed by subsequent exposure to any other following it in the list, but under no circumstances by any one preceding it. For example, pressure marks can be reversed by any one of the other three stimuli, while X-ray images are reversed only by light-shock and lamplight.

Experiments with Becquerel rays have given rather uncertain results. Pressure marks can be reversed by them, and they in turn can be reversed by lamplight, but these were the only two cases in which reversals were obtained, which makes it difficult to fit the rays into the series and still have the rule hold.

These experiments show that the effects of the different kinds of stimuli on the sensitive film are quite different. Much more experimental work will have to be done before any definite notion can be obtained as to the nature of the changes produced by the action of radiation of any sort, and it is hoped that the experiments described in this paper may prove suggestive to others. Doubtless an exhaustive study of the action of various chemical agents on the plate between the two exposures would throw much light on the cause of the reversals.

If I interpret the strain theory correctly, the application to these phenomena would be to assume that the light-shock produces a negative strain, while lamplight produces a positive strain, either of which yields an image on development. The reversal in cases where the lamplight follows the impression of the light-shock could be explained by assuming that the negative strain has to be undone before the positive strain can begin; consequently these parts of the plate lag behind the parts which have not received the light-shock. We should then have to assume that the positive strain once started can be continued by a stimulus which, acting first, would have produced a negative strain, in order to account for the fact that exposures to lamplight are not reversed by light-shocks. Moreover, it is difficult to explain on the strain theory that two different stimuli acting in succession may produce only the same effect as one of them acting alone. It appears to me that the strain theory would lead us to suppose that the negative strain produced by the first stimulus might be exactly neutralized by a stimulus which pro-

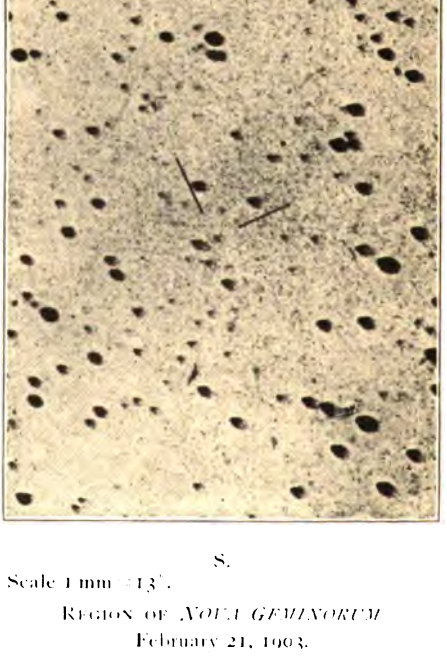
duces a positive strain, the plate returning to its original condition, *i. e.*, not darkening on development. This is never the case.

In cases where reversed lines appear in the spectrum which are suspected of being photographic in origin, *i. e.*, not true absorption lines, the following precautions should be taken. Repeat the exposure a number of times, using successively smaller diaphragms in front of the prism. If the reversal is ordinary solarization due to overexposure, it should disappear when the intensity of the light is sufficiently reduced. It seems to me that in such cases the reversed line should be bordered by bright edges, which does not seem to be the case in the photographs published by Professor Trowbridge. If the Clayden effect is suspected, the source of light should be examined with a revolving mirror, to determine whether a dual illumination is present. The speed of the mirror should not be too great, otherwise the phosphorescence (if it exists) which follows the very brief bright line flash, may be spread out to such an extent that no trace of it appears. This may account for the failure to obtain evidence of a dual illumination in the case of the heavy discharges in quartz tubes.

JOHNS HOPKINS UNIVERSITY,
Baltimore, Md.,
May 1903.

PLATE XXIII.

N.



S.

Scale 1 mm = 13″.

REGION OF *NOVA GEMINORUM*

February 21, 1903.

NOVA GEMINORUM.

AN EARLY PHOTOGRAPH AND PHOTOMETRIC MAGNITUDES.

By J. A. PARKHURST.

THE negative of the *Nova* region taken 1903, February 21, $8^h\ 22^m$ to $8^h\ 42^m$, C. S. T., was mentioned in *Bulletin* No. 19 of the Yerkes Observatory,[1] but the cut made to illustrate it was not successful and was therefore omitted. An untouched copy is reproduced here, Plate XXIII, showing a star of magnitude 14.5 apparently coincident with the present position of the *Nova*. The photometric magnitudes of four faint stars near the *Nova* are as follows:

Star No.	Mag.	Measures
5.....	13.11	3 nights
6.....	12.99	2 "
3.....	14.89	1 "
1.....	13.34	1 "

The numbers given for identification are those on Professor Barnard's drawing of the region on page 304 of the *Bulletin* No. 19. It will be seen that the *Nova* is a little brighter than star No. 3; its magnitude on February 21 is therefore estimated at 14.5. It will be noticed that the photograph agrees with Professor Barnard's micrometer measures in placing the *Nova* farther north than star No. 1, whereas the object taken for the *Nova* on the photograph by Mr. Dugan at Heidelberg February 16, and reproduced in *Popular Astronomy* for May 1903, page 260, is a little farther south than star No. 1, possibly due to the retouching of Mr. Dugan's print.

Following are the photometric magnitudes of the *Nova*, preceded by a list of standard stars and the comparison stars so far used. The standards, *l*, *g*, and *h*, were measured both at Har-

[1] ASTROPHYSICAL JOURNAL, 17, 300, May 1903.

vard (*H. C. O. Annals*, Vol. XLV) and at Potsdam, the difference between the means, 0.20 magnitude, agreeing closely with the systematic difference between the two catalogues; these stars seem therefore well adapted for standards. For the comparison stars, *b*, *c*, *d*, and *e*, the magnitudes given in the columns headed H. and P. are the means of three nights' comparison with the standards given above, H. being on the Harvard basis, and P. the Potsdam.

STANDARDS.

Harvard and Potsdam.

Star	B. D. No.	B. D. Mag.	H.C.O.	P.D.M.
l.....	+29 1307	7.2	7.36	7.22
g.....	30 1318	7.0	7.13	7.50
h.....	31 1303	7.0	7.23	7.60
	Mean		7.24	7.44

COMPARISON STARS.

Star	B. D. No.	B. D. Mag.	H.	P.
b.....	29 1342	8.2	7.79	7.99
c.....	29 1320	7.8	7.46	7.66
d.....	30 1306	8.6	9.09	9.29
e.....	30 1302	9.2	9.32	9.52

MEASURED MAGNITUDES OF THE *NOVA*, BASED ON THE HARVARD SCALE.

	G. M. T.	Mag.	Seeing		G. M. T.	Mag.	Seeing
1903, March	27.715	8.51	fair	1903, April	7.574	9.19	clouds, moon
	28.630	8.71			8.593	9.19	moon
	29.603	8.85			16.661	9.31	good
	30.673	8.76	fair		26.671	9.63	fine
	31.588	8.80	good		27.632	9.41	good
April	3.564	8.91	clouds		30.591	9.74	good
	4.582	8.96	moon	May	3.621	9.56	moon
	6.574	9.19	moon		6.634	9.50	
					7.593	9.84	moon, good

The resulting magnitudes of the *Nova* should be compared with Professor Barnard's (given elsewhere in this number), which are based on photometric measures of comparison stars, on the Harvard system.

Fluctuations, similar to those of *Nova Persei*, are clearly shown, as indicated in the measures given in *Bulletin* No. 19, and noted in Harvard *Circulars* Nos. 127 and 128.

YERKES OBSERVATORY,
May 9, 1903.

OBSERVATIONS OF *NOVA GEMINORUM*.

By E. E. Barnard.

My estimates of the magnitudes of the *Nova* were made by comparison of its light with neighboring stars, the magnitudes of which have been determined by Mr. Parkhurst with the photometer.

The instruments used were the finders of the forty-inch and twelve-inch refractors, and on the last few dates (May 7, 8, 9, and 10) the twelve-inch, as the star could not be seen in the finder because of moonlight and haze.

The observations of April 27 were made through a very hazy sky, and the estimations were more or less difficult.

At first the *Nova* was of a strong red color, but it has since become colorless. On March 29 there was not much color seen in the twelve-inch, though it was a little redder than *B. D.* $+29^{\circ}$ 1342. On April 3, with the same instrument, it was noted as "a little reddish;" April 22, also with the twelve-inch, it was colorless and there seemed to be a slight difference of focus. On April 27 and 28 no trace of color was seen with the forty-inch.

In the first observations the *Nova* showed no difference in focus from that of an ordinary star (see Astrophysical Journal, **17**, 302, 1903). In the later observations there seems to be a sensible change in the focus; this, however, has not been properly verified.

On April 27 careful observations for focus were made with the forty-inch, with the following results:

Focus for *Nova*, scale reading	-	2.36 in. (5 obs.).
Focus for star, scale reading	-	2.28 in. (5 obs.).
Difference, *Nova* − star	-	$= +0.08$ in. ($= 2.0$ mm).

The seeing was poor and the settings discordant for both objects. I think, however, that this indicates a real change of focus, for when either object was in the best focus the other was somewhat ill-defined. Cloudy weather has since prevented verification of this with the forty-inch.

On April 6 the crimson image seen on March 30 (*loc. cit.*, p. 303) was still present, though not so strong and definite. On April 27 it had entirely disappeared, the out-of-focus image of the *Nova* resembling that of an ordinary star. Cloudy weather had prevented any observation by me with the large telescope between April 7 and 27. I am therefore unable to say just when this change took place; it would perhaps not be far from the middle of April.

OBSERVED MAGNITUDES OF *NOVA GEMINORUM*.

	Central Standard Time.	Mag.
1903, March 27	12h 30m	8.00
29	9 0	8.81
30	9 0	8.82
31	7 20	8.84
April 3	8 0	8.96
4	7 40	9.04
6	9 0	9.15
7	10 20	9.22
9	8 0	9.12
16	7 40	9.12
17	9 0	9.20
22	7 50	9.20
24	8 15	9.92
26	8 30	9.81
27	8 30	9.90
28	8 30	10.18
May 3	8 20	9.80
6	8 10	9.80
7	9 10	9.87
8	8 30	9.77
10	8 10	9.77
18	8 30	10.02
19	8 30	10.07

At the observation of March 27 the star was very low and the estimated magnitude is unreliable.

YERKES OBSERVATORY,
May 11, 1903.

Minor Contributions and Notes.

PARALLAX OF THE BINARY SYSTEM δ *EQUULEI*.[1]

THE object of this communication is to present a value of the parallax of δ *Equulei* obtained by using as data the elements of the orbit derived from micrometrical measures and the relative velocity of the components in the line of sight furnished by spectroscopic observations. While the result given is merely provisional, one to be improved when better elements of the orbit are available, as they no doubt will be after two or three years more, yet it is thought that it has already a measure of certainty which places it in the list with the more accurately determined stellar parallaxes. Moreover, the manner of its derivation differs from previous determinations. The use of comparison stars and the uncertainties of their assumed distances are eliminated. In theory the method leads to an absolute value of the parallax, and not to a relative one, as is necessarily the case when comparison stars are employed.

The mathematical relations connecting the elements of the orbit of a binary, the relative velocity of its components in the line of sight, and the parallax of the system, have long been known. Applications have hitherto been wanting, for the reason that no double stars had been found which furnished all the data needed. δ *Equulei* has proven an exception. The elements of its orbit are approximately known, and in 1901, at the time of the last periastron passage, the relative velocity of the components in the line of sight was so great that the spectra of the two stars were displaced to a measurable extent.

In the *Monthly Notices* of the Royal Astronomical Society for March 1890, Professor Arthur A. Rambaut has given the following simple formula for computing the parallax of a binary from the data here considered:

$$\pi = \frac{la^2 \sqrt{1-e^2}\,\sin(\phi-\lambda)\sin i}{VPr\sqrt{1-e^2\cos^2\phi}}.$$

In this equation P denotes the periodic time, a the mean distance, e the eccentricity, i the inclination, r the radius vector, λ the angle

[1] *Lick Observatory Bulletin* No. 32.

between the line of nodes and the major axis of the true ellipse, ϕ the angle between the tangent to the orbit and the major axis, V the relative velocity of the components in the line of sight as furnished by the spectroscopic observations, l the velocity of the Earth in its orbit, and π the parallax of the system.

The elements of the orbit of δ *Equulei* are still imperfectly known. A nearly continuous record of the motion of the companion during the next three years is necessary to remove effectively the uncertainties which still exist in their values. Eight times since the discovery of this binary the companion has passed over the arc which it will describe during the next three years, and numerous observations relate to this portion of the orbit. But the measures are more or less discordant, and on this account additional ones are required to fix with certainty the direction and magnitude of the greatest apparent distance. When these measures are obtained and the elements of the orbit revised, we shall have the data which will give an accurate value of the parallax of the system. A consideration of the material at present available appears to show that the changes which will probably take place in the elements due to the revision of the orbit will not greatly alter the value of the parallax which may now be derived, and it is on this account that a provisional value is here presented.

During the past three years this binary has been followed closely at the Lick Observatory and much has been done to advance our knowledge of its orbit. It has been shown that the periodic time is about 5.7 years; that the last epoch of periastron passage was about 1901.5; that the apparent distance at periastron is about 0".15; that the inclination is greater than given by the trial elements which I printed in the *Publications of the Astronomical Society of the Pacific* for December 1900, and afterwards in Vol. V of the *Publications of the Lick Observatory*, being perhaps as much as 82° or 83°; and that the mean distance is probably a little larger than given by those elements. Further, it is known that the line of nodes and the projection of the major axis of the orbit are nearly coincident.

If we retain the line of nodes and the angle between the line of nodes and the major axis, the true ellipse as given by my published elements, and put the mean distance equal to 0".28 and the eccentricity equal to 0.46, we get 0".151 and 0".409 as the periastron and apastron distances.

The relative velocity of the components in the line of sight at the epoch of periastron passage, as determined by the observers using the

Mills spectrograph[1] was about 20.3 miles per second. At the same time the Earth's orbital velocity was about 18.2 miles per second. At periastron ϕ becomes 90° or 270°, and with $\lambda = 179°$, we have $\phi - \lambda = 91°$. Substituting these values in Professor Rambaut's formula we get

$$\pi = 0''.071.$$

It is evidently impossible to give the probable error of this result. Some of the quantities upon which it depends are much better determined than others. It happens that those which appear to be the most uncertain may vary considerably without greatly affecting the resulting value of the parallax. For example, considerable variations may take place in the values of i and λ without greatly changing the values of $\sin i$ and $\sin(\phi - \lambda)$ which appear in the formula. Of the other quantities those which appear the least certain are the mean distance and the eccentricity. From a consideration of all the past measures it appears highly improbable that the apastron distance can be as much as 0".05 greater or less than the value used above. If we vary the apastron distance and eccentricity together so as to keep the already well-determined periastron distance constant, and further retain the other elements of the computation as used above, we shall find that the apastron distance and parallax increase together, but the former at about four times the rate of the latter. From this consideration it appears improbable that the parallax will be altered so much as 0".012 owing to changes which take place in the values of a and e due to the revision of the elements of the orbit.

The mass of the system corresponding to the parallax given above, and the mean distance and the periodic time used in its derivation, is 1.89, the mass of the Sun being taken as unity. The components of this pair are slightly unequal in brightness, and perhaps also in mass. One may be as massive as the Sun, but it cannot much exceed it.

The mean distance at which these stars perform their revolutions is about four times that of the Earth from the Sun. But owing to the eccentricity of the orbit the range in distance is enormous. At periastron the stars are separated by a space only a little greater than two astronomical units, while at apastron it is five of these units.

It would form an interesting analytical problem to investigate the effect of tidal action upon the relative orbit of this system. The data are pretty well ascertained. Broadly speaking the stars have spectra

[1] *Bulletin of the Lick Observatory* No. 4, and *Publications of the Lick Observatory*, 5 211.

of the solar type, and being comparable with the Sun in mass, it would seem not unreasonable to assume that their densities are approximately the same as that of the Sun.

WILLIAM J. HUSSEY.

FEBRUARY 23, 1903.

TWO STARS WITH VARIABLE RADIAL VELOCITIES.

IN the course of our most recent work with the Bruce Spectrograph on stars having spectra of the *Orion* type the two stars u *Herculis* and 57 *Cygni* have been found to show large variations in their radial velocities.

The data for these stars are as follows:

u *HERCULIS* ($\alpha = 17^h\ 14^m$; $\delta = +33^\circ\ 12'$; Mag. = 4.9).

Plate	Date	Taken by	Velocity	No. of Lines	Measured by
C23	1903, February 19	A.	− 65	5	A.
C34	March 18	F.	− 44	3	A.
DB6	May 7	A.	+101	4	A.
DB17	May 9	A.	+ 98	3	A.

This star is of especial interest on account of being a photometric variable of irregular period. The lines of its spectrum are broad and diffuse, and the measures are consequently uncertain to the extent of several kilometers.

57 *CYGNI* ($\alpha = 20^h\ 50^m$; $\delta = +44^\circ\ 0'$; Mag. = 4.6).

Plate	Date	Taken by	Velocity	No. of Lines	Measured by
DB22	1903, May 16	F.	−114	4	F.
DB24	May 17	A.	− 23	3	F.

On plate 22 the magnesium line at $\lambda 4481.4$ appears to have a second component, which is displaced toward the red by an amount corresponding to a velocity of about + 56 km. No second component can be seen on plate 24. In this connection it may be stated that on different plates we often find marked differences in the distinctness of the lines in spectra of stars varying rapidly in velocity, doubtless in part due to the change of velocity during the exposure.

In the list of plates given above those belonging to series C were taken with the short-focus camera, to which reference has already been made in our communication in the April number of this JOURNAL.

The plates of series DB were taken with a dispersion of one prism and the regular camera of twenty-four inches focal length. For this apparatus two of the prisms of the Bruce spectrograph are removed, and auxiliary castings added to support the camera tube at the necessary angle in reference to the first prism. Especial attention was given to the stability of the support, and the results obtained are entirely satisfactory in this respect. The scale of the plates is, of course, very closely one-third that given by the three prisms. In the case, however, of stars which have wide and diffuse lines in their spectra, the narrowing of these lines, and the consequent increase in the accuracy of the measures upon them, largely counteracts the effect of the smaller scale, so that the final values often compare favorably with those obtained with the use of the higher dispersion. The considerable increase in the number of measurable lines, through the additional extent of spectrum which is brought into focus, also contributes materially to this result.

EDWIN B. FROST AND WALTER S. ADAMS.

YERKES OBSERVATORY,
May 19, 1903.

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THE

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THE

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THE CONSTRUCTION OF A SENSITIVE GALVANOMETER FOR SPECTRO-BOLOMETRIC PURPOSES.

By C. G. ABBOT.[1]

SINCE 1896 the galvanometers used for bolographic purposes at the Smithsonian Astrophysical Observatory have been more or less improved every year at the hands of the Observatory staff, with the encouragement and approval of the director, Secretary Langley, who has so kindly introduced this paper, which is a description of the results so far reached. For bolographic purposes reflecting galvanometers of low resistance and comparatively short time of swing are best suited. Solar-energy spectra are now being taken at the Smithsonian Observatory with a galvanometer of 1.6 ohms resistance, usually employed at 1½ seconds time of single swing of the needle; though for certain very delicate experiments the time of single swing is sometimes as high as 10 seconds. It may then be said that it is aimed to measure the least possible current with a galvanometer of 1.6 ohms resistance at 10 seconds single swing of the needle. It is well to make

[1] A paper read before Section B of the American Association for the Advancement of Science, December 30, 1902, after introductory remarks by S. P. Langley, Secretary of the Smithsonian Institution. It is here published with his permission.

here a distinction between computed sensibility and working sensitiveness, for it may happen that an instrument of less computed sensitiveness is capable of measuring smaller currents than another whose high figure of sensibility is overbalanced by unsteadiness of its needle system. This consideration leads me to suggest that possibly it might be fairer to compare galvanometers on a basis of the least current which can be read with them, or at least to give this in addition to the usual statement of computed sensibility.

In common with other laboratories, the Smithsonian Observatory has greatly profited by the introduction of Professor Boys' quartz fibers and by diminishing the size of the needle system and mirror of the galvanometer. These improvements were introduced about 1893 in the galvanometer described by Mr. Wadsworth[1] and used for bolographic purposes up to the autumn of 1896.[2]

In September 1896 the Observatory was visited by Professor Kayser, of Bonn, who expressed the belief that a galvanometer of considerably greater sensitiveness might be employed. He referred especially to the excellent instrument of Paschen, and to a galvanometer which he had seen at Baltimore, of somewhat similar design to Paschen's. Acting upon this suggestion, Mr. Langley addressed a letter of inquiry to Professor Paschen, and he was so good as to send in reply quite an account of his instrument. I also went to Baltimore and saw the galvanometer being set up by Messrs. Mendenhall and Saunders. These instruments seemed to gain in sensitiveness chiefly by diminishing the dimensions of the needle system, and as an aid to this the dimensions of the coils were also small.

Beginning with the autumn of 1896 experimental and theoretical work has gone on in the Smithsonian Observatory at frequent intervals, with the object of improvement in all the

[1] *Phil. Mag.*, 38, 553–558, 1894.

[2] As last employed with a needle system of 31 milligrams weight, constructed by the writer, this instrument of about 20 ohms resistance gave a deflection of 1 millimeter on a scale at 1 meter with a current of 6×10^{-10} ampères when the time of single swing was 4 seconds. At this time of swing the needle was ideally steady, so that deflections of 0.1 millimeter could readily be observed.

branches of galvanometer construction, including the coils, the needle system, the case, and the accessories. In what follows the principal results of this study will be stated.

THE COILS.

Best resistance.—The bolometric circuit with which the galvanometer is connected, consists, as is well known, of a balanced Wheatstone's bridge. In our practice one of the four resistances of this bridge is a fine flattened platinum wire subjected to changes of temperature through the absorption of radiation. For the sake of symmetry a second one of the four resistances is a similar strip of platinum close to the first, but protected from radiation by means of diaphragms. The other two resistances are coils of platinoid wire. The battery is connected between the junction of the two coils and the junction of the two strips, while the galvanometer is connected between the junction of one strip with one coil and the other corresponding point. It is shown in the *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. I, p. 246, that the condition of maximum galvanometer deflection requires the resistance of the two coils to be large in comparison with the strips.

Let a and $\left(a+\frac{a}{m}\right)$ be the resistances of the two bolometer strips; (m being a large number); na the resistance of each balancing coil; G the resistance of the galvanometer; C the current in the bolometer strip whose resistance is a; g the current flowing through the galvanometer; Δ the deflection of the galvanometer, and k a constant.

Then

$$g=\frac{C}{m}\frac{1}{2+\frac{G}{a}\left(\frac{1+n}{n}\right)}. \qquad (1)$$

When, as with the coils shortly to be described,

$$\Delta=kgG^{0.45}, \qquad (2)$$

we have by substitution

$$\Delta=k\frac{C}{m}\frac{G^{0.45}}{2+\frac{G}{a}\left(\frac{1+n}{n}\right)}. \qquad (3)$$

In this expression Δ is a maximum with respect to n when n is infinitely great, and with respect to G when

$$\left(\frac{1+n}{n}\right)\frac{G}{a} = 1.64 \ . \tag{4}$$

If n is large, this becomes approximately

$$\frac{G}{a} = 1.64 \ . \tag{5}$$

The two following tables indicate how much is lost by departure from these conditions of maximum deflection:

n	¼	1	2	3	4	∞
Δ	44	71	82	87	89	100

$\left(\frac{1+n}{n}\right)\frac{G}{a}$	0.0164	0.164	0.82	1.64	3.28	16.4	164
Δ	23	60	95	100	94	56	18

It appears that the conditions of maximum sensitiveness of the bolometric circuit as used here are closely approximated when the balancing coils are upward of four times the resistance of the bolometer strips, and the galvanometer resistance is not less than six-tenths, or more than four times the resistance of the bolometer strip.

At the Smithsonian Observatory best results have been obtained with bolometers of comparatively low resistance, for, unless in an air-tight case, very thin strips are much affected by air-currents, so that a perpetual tremor of the galvanometer is observed when there is moderate wind. At present a bolometer with strips of 4 ohms resistance is chiefly employed, but occasionally a wide bolometer of only 0.8 ohms resistance is substituted. Accordingly a galvanometer of from 2 to 3 ohms resistance would be most serviceable, but the actual resistance of the galvanometers now in use is 1.6 ohms.

A four-coil galvanometer of a given total resistance G, may have its separate coils of resistance $4G$, G, or $\frac{G}{4}$, according to

the manner in which they are connected; and for coils whose force at the center varies with the 0.45 power of the resistance, the relative efficiency of these three ways of producing the total resistance G is in the ratio of the numbers 87, 93, and 100. Thus it appears that the resistance of a four-coil galvanometer may be varied in three steps from 1 to 16 without greatly altering its efficiency. Again a galvanometer of a given resistance may be made by connecting in series two, four, eight, sixteen, or even more such coils. The efficiency of these several arrangements of the given resistance, so far as the force at the center of the coils is concerned, is in the ratio of the numbers 100, 68, 49, 32. There is a slight compensation for this diminution of efficiency as the coils get smaller, arising from the greater effect of the outer windings of each coil upon the magnets at the center of its neighbor, but the only considerations which warrant a multiplication of coils are some which concern the efficiency of the magnet system.

Best form of coils.—Maxwell[1] has shown that the contour of the cross-section of the coil should be of the form determined by the equation

$$r^2 = H^2 \sin\theta \ , \qquad (6)$$

where r is the length of a radius making the angle θ with the axis of the coil, and H the value of r when $\theta = 90°$. Although the efficiency of the winding is not very greatly impaired by considerable variations from the best form of coil section, it is so easy to follow it closely that this form has always been employed here. In order to leave room for the needle system, a space is left unwound at the center and front of each coil as indicated by dotted lines in the accompanying diagram (Fig. 1).

Best sizes of wire.—The reader is referred to *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. I, p. 248, for the derivation of equations required to determine the radius of a coil of the above form containing a given length of wire of a given diameter and gain of diameter by insulation, and the force such a coil exerts at its center. Maxwell has shown[2]

[1] MAXWELL, *Electricity and Magnetism*, Vol. II, paragraph 718.

[2] *Ibid.*, paragraph 719.

that the wire should increase in diameter from the center outward, and it has been customary therefore to wind the coils in several sections, with larger sizes toward the outside of the coil. Although there would be a slight gain by using a larger number, it has been the practice here to wind the coils in three sections as shown in the accompanying diagram (Fig. 1), which is drawn to scale to illustrate the best dimensions for a three-section coil of 5 ohms resistance. The following table is for the most part abridged from Table 32 of the Astrophysical Observatory *Annals*, Vol. I, and shows the diameters of wire, gain of diameter by single white silk insulation,[1] lengths of wire, external radii of sections and forces exerted at the center by the most efficient coils of various given resistances, when wound of wire of a single size or in three sections of wire of different diameters. The coils of maximum efficiency have been selected from a large number whose constants have been computed.

Total Resistance	Diameter of Wire in Sections			Gain of Diameter by Insulation	Length of Wire in Sections			External Radii of Sections			Forces at Center of Coil			Total Force at Center of Coil
	d_1	d_2	d_3	t	L_1	L_2	L_3	H_1	H_2	H_3	F_1	F_2	F_3	F
ohms	cm	cm	cm	cm	cm	cm	cm	cm	cm	cm				
0.1	0.0405	.....	.....	0.0038	79.1	.....	....	0.55	....	....	....	....	....	104
"	0.0202	0.0405	0.0511	"	6.6	26.4	41.6	0.36	0.42	0.60	54	45	42	141
"	0.0101	0.0202	0.0405	"	81	328	1318	0.32	0.60	1.30	298	321	301	920
"	0.0101	0.0255	0.0644	"	81	521	3332	0.32	0.75	2.26	298	359	290	947
25	0.0160	.....	.....	"	3092	.....	.....	1.00	....	....	....	....	....	1467
"	0.0080	0.0160	0.0321	"	256	1031	4144	0.38	0.74	1.63	671	670	605	1946
"	0.0101	0.0202	0.0405	"	407	1639	6589	0.46	0.95	2.16	754	653	564	1971
50	0.0160	.....	.....	"	6185	.....	.....	1.24	....	....	....	....	....	1927
"	0.0080	0.0160	0.0321	"	513	2062	8288	0.45	0.91	2.03	991	913	784	2688
"	0.0101	0.0202	0.0405	"	815	3278	13180	0.55	1.17	2.70	1095	898	717	2710
250	0.0101	.....	.....	"	12225	.....	.....	1.23	...	....	....	....	....	4007
"	0.0050	0.0101	0.0202	"	1017	4075	16390	0.46	0.91	1.96	1871	1798	1645	5314
"	0.0050	0.0127	0.0321	"	1017	6483	41440	0.46	1.14	3.38	1871	2017	1617	5505
500	0.0101	.....	.....	"	24450	....	.	1.53	....	....	....	....	....	5398
"	0.0050	0.0101	0.0202	"	2033	8150	32780	0.555	1.12	2.45	2731	2306	2126	7253
"	0.0050	0.0127	0.0321	"	2033	12970	82880	0.555	1.42	4.23	2731	2677	2068	7476

Taking the best coils given in the table, the total force exerted at the center is closely proportional to the 0.45 power

[1] Wire of the diameters given (which are those of standard sizes of the B & S wire gauge) can be procured with white silk insulation of the given thickness from the firm of A. F. Moore & Co., of Philadelphia.

of the total resistance, Coils composed of three sections of best sizes of wire give about 1.4 times the force at center given by a coil of the best single size of wire of the same total resistance.

Method of winding.—The method I have employed in winding such coils is as follows: A brass mandrel is prepared as shown in the diagram (Fig. 1). Where coils of large resistance are to be wound the ends of wires of the several sections are brought out at the back of the coil and soldered after its completion, but for 0.1-ohm coils it is better to solder the wires together before winding. The wire is wound in a solution of shellac, and the sections are formed correctly and held in place till the shellac dries by winding thread against the back of the section as it grows in radius. After the coil is complete it is boiled in carnauba wax to make it rigid, after which the front plate of the mandrel is warmed and removed, and then the mandrel itself is warmed while the coil is held downward in cold water till it can be drawn from the mandrel.

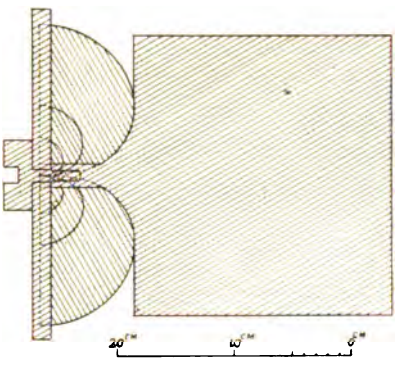


FIG. 1.

THE NEEDLE SYSTEM.

The great sensitiveness of recent galvanometers of the Thomson type in comparison with those of twenty years ago is much more due to the small dimensions of the needle system, than to improvements in the coil construction. Paschen[1] has emphasized this point, and he and others had constructed galvanometers of great sensitiveness prior to the time when the present investigations began. More recently Mendenhall and Waidner,[2] in a valuable paper on galvanometer construction, have described more thorough studies of the construction of needle systems, and point out that the useful diminution of dimensions cannot proceed as far as would appear from Paschen's paper.

[1] *Zeitschrift für Instrumentenkunde*, 13, 13, 1893.

[2] *Am. Jour. Sci.* (4), 12, 249, 1901.

In their construction of magnet systems these authors are guided by extensive experiments on magnets of different forms, but they have not given exact directions for the dimensions and arrangement of the whole needle system to give highest sensitiveness. In what follows I shall endeavor to furnish exact information of this kind.

If we suppose a single short magnet suspended horizontally at the center of a coil to be influenced in a direction parallel to the plane of the coil by a magnetic field of strength H, and to be influenced at right angles to the plane of the coil by a current field of strength F, it will take up a direction making an angle θ with the plane of the coil such that

$$\tan \theta = \frac{F}{H} . \tag{7}$$

Let T be the time of swing of the suspension (undamped), M its magnetic moment, and I its moment of inertia, and let F be small as compared with H, then

$$T^2 \propto \frac{I}{MH} . \tag{8}$$

Combining (7) and (8)

$$\tan \theta \propto \frac{FMT^2}{I} . \tag{9}$$

For a given current and time of swing therefore

$$\tan \theta \propto \frac{M}{I} . \tag{10}$$

It is usual for purposes of astaticism to employ more than one group of magnets, with poles in the several groups oppositely directed. Suppose n such groups each composed of p magnets of moment M and let ΣMp be their algebraically combined magnetic moment tending to direct the system in a given sense, when $F = 0$, while nMp is their total magnetic moment without regard to sign. Let each group hang at the center of a coil whose field strength is f, and let H, T, and I have the same significance as before.

Then

$$\tan \theta = \frac{npfM}{H\Sigma Mp} . \tag{11}$$

If, as before, $npfm$ is small as compared with $H\Sigma Mp$

$$T^2 \propto \frac{I}{H\Sigma Mp} . \qquad (12)$$

Whence for a given current and time of swing

$$\tan\theta \propto \frac{npM}{I} . \qquad (13)$$

The moment of inertia I falls naturally in two portions, viz., the non-magnetic part which may be called I_n, and the magnetic part which is equal to npI_m where I_m is the moment of inertia of each separate magnet. Let $np = N$ the total number of magnets.

Thus the expression (13) becomes

$$\tan\theta \propto \frac{NM}{NI_m + I_n} . \qquad (14)$$

Of I_n all that need be said at present is that obviously it should be as small as practicable, and may be regarded as a known constant. If then M and I_m can be expressed in common terms the conditions of maximum deflection can be determined. It is easy to express I_m in terms of the linear dimensions of the magnets; thus for a magnet of length $2a$, weight w, and radius r cemented to a thin vertical glass stem, the moment of inertia is approximately $\frac{wa^2}{3}$ when $\frac{a}{r}$ is not less than 7.

Magnetic moment in terms of linear dimensions.—In order to find an expression for the magnetic moment in terms of the linear dimensions of the magnet, many experiments have been conducted here, which have led to a useful expression of this kind. As the method of measuring relative magnetic moments is useful in testing magnets used in needle systems, I shall briefly describe it. A single short magnet provided with a mirror is suspended in a field controlled in such a way that the time of swing of the suspension is raised to one or two seconds. The magnets to be tested are then brought up from the rear, end on, at right angles to the suspended magnet, at the same height with it and to a fixed distance which should be large compared with the length

of the magnets used. In this case the deflections of the suspension are directly proportional to the magnetic moments of the magnets approached.

In the experiments the relative magnetic moments of numerous magnets of different kinds, forms, and sizes were thus determined. The magnets varied in weight between 368 mg and 0.06 mg; in length between 22 mm and 0.8 mm; in diameter between 1.7 mm and 0.075 mm. Some were flat, some short and thick, others as much as fifty times as long as their diameters. Not only were they tried separately, but in combinations of from two to twenty, separated by spaces of from zero to five diameters, sometimes magnetized separately and then approached, at other times magnetized together.

The following table illustrates the advantage of dividing a magnet of a given weight into a number of thinner ones. The magnets were all from the same bar of steel tempered at once and magnetized to saturation:

TABLE I.

Designation of Magnet	Length mm	Width mm	Thickness mm	Weight	Magnetic Moment (Arbitrary Unit)
A_1	6.1	3.0	0.35	0.0425	7.3
A_2	6.1	0.88	0.35	.....	4.9
A_3	6.1	0.88	0.35	Total weight	
A_4	6.1	0.88	0.35	of four	
A_5	6.1	0.88	0.35	0.0507	
A_2 and A_3 placed parallel and a little apart					9.3
A_2 A_3 and A_4 do. do.					13.4
A_2 A_3 A_4 and A_5 do. do.					18.0

We see that the four magnets had a combined magnetic moment two and one-half times as great as the single one of five-sixths their combined weight. Other experiments showed that a given weight of steel continued to increase in magnetic moment with increasing ratio of length to mean diameter even when the ratio was 50 to 1. A round or square cross-section appears preferable to a flat one.

Thoroughly to determine the connection between magnetic

moment and dimensions, a certain piece of steel was drawn into wire of four different sizes. Pieces were taken from each wire and all hardened at the same time. These were then magnetized to saturation together in a long helix. After determining the magnetic moment of each they were broken up into short lengths and again magnetized as before and the magnetic moment of each piece determined separately. The results are given in the following table:

TABLE II.

Designation	Length $2a$ cm	Diameter $2r$ cm	Weight W gr	Magnetic Moment M (Arbitrary Unit)	Ratio $\frac{2a}{2r}$	Ratio $10^5 \frac{W}{M}\frac{2a}{2r}$	Deviations from Mean
A_1	0.80	0.0266	0.00297	130.0	30.1	68.7	−29.2
A_2	0.30	0.0266	0.00111	9.7	11.3	117.4	+19.5
A_3	0.13	0.0266	0.00048	1.8	4.9	130.0	+32.1
B_1	0.73	0.0181	0.00138	59.0	40.3	93.5	− 4.4
B_2	0.35	0.0181	0.00066	19.0	19.4	67.0	−30.9
B_3	0.13	0.0181	0.00025	1.3	7.2	138.0	+40.1
B_4	0.08	0.0181	0.00015	0.7	4.4	94.6	− 3.3
C_1	0.33	0.0137	0.00037	8.4	24.1	94.4	− 3.5
C_2	0.08	0.0137	0.00009	0.5	5.8	96.9	− 1.0
D_1	0.50	0.0075	0.00015	9.4	66.6	106.4	+ 8.5
D_2	0.19	0.0075	0.00006	2.1	25.4	70.0	−27.9
					Mean	97.9	18.1

It will be seen that in this series of experiments the weight of the magnets varied fifty fold, the ratio of length to diameter varied from 66.6 to 4.4, and the magnetic moment varied 250 fold. In all this range of values the magnetic moment was proportional to the product of the weight by the ratio of length to diameter, within an average deviation of about 18 per cent. While this average deviation is large, it does not indicate a departure from the numerical relation just pointed out, for there appears on the whole no tendency for the deviations to become positive or negative either for the heavier or lighter needles or for those longest or shortest in proportion to their diameter. The large deviations may reasonably be explained as caused by differences in the magnetic quality of the steel, as will appear in a later page. Numerous experiments not here given have con-

firmed this general result, and in some cases series have been considerably more accordant than the one here given. It will be interesting to see whether the results of Mendenhall and Waidner support this relation. The following table is roughly computed from the series of magnets H and A given by them.[1]

TABLE III.

Designation	Length $2a$	Mean Diameter[2] $2r$	Weight W	Magnetic Moment M	Ratio $\frac{2a}{2r}$	Ratio $100\,\frac{W}{M}\,\frac{2a}{2r}$	Deviations from Mean
	cm	cm	g				
H_1	0.530	0.010	0.000430	0.0258	53.0	88.5	+23.6
H_2	0.415	0.010	0.000337	0.0184	41.0	75.2	+10.3
H_3	0.316	0.010	0.000251	0.0122	32.0	65.8	+ 0.9
H_4	0.195	0.010	0.000155	0.0048	19.0	61.3	− 3.6
H_5	0.124	0.010	0.000100	0.00215	12.0	55.9	− 9.0
H_6	0.102	0.010	0.000081	0.00143	10.0	56.5	− 8.4
H_7	0.065	0.010	0.000053	0.00054	6.5	62.9	− 2.0
					Mean	64.9	8.3
A_1	0.335	0.024	0.00157	0.0407	13.0	50.0	+ 2.2
A_2	0.229	0.024	0.00105	0.0198	8.8	46.9	− 0.9
A_3	0.167	0.024	0.00078	0.0100	6.4	50.0	+ 2.2
A_4	0.080	0.024	0.00036	0.0031	3.2	37.7	−10.1
A_5	0.153	0.017	0.00034	0.0049	9.0	59.9	+12.1
					Mean	47.8	5.5

These results show closer agreement to the relation above given than does the series of observations before quoted. There appears, however, a tendency to departure from the formula for magnets more than forty and for those less than five times as long as their diameter. I shall therefore restrict my statement as follows: For magnets of a given steel tempered and magnetized to saturation under like conditions, and whose length is between five and forty times their diameter, the magnetic moment is proportional to the product of the weight by the ratio of length to diameter.

Mutual action of magnets.—My observations have led me to a

[1] *Am. Jour. Sci.*, 12, 256, 1901.

[2] The square root of the product of width and thickness.

different view of the seriousness of mutual action between magnets in needle systems from that arrived at by Mendenhall and Waidner, who incline to think it of little consequence. In the table of experimental results which they give the distance between centers of the magnets was never less than 1.0 mm and the magnets were flat and were 0.8 mm wide. Had they been turned up edgewise and approached closer, I think the mutual effect would have greatly increased. The following experiments were made with magnets of round wire about 1.4 mm long and 0.08 mm in diameter, being of the same dimensions that I am accustomed to use for needle systems. Ten of these were magnetized to saturation in a long helix, and gave (in arbitrary units) magnetic moments of 15, 13, 15, 12, 14, 13, 14, 13, 11, and 17, respectively, when measured separately, so that the arithmetical sum of their moments was 137 units. When placed in a bunch parallel and touching each other, the combined moment was only 35 units. The magnets were then again tested separately and had permanently decreased in magnetic moment to only five units each. They were then built into a system with spaces of three diameters between them, and were re-magnetized in the helix. This system gave a combined moment of 105 units. At two diameters and one diameter separation the moment became 95 and 74 units, respectively. Further experiments showed that similar results were reached whether the magnets were first approached and then magnetized, or *vice versa*. With spaces of five diameters between the magnets the loss of combined moment by mutual effect was only about 3 per cent.

Comparative magnetic moment of different samples of steel.— Different kinds of steel are unequally efficient for magnet systems. The best kind tried here was a small bar of tungsten steel of a composition unknown to me, which was given to me at Johns Hopkins University in Baltimore. As compared with Stubs' tool steel, this made up into magnets of twice the magnetic moment for equal weight and similar form. I have used it exclusively in recent galvanometers. But it is not alone samples of steel of different composition, but different samples

from the same bar, that show unequal magnetic value. Thus among 140 magnets of tungsten steel wire 0.008 cm in diameter and all between 0.125 and 0.135 mm long, broken from the same wire, tempered at the same time, and magnetized in a long helix at once, the magnetic moment was found to vary between the limits 13 and 35. Sixty-four, however, were selected whose average magnetic moment was 30 on this arbitrary scale, and most of the remainder were between the limits 20 and 25. It is undoubtedly to this variation of small samples from the same bar that the large deviations in Table II are due.

The following is a summary of the general results reached in these experiments on magnetic moment.

1. Rowland's tungsten steel made the strongest magnets of all the kinds tried, and proved twice as good as Stubs' tool steel.

2. A given weight of steel used to compose a single magnet has a greater magnetic moment if square or round in cross-section than if flat, and the magnetic moment is greater the greater the ratio of the length to the diameter.

3. A given weight of steel of a given length is increased in magnetic moment the more it is subdivided lengthwise to form separated magnets.

4. For magnets of the same steel between five and forty times as long as their diameter, the magnetic moment is given by the relation $M = K' \frac{Wa}{r}$, where M is the magnetic moment, W the weight, a the half length, r the radius, and K' a constant for the given kind of steel. This important relation was found to hold throughout the great range of weights and forms of magnets examined.

5. A number of magnets placed parallel in a group reduce each other's magnetic moments by about one-thirtieth if separated by spaces of 5 diameters, and the loss becomes about one-third at two diameters. It is undesirable to have the spaces less than three diameters.

6. It is immaterial whether the magnets are first magnetized and then approached, or first approached and then magnetized, for the loss is not recovered when the magnets are separated.

We now are in position to determine the best construction of needle systems; for, remembering that the magnetic moment is proportional to $\frac{NWa}{r}$, the moment of inertia to

$$\frac{NWa^2}{3} + I_N ,$$

and the deflection of the galvanometer to their quotient we have

$$\tan\theta \propto \frac{\frac{NW\frac{a}{r}}{}}{\frac{NWa^2}{3} + I_N} . \qquad (15)$$

I have used this formula to compute numerical values for a series of systems, selecting for construction one which is as heavy as possible without sacrificing more than one-tenth in sensitiveness as compared with the most sensitive. In this way better results are reached than by substituting for W in terms of the linear dimensions and specific gravity, and determining analytically the conditions of maximum sensitiveness.

We see by the formula that if the total weight of the system, the length of the needles, and the non-magnetic moment of inertia are all kept constant, the deflection increases indefinitely by diminishing the diameter of the magnets. But a practical limit is soon reached in this course of procedure, for in order to remain effective, the magnets must be separated by three or four diameters, and thus the space available for them is soon taken up. In practice I now employ magnets 0.008 centimeters in diameter.

By inspection of the formula it is also apparent that if the length and radius of the magnets and the non-magnetic moment of inertia all remain unchanged, an increase in the number of magnets increases the deflection. The great advantage of increasing the number is, however, that the non-magnetic moment of inertia can also be increased at the same time, and thus both the weight and rigidity of the needle system is augmented without loss of sensitiveness, so far as the needle system is concerned, and with a real gain in steadiness. But the limitation of space at the center of the coils bars indefinite progress in this

direction. In the two galvanometers now in use at the Observatory this increase in number of magnets has been effected by increasing the number of groups of magnets from two to eight, with an attendant increase of the number of coils from four to sixteen.[1] As shown at a former page, this increase in the number of coils diminishes the effectiveness of the current in the ratio of 68 to 32.

The "constant" of one of these sixteen-coil instruments expressed in ampères for a deflection of 1 mm on scale at 1 meter with time of single swing of 10 seconds and total resistance only 1.6 ohms is 5×10^{-11}.[2]

It is well to remark that, while the sixteen-coil instrument involves a deliberate sacrifice in the efficiency of the coils, it gains greatly in its better astaticism and the steadiness of the needle system. The least current which can be read with it as now arranged is about 1×10^{-12} ampères.

For a region free from prejudicial magnetic disturbances it would be better to retain the four-coil form of galvanometer. By making the coils rather large the interior unwound space could be a centimeter or more in diameter without great sacrifice of current effectiveness, and a large number of needles could be placed in each group.

Building of needle systems.—The method of building up a needle system of fifty or more magnets deserves a little explanation. In the first place the steel wire is cut off in three-quarter-inch lengths and hardened glass hard between thin

[1] A needle system of 10 mg weight having 48 needles each 0.132 cm long and 0.0082 cm in diameter, fixed to one side only of a stem 12 cm long and weighing complete with magnets and mirror 8 mg, is now in use. The several moments of inertia of its materials are: of magnets, 0.0000045 gr cm; of mirror, 0.0000005; of stem, 0.0000012; of shellac, 0.0000002. The practice of putting needles on both sides of the stem far enough apart to avoid loss of magnetic moment by mutual action increases the moment of inertia of the system too much to be of advantage.

[2] One needle system of 2 mg weight for a four-coil instrument of 1.4 ohms at 1.5 seconds single swing gave a millimeter deflection on the scale at one meter distance, with a current of 4×10^{-10} ampères. If we compute from this to a ten-second single swing the result is 9×10^{-12}, and if to a ten-second complete period the result is 24000 on the Ayrton-Mather scale. But both these computed values are illusory, for with atmospheric pressure the deflection of light systems is not even approximately proportional to the square of the time of swing. As shown a little later, this relation holds closely for galvanometers from which the air is exhausted to a pressure of 0.2 mm of mercury.

plates of steel. I have never softened the wire at all, although I have heard that there is a slight advantage in tempering it. The needles are then cut off from the glass-hard wire by means of a simple shears, illustrated in the diagram (Fig. 2), which cuts all of nearly equal length. These short pieces, more in number than are actually required, are next measured under the microscope. Each piece is then stuck like a pin in the end of a little square of cardboard, on which is marked its length;

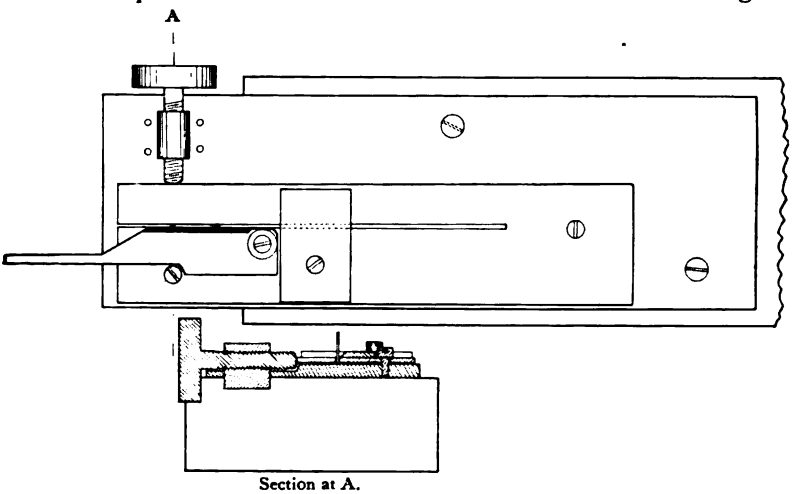


FIG. 2.

all the cardboards are put upon a rod and inserted together in a magnetizing helix, and the magnets are strongly magnetized to saturation. The magnetic moment of each is then determined and the weak ones are eliminated, and, from the remainder, the groups are selected so that total magnetic moment and total weight of all the groups shall be equal.

The groups are built up on a form shown in the accompanying diagram (Fig. 3), consisting of two straight-edged glass strips of equal thickness separated by a space of about 0.8 mm and stuck down to a piece of plate glass. In the crack between the straight edges runs a wire connected to binding posts, one of which is movable and connected to a spring so as to stretch the wire. The little magnets are laid in sugar syrup across the crack from edge to edge, and their distance apart is carefully

adjusted while the sugar is kept moist by breathing upon it; after which the syrup is allowed to harden. A straight glass stem is now drawn of the desired weight, and upon this at intervals corresponding with the distance between the groups of needles are placed little portions of shellac. The stem is now laid upon the needles and a proper current of electricity is passed through the wire underneath till it is seen with a hand glass that the shellac has run around every magnet, but without running out toward

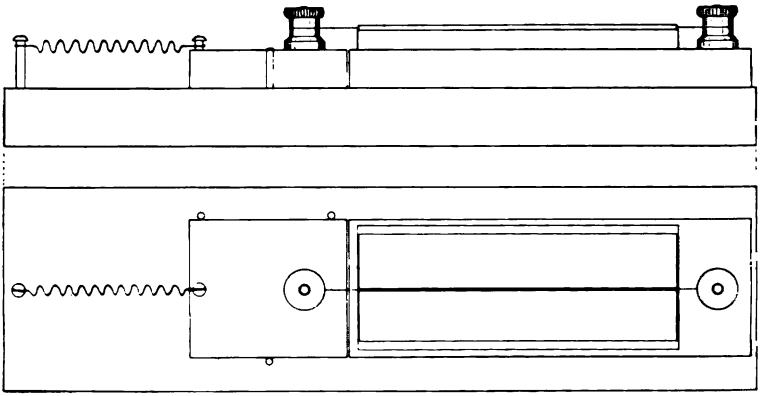


FIG. 3.

their ends. After the shellac cools, the needle system is dissolved off from the sugar bed with water, carefully washed, and is ready for the mirror. Such a needle system is almost mechanically perfect. I find no occasion with eight group needles to astaticize, but I do re-magnetize by the aid of an electro-magnet of sixteen poles.

The mirror is a fragment of microscope cover glass, selected from a large number of pieces of glass about 1×1.5 mm cut out with a dividing engine. These are all coated with metal on both sides, and their optical quality is tested by mounting each on a wad of cotton and reflecting with each mirror in succession the rays from a slit. Both surfaces are tested and each with both long sides and short sides vertical, so that the chance of obtaining a pretty good figure is quite considerable.

Galvanometer support and accessories.—In a city two well-known

kinds of disturbance are ever active, namely, mechanical jarring and electro-magnetic influences. The former of these may be communicated either by sound-waves in air or by direct ground vibrations. To remove the latter the Julius three-wire suspension is efficient, but may be combined advantageously with mercury flotation. Such a combination is employed for both the galvanometers at the Astrophysical Observatory, and its chief features will be found described in Vol. I of the Observatory *Annals*, and in the Appendix to the *Report of the Secretary of the Smithsonian Institution* for 1900 and 1901, pp. 100 and 122 respectively. I will merely add here that as a result of a series of experiments it was found best to use glycerine to damp the horizontal vibrations of the three-wire suspension, as this liquid appears to damp best in proportion to its transmission of ground tremors.

For the destruction of the disturbing vibrations of sound within the galvanometer case, and for a further purpose shortly to be spoken of, the most recent galvanometer case has been made air tight and is used at an air pressure of less than ½ mm of mercury.

Magnetic shielding.—Magnetic shielding is now employed for the most sensitive galvanometer. Three shields are made use of, whose dimensions are in general accord with the recommendations of Du Bois.[1] The outer one is a two-inch-thick cast-iron box open at top and bottom and about six feet high and three feet square. Within this hangs the heavy table upon which rests the galvanometer. Upon this table are two concentric soft cast-iron cylindrical shields. The outer is 30 cm high, 15 cm outside, and 13 cm inside diameter. The inner, symmetrically placed with regard to the outer and fastened to it, is 20 cm high, 8.7 cm in exterior, and 7.5 cm in inside diameter. Both shields are lifted up by means of a cord when one wishes to examine or adjust within the galvanometer. The latter is in the basement of the laboratory and the beam of light for reading it is brought down to the mirror from a slit 3 m above. A small side mirror within the galvanometer case at an angle of 45° to the vertical reflects

[1] *Wied. Ann.*, 65, 8, 1898.

the beam upon the needle system, and returns it to the scale or photographic plate 4 m away in the room above.

A number of strong control magnets within the outer iron box outside the inner shield are used to adjust the time of swing and position of zero. As a constant temperature is essential to freedom from drift when the galvanometer is used at a high time of swing, automatic temperature regulation and other precautions are resorted to.

With this galvanometer with its relatively heavy needle system — protected so thoroughly from shaking by a mercury-floated Julius suspension system weighing nearly a ton, and from magnetic disturbances by three iron shields — great steadiness is of course obtained, and consequently it is practicable to use a high time of swing. But, as is well known, the damping of the air generally prevents the deflection of a modern galvanometer from increasing anything like the square of the time of swing, when the time of single swing is above 1 second, and the air has therefore been exhausted. It was found that little gain resulted till a pressure of less than 1 cm was reached, but at 0.8 mm mercury pressure the law of squares held to 2.5 seconds single swing, and at 0.2 mm the deflection remained proportional to the square of the time of swing up to a time of single swing of 5.5 seconds. The needle system was found to be almost surprisingly steady.

This brings the account down to September, 1902. Since then attention has been wholly diverted from the great galvanometer. It will illustrate the working sensitiveness of an instrument measuring currents of 1×10^{-12} ampères, when I say that the difference of radiating power between an observer's black coat distant 2 m from the balanced bolometer, and the walls of the room which his coat momentarily hid, threw the spot of light off the scale some 40 cm, while the observer's naked hand within a meter of the bolometer turned the galvanometer needle round and round.

In conclusion I wish to say that in the work above described I have had the co-operation of Mr. F. E. Fowle, Jr., of the Astrophysical Observatory.

WASHINGTON, D. C.,
May 1903.

ON FORMULÆ FOR SPECTRUM SERIES.

By A. FOWLER and H. SHAW.

IN spite of the admirable work on spectrum series which has been done by Rydberg and by Kayser and Runge, the formulæ which they have employed to express the relation between the lines which constitute a series can, in many cases, be regarded only as rough approximations, more particularly for series of the principal and second subordinate types. The present paper gives the results of an attempt to bring the results of calculation into better accordance with those of observation by employing modified formulæ.

After several trials, the most accurate representations of various series were obtained by further generalizing the modifications of Balmer's formula which have already been suggested or employed by others. Two of these formulæ seem to merit a somewhat detailed discussion.

Employing Rydberg's convenient notation so far as possible, the first may be written:

$$n = n_\infty - \frac{C}{m^p - m_0}, \qquad \text{(I)}$$

where m takes successively the values 2, 3, 4, $\cdots$, n is the oscillation frequency corresponding to the different values of m, and n_∞, C, m_0, and p are four constants to be determined from four lines for which n and m are known; n_∞ is the "convergence frequency" of the series, or the value which n assumes when m is infinite. This formula may be considered as a variation of the generalization of Balmer's formula suggested by Ames,[1] which may be written

$$n = n_\infty - \frac{C}{m^2 - m_0},$$

in which m is taken to be a whole number.

The second equation investigated is

$$n = n_\infty - \frac{C}{(m + \mu)^2 - m_0}, \qquad \text{(II)}$$

[1] *Phil. Mag.*, 30, 47, 1890.

where n_∞, C, μ, and m_0 are the four constants to be calculated for each series. This may be regarded as a combination of the formula of Ames with that of Rydberg,[1] the latter being

$$n = n_\infty - \frac{C}{(m+\mu)^2},$$

in which m is a whole number, but μ is usually a fraction, and C has the value 109675 for all series.

Formula II may be regarded geometrically as the equation to a hyperbola of the second degree with respect to axes parallel to the asymptotes, the co-ordinates being represented by the values of n and $(m+\mu)^2$, while n_∞ and m_0 are the co-ordinates of the point of intersection of the asymptotes. Similarly, Formula I may be looked upon as the equation to a hyperbola of the pth degree.

In terms of wave-lengths, the two formulæ may be written:

$$\lambda = \lambda_0 + \frac{C'}{m^p - m'}. \qquad (Ia)$$

$$\lambda = \lambda_0 + \frac{C''}{(m+\mu)^2 - m''}. \qquad (IIa)^2$$

DETERMINATION OF THE CONSTANTS.

The determination of the constants of the two formulæ is less tedious than might at first appear. Designating the four given lines by n_1, n_2, n_3, n_4, and the corresponding values of m by m_1, m_2, m_3, m_4, the index p in Formula I may be derived by a few trials from the relation

$$\frac{(n_4 - n_1)\ (n_3 - n_2)}{(n_2 - n_1)\ (n_4 - n_3)} = \frac{(m_4^p - m_1^p)\ (m_3^p - m_2^p)}{(m_2^p - m_1^p)\ (m_4^p - m_3^p)}.$$

If many series are to be investigated, it is convenient to construct curves showing the values of the expression on the right

[1] "Recherches sur la constitution des spectres d'émission des éléments chimiques," *Kgl. Svenska Vet.-Akad. Handl.*, **23**, 11, 1890.

[2] Since completing the present paper we have found that this formula has been previously employed by Rummel (*Proc. Roy. Soc. Victoria*, **10**, 75, 1897; **12**, 15, 1899), who, however, does not appear to have noticed that the equation is of the same form whether expressed in wave-lengths or frequencies.

The work of Rummel appears to have received far less attention than it deserves. This is perhaps partly due to the fact that in an earlier paper (*Proc. Roy. Soc. Vict.*, **9**, 260, 1897) the formula which he employed was simply that of Ames transposed to wave-lengths (which applies with reasonable accuracy to only a few series), and to his continued use of the symbol n (corresponding with m in the above formulæ) in his later papers for a number which may have a fractional value.

for various values of p and for different groups of values of m. From such curves the value of p may usually be read with a sufficient degree of accuracy, and when p has been determined the other constants are easily derived.

The value of $(m_1 + \mu)$ in Formula II is determined directly from the relation

$$(m_1 + \mu) = \sqrt{\frac{a}{a-3}} - 1.5 ,$$

where

$$a = \frac{(n_4 - n_1)\ (n_3 - n_2)}{(n_4 - n_3)\ (n_2 - n_1)} .$$

and n_1, n_2, n_3, n_4 refer to four consecutive lines. If four lines which are not consecutive are employed in the calculation of the constants, the above equation will not hold, but another of similar character may easily be obtained. When μ has been ascertained, the other constants readily follow; or, if desired, n_∞ may be determined independently.

In view of the possible physical significance of the term μ, it may eventually be desirable to adjust its value by the addition of an integer, when necessary, so that $m = 1$ always gives the first positive value of n, or, as in the case of hydrogen, makes $n = 0$. For the present this has not been considered necessary.

EXAMPLES.

As already remarked, it is chiefly in the case of series of the principal and second subordinate types that the older formulæ are most defective, and it may now be added that this is especially marked when it is attempted to include the least refrangible line of a series. The following examples illustrating the two formulæ under investigation have accordingly been chiefly selected from series of these types. In the first instance, the equations given are those calculated from the first four lines of each series.

Unless otherwise stated, the frequencies given in the tables have been derived from Kayser and Runge's wave-lengths (as given in Watts's *Index of Spectra*), corrected to vacuum by the revised table drawn up by the same observers.[1]

[1] *Abh. Königl. Preuss. Akad. Wissensch.*, Berlin, 1893; WATTS's *Index*, App. E, p. 52.

For the helium series, the frequencies already corrected to vacuum, are those given by Runge and Paschen.[1]

In the tables the numbers in brackets refer to the values of m; $O-C$ indicates observed minus computed values.

The differences $O-C$ in the case of Kayser and Runge's calculations, unless otherwise stated, are quoted from Professor Kayser's recent book (*Handbuch der Spectroscopie*, Vol. II), where the formulæ and tables are conveniently brought together.[2] Those resulting from the use of Rydberg's formula are taken directly from Rydberg's general work ("Recherches," etc.), but it should be pointed out that these would probably be slightly modified if recalculated with the later value of the constant C, namely 109675 in place of 109721.6.

THE PRINCIPAL SERIES OF SODIUM.

For the less refrangible components of the double lines which constitute the principal series of sodium lines, the equations derived from the first four lines are:

Formula I: $$n = 41510.37 - \frac{92513.4}{m^{1.917} - 0.02613}.$$

Formula II: $$n = 41449.92 - \frac{110875.4}{(m + 0.1935)^2 - 0.284956}.$$

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$ K. and R.'s Formula (In Tenth-Meters)	$O-C$[4] K. and R. (In Tenth-Meters)	$O-C$ Rydberg (In Tenth-Meters)
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters			
16955.15	(2) +0.14	−0.049	(2) +0.03	−0.010	+77.46	+1.0	+3.0
30265.61	(3) −0.09	+0.010	(3) −0.01	+0.001	0.00	−1.6	−7.8
35041.12	(4) −0.03	+0.002	(4) 0.00	0.000	0.00	+1.0	+0.7
37295.34	(5) −0.02	+0.001	(5) 0.00	0.000	0.00	+0.7	+1.2
38539.66[3]	(6) −0.04	+0.002	(6) +1.81	−0.121	+ 0.06	+0.1	+2.9
39209.06[3]	(7) −1.15	+0.074	(7) +3.67	−0.237	+ 0.17	−0.4	.....
39793.70[3]	(8) −5.90	+0.370	(8) +2.39	−0.160	+ 0.65	−0.4	.....

[1] *Sitz. d. K. Akad. der Wiss.*, Berlin, July, 1895, pp. 639, 759; ASTROPHYSICAL JOURNAL, January 1896.

[2] The equation generally employed by Kayser and Runge is $N = A\,B\frac{\beta}{m^2} - \frac{C}{m^4}$, where A, B, C, are three constants to be separately determined for each series, and m is a whole number.

[3] Observed as single lines.

[4] The errors in this column are those resulting from a formula in which all the lines are used in the calculation of constants (KAYSER, *Handbuch der Spect.*, Vol. II, p. 521).

PRINCIPAL SERIES OF POTASSIUM.

For the less refrangible components of the principal series of double lines of potassium the formulæ and results are as follows:

Formula I: $n = 34927.31 - \frac{195630.4}{m^{2.2182} - 2.5224}$.

Formula II: $n = 35047.96 - \frac{115321.9}{(m + 0.459)^2 - 0.81980}$.

Oscillation Frequency in Vacuo	*O−C* Formula I		*O−C* Formula II		*O−C* K. & R.'s Formula In Tenth-Meters	*O−C* Rydberg In Tenth-Meters
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters		
12984.69	(3) 0.00	0.000	(2) −0.03	+0.020	+161.30	−17 0
24700.46	(4) −0.01	+0.002	(3) +0.02	−0.003	0.00	−22.7
28998.41	(5) 0.00	0.000	(4) 0.00	0.000	0.00	−6.6
31068.72	(6) 0.00	0.000	(5) 0.00	0.000	0.00	+1.6
32224.22	(7) −0.91	+0.087	(6) −4.06	+0.390	+0.27	+4.0
32940.18	(8) +4.53	−0.417	(7) −4.01	+0.369	+0.23	+4.3
33409.17	(9) +6.58	−0.588	(8) −8.45	+0.755	+0.68	+6.6
33736.46	(10)+11.18	−0.978	(9)−10.67	+0.934	+1.05	+7.3
33971.45	(11)+14.24	−1.234	(10)−14.33	+1.242	+1.45	+6.7

THE PRINCIPAL SERIES OF LITHIUM.

For this series the formulæ and results are as follows:

Formula I: $n = 43446.07 - \frac{116717.5}{m^{2.032} - 0.00056}$.

Formula II: $n = 43470.35 - \frac{108764.1}{(m + 0.931)^2 + 0.07855}$.

Oscillation Frequency in Vacuo	*O−C* Formula I		*O−C* Formula II		*O−C* K. & R. In Tenth-Meters	*O−C* K. & R.[2] In Tenth-Meters	*O−C* Rydberg In Tenth-Meters
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters			
14903.08	(2)+0.25	−0.111	(1)−0.09	+0.040	+108.12	0.00	+0.4
30924.54	(3)−0.16	+0.017	(2)+0.06	−0.006	0.00	0.00	+2.2
36467.48	(4)−0.05	+0.004	(3)+0.02	−0.001	0.00	0.00	+0.5
39011.60	(5)−0.03	+0.002	(4)+0.02	−0.001	0.00	+0.66	−0.8
40389.99	(6)+5.45	−0.350	(5)+4.68	−0.287	−0.20	+0.95	−0.3
41215.53	(7)+7.77	−0.457	(6)+5.57	−0.327	−0.01	+1.44	−0.4
41749.29	(8)+9.54	−0.550	(7)+5.92	−0.338	+0.29	+1.80	−0.4
42112.35[1]	(9)+9.41	−0.530	(8)+4.25	−0.238	+0.75	+2.18	−0.6

[1] Based on observations by Messrs. Liveing and Dewar.

[2] The errors in this column are those which result when the constants are calculated from the first three lines.

THE SERIES OF MAGNESIUM TRIPLETS.

For the middle components of the triplets which constitute the first subordinate series of magnesium lines the following formulæ have been derived from the first four lines of the series:

Formula I: $n = 39762.68 - \dfrac{126484.3}{m^{2.058} - 0.34427}$.

Formula II: $n = 39790.41 - \dfrac{109684.5}{(m + 0.840)^2 - 0.06227}$.

Oscillation Frequency in Vacuo	O−C Formula I		O−C Formula II		O−C K. & R.'s Formula In Tenth-Meters	O−C Rydberg's Formula In Tenth-Meters
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters		
26085.57	(3) 0.00	0.000	(2)+0.02	−0.003	−0.01	+0.5
32320.40	(4)+0.01	−0.001	(3)−0.01	+0.001	−0.01	−1.1
35095.68	(5) 0.00	0.000	(4) 0.00	0.000	+0.31	−0.1
36568.50	(6)+0.01	−0.001	(5) 0.00	0.000	0.00	−0.6
37444.55	(7)+2.29	−0.162	(6)+1.67	−0.118	−0.41	+0.9
38004.23	(8)+1.71	−0.118	(7)+0.12	−0.008	−0.46	+0.4

For the least refrangible components of the second subordinate series of triplets the formulæ are:

Formula I: $n = 39699.68 - \dfrac{176665.9}{m^{2.175} - 2.25379}$.

Formula II: $n = 39794.75 - \dfrac{115447.6}{(m + 0.555)^2 - 0.89949}$.

Oscillation Frequency	O − C Formula I		O − C Formula II		O − C K. & R.'s Formula (In Tenth-Meters)	O − C Rydberg's Formula (In Tenth-Meters)
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters		
(b1) 19285.44	(3) 0.00	0.000	(2)+0.03	−0.008	+53.22	+3.9
29000.21	(4)−0.01	+0.001	(3)−0.02	+0.002	−0.03	−13.0
33978.46	(5)−0.01	+0.001	(4)+0.01	+0.001	−0.05	−2.2
35041.23	(6)+0.04	−0.003	(5) 0.00	0.000	−0.02	+2.5
37047.87	(7)−0.18	+0.013	(6)−2.64	+0.198	+0.11	+3.8
37734.99	(8)+1.80	−0.127	(7)−4.76	+0.336	+0.27	+4.8

RYDBERG'S SERIES OF MAGNESIUM.

There is another series of magnesium lines to which attention was first drawn by Rydberg[1] for which the formulæ of Rydberg

[1] *Öfversigt af Kongl. Vet.-Akad. Forhandl.* Stockholm, 1893.

and of Kayser and Runge give very unsatisfactory results. For this series Rydberg has employed the modified formula:

$$n = n_\infty - \frac{B}{(m+\mu)^2} - \frac{C}{(m+\mu)^4} .$$

It is interesting to find that the new formulæ may be applied to the series without change. As the lines are somewhat nebulous, it is possible that the wave-lengths of the more refrangible members of the series, which are also feeble lines, are not very precise, and in fact for the last two lines the limit of error stated by Kayser and Runge is 1.0 tenth-meter.

Formula I: $n = 26633.03 - \frac{88847.1}{m^{1.925} + 2.10247}$.

Formula II: $n = 26601.49 - \frac{107071.4}{(m + 0.2304)^2 + 2.13282}$.

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$ Rydberg's Modified Formula
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	
18082.33	(3)−0.01	+0.003	(3) 0.00	0.00	+0.019
21255.70	(4) 0.00	0.000	(4) 0.00	0.00	−0.009
22970.71	(5) 0.00	0.000	(5) 0.00	0.00	−0.004
23986.85	(6)−0.01	+0.002	(6) 0.00	0.00	0.000
24633.15	(7)−1.01	+0.170	(7)−0.53	+0.09	+0.200
25074.10	(8)+3.61	−0.570	(8)+5.00	−0.79	−0.480

THE TWO SUBORDINATE SERIES OF HELIUM DOUBLES.

For the less refrangible components of the first subordinate series, the formulæ and results are as follows:

Formula I: $n = 29222.11 - \frac{110093.6}{m^{2.0015} + 0.00332}$.

Formula II: $n = 29222.89 - \frac{109697.9}{(m + 0.996)^2 + 0.009151}$.

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C^1$ K. & R.'s Formula		$O-C^2$ Rydberg's Formula	
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters
D_3 17014.13	(3) 0.00	0.000	(2)+0.02	−0.007	0.00	0.00	−9.43	+3.263
22356.97	(4) 0.00	0.000	(3)−0.01	+0.002	0.00	0.00	+2.40	−0.480
24829.56	(5) 0.00	0.000	(4) 0.00	0.000	0.00	0.00	+0.74	−0.119
26172.44	(6) 0.00	0.000	(5) 0.00	0.000	−0.13	+0.02	+0.25	−0.036
26981.92	(7)−0.08	+0.011	(6)−0.10	+0.014	−0.39	+0.05	−0.01	+0.001
27507.23	(8)−0.11	+0.013	(7)−0.16	+0.021	−0.60	+0.08	−0.12	+0.014
27867.34	(9)−0.12	+0.015	(8)−0.20	+0.026	−0.78	+0.10	−0.17	+0.022
28124.73	(10)−0.02	+0.003	(9)−0.43	+0.054	−1.09	+0.14	−0.36	+0.046
28315.49	(11)−0.05	+0.006	(10)−0.21	+0.026	−1.00	+0.13	−0.16	+0.020
28460.56	(12)+0.29	−0.045	(11)−0.08	+0.010	−0.94	+0.12	−0.01	+0.001
28573.37	(13)+0.19	−0.023	(12)−0.06	+0.007	−0.99	+0.12	+0.03	−0.004
28662.74	(14)+0.10	−0.012	(13)−0.17	+0.021	−1.16	+0.14	−0.08	+0.010
28735.00	(15)+0.20	−0.024	(14)−0.10	+0.012	−1.14	+0.14	0.00	0.000
28794.33	(16)+0.48	−0.057	(15)+0.15	−0.018	−0.93	+0.11	+0.26	−0.031
28843.27	(17)+0.50	−0.060	(16)+0.12	−0.014	−0.99	+0.12	+0.24	−0.029
28881.93	(18)−1.86	+0.222	(17)−2.25	+0.268	−3.39	+0.41	−2.12	+0.253
28919.43	(19)+0.94	−0.112	(18)+0.53	−0.063	−0.64	+0.08	+0.66	−0.079

For the second subordinate series the formulæ for the less refrangible components, as calculated from the first four lines, are as follows:

Formula I: $$n = 29176.95 - \frac{142019.0}{m^{2.103} - 0.62765}.$$

Formula II: $$n = 29227.70 - \frac{110377.6}{(m + 0.724)^2 - 0.099836}.$$

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$ K. & R.'s Formula		$O-C^4$ Rydberg's Formula	
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters
14149.48	(3)+ 0.01	−0.005	(2)−0.02	+0.011	−47.85	+24.00	−44.86	+22.660
21210.94	(4) 0.00	0.000	(3)+0.01	−0.002	0.00	0.00	− 6.70	+ 1.500
24259.39	(5) 0.00	0.000	(4)−0.01	+0.002	0.00	0.00	− 1.41	+ 0.241
25848.55	(6) 0.00	0.000	(5) 0.00	0.000	0.00	0.00	− 0.16	+ 0.024
26780.01	(7)+ 0.84	−0.117	(6)−0.36	+0.050	+ 1.38	− 0.19	+ 0.11	− 0.015
27373.71	(8)+ 1.27	−0.168	(7)−0.78	+0.103	+ 3.44	− 0.46	+ 0.18	− 0.024
27774.08	(9)+ 4.04	−0.524	(8)−1.44	+0.187	+ 5.37	− 0.70	− 0.03	+ 0.004
28057.39	(10)+ 6.35	−0.807	(9)−1.75	+0.222	+ 7.51	− 0.95	+ 0.05	− 0.006
28264.92	(11)+ 8.55	−1.064	(10)−2.40	+0.299	+ 9.28	− 1.16	− 0.03	+ 0.004
28421.47	(12)+ 10.64	−1.324	(11)−2.62	+0.326	+10.76	− 1.33	− 0.17	+ 0.024
28543.27	(13)+ 13.41	−1.634	(12)−2.25	+0.274	+12.82	− 1.57	+ 0.47	− 0.057
28638.93	(14)+ 15.31	−1.865	(13)−2.43	+0.296	+14.10	− 1.72	+ 0.52	− 0.063
28714.37 [3]	(15)+15.99	−1.948	(14)−3.97	+0.484	+13.85	− 1.69	− 0.82	+ 0.100

[1] For all the helium series Runge and Paschen use the formula:

$$n = A - \frac{B}{m^2} - \frac{C}{m^3}.$$

[2] Rydberg does not appear to have published calculations for the helium lines, but he has published statements of the convergence frequencies of the series (Astrophysical Journal, 4, 94, 1896) and of the values of the term μ in his equation (*Ibid.*, 6, 234, 1897). By making use of these we get for this series the formula

$$n = 29222.70 - \frac{109675}{(m + 0.996084)^2},$$

and it is from this that the tabulated errors have been derived.

[3] Marked "doubtful" in Runge and Paschen's table.

[4] See note to previous table; the errors stated have been calculated from the equation

$$n = 29222.63 - \frac{109675}{(m + 0.701464)^2}.$$

GENERAL DISCUSSION.

The foregoing examples suffice to demonstrate that series which present great departures from calculation when the older formulæ are employed can be better represented by either of the two formulæ investigated, but few of them permit a final comparison of their relative merits or of the ultimate accuracy of which they are capable. In some of the series there are probably small errors of observation which account in part for differences between the observed and calculated frequencies, and in most of them comparatively few lines have been traced. For helium, however, the case is different; many lines have been observed, and as they are also well defined, the positions determined by Runge and Paschen are probably accurate to a high degree.

All the helium series are not, however, well adapted to test the different formulæ. The first subordinate series of doubles, for example, is very closely represented even by the simplest generalization of Balmer's formula for hydrogen, the equation calculated from lines for which m has the values 3, 4, 5, being

$$n = 29225.58 - \frac{109901.37}{m^2} .$$

With this formula the differences between the observed and calculated frequencies for values of m as far as 17 do not differ by more than 2.13. As all the formulæ under investigation are attempted generalizations of Balmer's law, it is clear that such a series as this cannot be effectively employed for their comparison, and it will be seen by reference to the table that the differences between the observed and calculated frequencies are very small throughout. It is the second subordinate series of double lines of helium which at present permits the best comparison of the formulæ.

From the table already given for this series it appears that Formula I, when calculated for the first four lines, gives errors comparable with those of Kayser and Runge's formula for lines corresponding to the greater values of m, but has the advantage of correcting the large error in the case of the first line. Still, this cannot be considered an accurate formula for the series,

especially as the convergence frequency is notably smaller than that derived from the first subordinate series. Nevertheless it is instructive to observe the effect of calculating the constants from lines more widely separated than the first four; taking lines for which $m = 3, 4, 6$, and 8, we get equation I*b* stated at the head of the following table. Again, reducing p to 2.09 and calculating the other constants from the first three lines, we get equation I*c*.

The results are as follows:

Formula I*a*: $$n = 29176.95 - \frac{142019.0}{m^{2.103} - 0.62765}.$$

Formula I*b*: $$n = 29189.83 - \frac{140337.9}{m^{2.0947} - 0.65602}.$$

Formula I*c*: $$n = 29200.77 - \frac{139489.5}{m^{2.09} - 0.66775}.$$

Oscillation Frequency in Vacuo	$O-C$ Formula I*a* In Frequency	$O-C$ Formula I*b* In Frequency	$O-C$ Formula I*c* In Frequency
14149.48	+ 0.01[1]	−0.02[1]	0.00[1]
21210.94	0.00[1]	+0.02[1]	0.00[1]
24259.39	0.00[1]	+0.61	0.00[1]
25848.55	0.00[1]	−0.01[1]	−1.66
26780.61	+ 0.84	−0.36	−3.13
27373.71	+ 1.27	0.00[1]	−3.76
27774.08	+ 4.04	−1.62	−3.96
28057.39	+ 6.35	+1.98	−3.38
28264.92	+ 8.55	+3.31	−2.67
28421.47	+10.04	+4.64	−1.86
28543.27	+13.41	+6.67	−0.20
28638.93	+15.31	+8.23	+0.89
28714.37	+15.99	+8.26	+0.59

The errors may in this way be generally reduced and differently distributed, but it is clear that the series cannot be represented by Formula I with an accuracy equal to the probable accuracy of the observations.

For the same series, Formula II, as derived from the first four lines, is decidedly more accurate than Formula I derived from the same four lines, and the convergence frequency is also very nearly equal to that of the first subordinate series. The regular increase in the differences $O - C$ suggests that the formula might

[1] Used in calculation of constants.

be improved by taking a value of μ slightly different from that given by the first four lines. Thus, by calculating the constants from lines for which $m = 2$, 3, 5, and 7, the value of μ becomes 0.716, and the equation derived is

$$n = 29224.24 - \frac{110040.6}{(m + 0.716)^2 - 0.07766} .$$

Here the convergence frequency differs very slightly from that of the first subordinate series, and the differences $O - C$ are greatly reduced, as shown in the following table:

Observed Frequency in Vacuo	$O-C$ Formula II Above	
	In Frequency	In Tenth-Meters
14149.48	−0.01	+0.005
21210.94	0.00	0.000
24259.39	−0.24	0.041
25848.55	0.00	0.000
26780.61	+0.03	−0.004
27373.71	0.00	0.000
27774.08	−0.28	+0.036
28057.39	−0.32	+0.041
28264.92	−0.49	+0.061
28421.47	−0.72	+0.091
28543.27	−0.17	+0.021
28638.93	−0.20	+0.024
28714.37	−1.61	+0.196

The differences between calculation and observation thus become almost insignificant, never amounting to more than 0.1 tenth-meter, except in the case of the last line, which is marked "doubtful" by Kayser and Runge.

Of the other examples given, the principal series of potassium is about the only one which is sufficiently extended to be used in a minute comparison of the formulæ, but in this case the wave-length determinations are probably much less precise than those of helium. From the table already given it will be seen that when the constants are determined from the first four lines, the two formulæ investigated in this paper represent the series with almost equal accuracy, and much more closely than the older formulæ. As in the case of the second subordinate series of helium, however, it is found that changes in the constants of Formula I do not result in quite so accurate a representation of

the series as can be obtained by a change of constants in Formula II. Thus by calculating the constants of the latter from the first, second, fourth, and six lines, we get the equation:

$$n = 35031.60 - \frac{113919.3}{(m + 0.430)^2 - 0.7380};$$

and the differences $O - C$ compared with the limits of error of observation stated by Kayser and Runge are as follows:

Observed Frequency in Vacuo	$O-C$ Formula Above		Limit of Error of Observation in Tenth-Meters
	In Frequency	In Tenth-Meters	
12984.69	+0.99	−0.58	5.00
24700.46	−0.06	+0.01	0.03
28998.41	−1.59	+0.19	0.03
31068.72	−0.04	+0.004	0.03
32224.22	−1.96	+0.19	0.10
32940.18	+0.11	−0.01	0.10
33409.17	−2.58	+0.23	0.15
33736.46	−3.35	+0.29	0.20
33971.45	−5.80	+0.50	1.00

When the errors are distributed in this way, it will be seen that they are on the whole scarcely too great to be considered as arising from errors of observation.

The general result of the comparison, therefore, is that, while both the formulæ investigated represent the observed series very closely, the most accurate general formula appears to be:

$$n = n_\infty - \frac{C}{(m + \mu)^2 - m_0}.$$

With this formula, if the constants are calculated so as to distribute the errors, the computed differ from the observed positions by amounts which perhaps do not exceed the probable errors of the observations at present available.

Royal College of Science, London,
March 30, 1903.

THE VARIABLE STAR 6871 *V LYRAE*.

By J. A. PARKHURST.

THE variability of this star was announced by Anderson in 1895[1], this being one of many variables which are very faint for the greater part of their period, and yet have been detected by this careful observer with telescopes of two and three inches aperture. In the place just cited, magnitudes are given from the Harvard photographs from July 24, 1892, to October 6, 1894. Visual observations have been published by Yendell,[2] H. M. Parkhurst,[3] and the writer,[4] covering the maxima of each year from 1893 to 1900, inclusive. Approximate data for the minima of 1899 and 1900 are given in the last reference. Photometric magnitudes of comparison stars brighter than the twelfth magnitude have also been published by H. M. Parkhurst,[5] with which those given in this paper are in fair agreement.

INSTRUMENTS.

These are the same as given in my paper on *X Cephei*[6]—a 157 mm reflector, a 305 mm, and the 101 cm refractor of the Yerkes Observatory; the equalizing wedge photometer being attached to each of these instruments. The photograph of the field reproduced in Plate I was taken November 30, 1902, with an exposure of thirty-six minutes, from $6^h\ 24^m$ to $7^h\ 0^m$ Central Standard Time. The faintest stars shown on the plate are about 16.5 magnitude, though none fainter than ϵ, 15.80 magnitude, have been measured in this field.

POSITION OF THE VARIABLE.

The variable was connected on two nights with the comparison stars *a*, *b*, and *c*, whose *Durchmusterung* numbers will be

[1] *A. N.*, **137**, 235.

[2] *Astronomical Journal*, **15**, 157, 1895; **17**, 27, 1896.

[3] *Ibid.*, **17**, 65, 1897; **18**, 100, 1897; **19**, 190, 1899.

[4] *Ibid.*, **17**, 102, 1897; **18**, 142, 1898; ASTROPHYSICAL JOURNAL, **14**, 174, 1901.

[5] *Ibid.*, **17**, 65, 1897.

[6] ASTROPHYSICAL JOURNAL, **17**, 48, 1903.

found in Table II. The places of these stars for 1875 were taken from the Cambridge, England, *Astronomische Gesellschaft* Catalogue. The results were as follows:

Star	*a*	*b*	*c*
R.A. 1875	$19^h\ 3^m\ 1^s.02$	$4^m\ 8^s.51$	$4^m\ 10^s.80$
ΔR.A.	$+1\ 9.93$	$+0\ 3.27$	$+0\ 0.30$
R.A. *V*	$19\ 4\ 10.95$	$4\ 11.78$	$4\ 11.10$
Mean *V* 1875	$19\ 4\ 11.28$		
Dec., 1875	$+29°\ 36'\ 16''.2$	$42'\ 43''.9$	$17'\ 48''.7$
Δ Dec.	$-8\ 42.4$	$-15\ 10.5$	$+9\ 43.3$
V	$+29\ 27\ 33.8$	$27\ 33.4$	$27\ 32.0$
Mean *V* 1875	$+29\ 27\ 33.1$		

For convenience the following are added:

POSITION OF 6871 *v LYRAE*

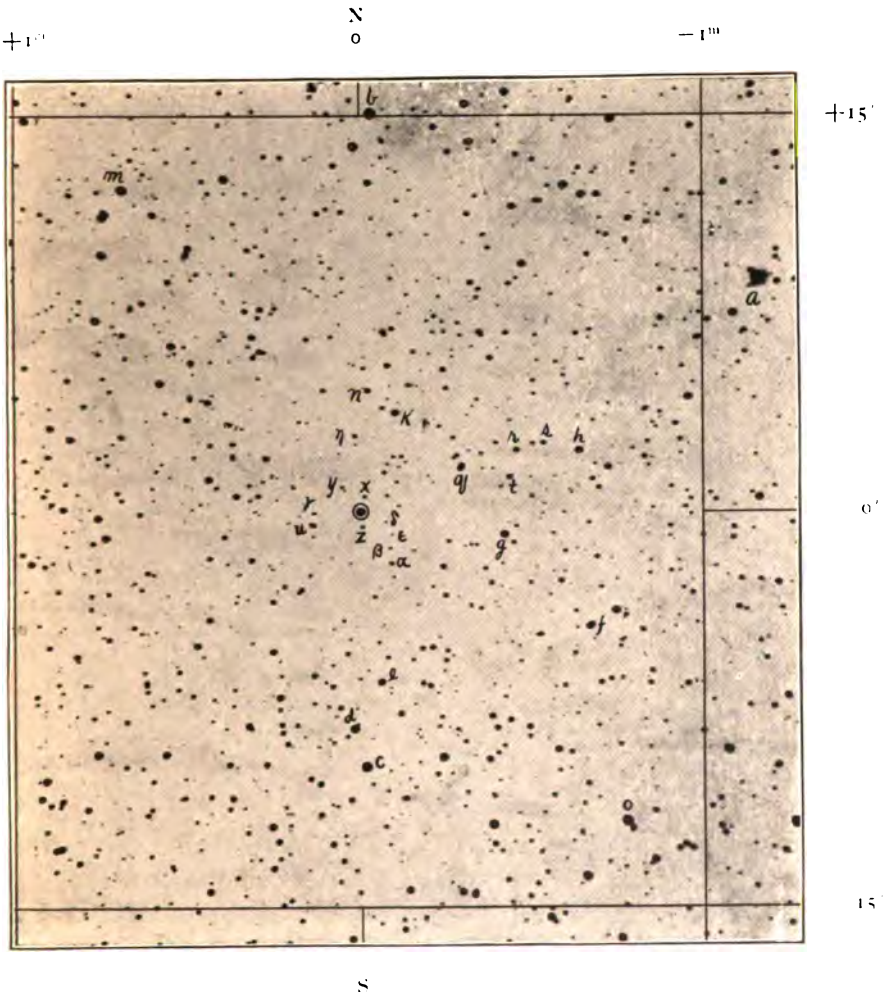
R.A. $19^h\ 3^m\ 24^s.5$, Dec. $+29°\ 25'\ 42''$ (1855).
$5\ 9.9$, $29\ 52$ (1900).

COMPARISON STARS.

The numerical data for the comparison stars are given in Tables I and II, which require little explanation. The positions of all the stars are shown on the chart. The co-ordinates from the variable for the stars brighter than twelfth magnitude were measured with the filar micrometer on the 6-inch reflector; the faint stars near the variable were measured with the 40-inch refractor, and all positions were checked from the photograph. The light scale was formed in the usual manner, from comparisons of the variable by Argelander's method; the results are given in the fourth column. The zero point of this scale, the star ϵ, is not the limit of vision of the 40-inch refractor, but simply the faintest star needed in the visual comparisons. The fifth column shows the results of the photometer measures which are given in detail in Table III.

The photometric magnitudes are based on the following standard stars:

PLATE I.



6871 *V LYRAE*

($19^h\ 5^m\ 9^s.9$; $+29°\ 29'\ 52''$)

Star	B.D.		1855		Magnitudes		
	No.	Mag.	R.A.	Dec.	P.DM.	Harvard	
						24	45
A	+29° 3472	6.5	19h 0m 7s.6	+29° 41'.9	6.45	6.46	6.77
B	+30 3425	7.8	1 54.4	+30 1.0	....	8.06	....
C	+30 3438	7.0	3 14.7	+30 4.4	6.94	6.86	6.90

The magnitudes in the last three columns are, respectively, from the *Potsdam Photometric Durchmusterung*, *Harvard College Observatory*, Vol. 24 (*Meridian Photometry*), and Vol. 45 (*Photometric Durchmusterung*). The magnitudes used were those of Harvard 24, as it contained all three of the stars and differed but little from the *P.DM*, the difference in the systems being only 0.035 mag. In the sixth column of Table I are given the magnitudes of such of the stars as were not measured with the photometer. These were read off from the "Magnitude Curve," Fig. 1, in a manner to be described later.

TABLE I.

Comparison Stars for *V Lyrae*. In order of Right Ascension.

Star	Co-ordinates from V		Light Scale	Magnitude		Star	Co-ordinates from V		Light Scale	Magnitude	
	R. A.	Dec.		Measured	From Curve		R. A.	Dec.		Measured	From Curve
	s	° '	steps				s	° '	steps		
a	−70	+ 8 43	....	8.29	.....	δ	− 4	− 0 25	1.5	15.60	.....
o	−45	−11 54	37.6	9.35	.....	e	− 3	− 6 26	29.2	.....	11.27
f	−44	− 3 49	28.1	11.18	.....	b	− 2	+15 11	44.3	8.54	.....
k	−38	+ 2 24	28.4	.....	11.40	n	− 2	+ 4 36	26.0	12.20	.....
s	−31	+ 2 40	....	.....	.....	x	− 1	+ 0 35	....	.....	14.8
r	−27	+ 2 25	19.0	.....	12.88	c	− 1	− 9 43	35.9	10.05	.....
t	−26	+ 1 18	....	.....	.....	z	− 1	− 0 34	8.0	14.76	.....
g	−25	− 0 52	25.4	11.22	.....	η	+ 1	+ 2 54	....	13.35	.....
q	−17	+ 1 42	23.0	.....	12.30	d	+ 1	− 8 8	31.8	.....	10.80
l	− 7	+17 48	....	.....	.....	y	+ 3	+ 0 59	....	.....	.....
h	− 5	+ 3 49	27.0	11.91	.....	γ	+ 8	− 0 3	7.7	.....	14.70
e	− 6	− 0 50	0.0	.....	15.80	u	+ 8	− 0 29	16.4	13.08	.....
a	− 5	− 2 00	15.3	.....	13.52	m	+40	+12 00	37.0	9.81	.....
β	− 5	− 1 21	7.1	.....	14.78						

Table II gives the *Durchmusterung* numbers of part of the stars, also a comparison between the *Durchmusterung* magnitudes and the photometric measures of H. M. Parkhurst and the writer. There is a systematic difference of about 0.15 mag.

between the two latter, which can be explained by the fact that they are based on a different set of Meridian Photometer stars. Besides this, there is a difference of 0.60 mag. in the star *b*, which has a range of over a magnitude from the *Durchmusterung* value. The photographic magnitude of *b* is about 9.2, showing a considerable color and probable fluctuations or variation.

TABLE II.

Comparison with the *Durchmusterung* and Measures by H. M. Parkhurst.

Star	"Durchmusterung"		H. M. Parkhurst		J.A. Parkhurst
	Number	Mag.	Letter	Mag.	
a	+29° 3483	8.0			8.29
o	3486	9.2			9.35
l	3490	8.4			....
b	3492	8.1	*J*	9.14	8.54
c	3493	9.3	*IX*	9.91	10.05
d	3494	9.5	*Z*	10.70	10.80
m	+29 3498	9.2	*W*	9.67	9.81
f	....	...	*b*	10.97	11.18
h	....	...	*c*	11.02	11.40
g	....	...	*a*	10.95	11.22
e	....	...	*e*	11.28	11.27

The photometric measures of the comparison stars are given in Table III. The arrangement of this table and the methods of reduction are similar to those in my previous paper on *X Cephei*. For convenience I will repeat here that the stars used as standards are given first in each part of the table, with their magnitudes in bold-faced type. The mean of the scale readings and the mean of the magnitudes of these standard stars are taken as the zero points. For the stars whose magnitudes are sought, the difference in scale readings is converted into magnitudes by the use of the wedge constant given at the foot of each set, and added to the zero magnitude, giving the quantities in the last column. The magnitudes of the comparison stars brighter than 11.5 were measured with the 6- and 12-inch telescopes, and the mean of the results given after the set for October 5, 1902. From these as standards, the stars *n* and *k* were measured with the 12-inch, and finally the four fainter stars were measured with

the 40-inch. The set of measures made October 5, 1902, requires explanation, since a cell holding two absorption glasses was interposed in the cone of rays of the real star while measuring the standards *A*, *B*, and *C*. The absorption caused by these two glasses has been measured by standard stars and also by the "wheel photometer," the resulting value, 1.70 mag., being used in this work. The mean magnitude of the three standard stars, 7.13, becomes 8.83 when the absorption glasses are used. It will be seen that the results are in good accord with those taken without the absorption glasses.

TABLE III.

1900, August 4; 6-inch. Wedge II.

Star	Scale Readings								Mean Scale Reading	Magnitude
	First				Second					
A	11.8	10.5	10.5	8.8	12.2	9.9	10.9	11.0	10.60	6.46
B	20.0	20.9	21.2	20.1	22.2	22.0	20.9	21.5	21.12	8.06
C	15.0	14.1	14.8	14.4	14.9	13.1	14.2	13.1	14.20	6.86
a	25.2	23.6	24.2	23.9	23.6	22.8	23.8	22.0	23.64	8.21
b	25.0	25.0	25.3	25.2	24.2	24.6	24.7	25.9	24.99	8.39
m	37.2	35.1	36.1	35.7	34.1	33.9	34.6	33.6	35.04	9.70
c	34.4	36.3	37.0	35.9	37.2	38.9	37.7	37.9	37.04	9.96
o	32.9	29.2	30.9	32.6	32.4	31.9	31.9	31.9	31.72	9.26

Wedge constant, 0.130 mag.

1900, October 13; 6-inch. Wedge II; seeing fine.

C	12.2	10.0	9.5	9.8	8.8	8.2	9.75	6.86
B	17.8	18.8	19.5	17.1	17.2	17.3	17.95	8.06
a	20.1	21.1	20.9	21.0	21.2	21.9	21.04	8.39
b	23.9	23.5	24.2	22.5	24.3	22.1	23.42	8.70
m	33.5	32.6	33.0	32.3	32.2	32.6	32.70	9.91
c	35.0	34.8	34.7	33.0	34.0	35.0	34.42	10.13
o	29.1	28.3	28.8	28.0	29.9	30.1	29.03	9.43
g	42.9	45.1	43.5	....	....	....	43.83	11.36
f	42.0	43.2	43.3	....	....	....	42.50	11.18
k	48.5	50.7	50.2	....	....	....	49.80	12.13
v	36.7	39.3	37.5	....	....	....	37.83	10.38

Wedge constant, 0.130 mag.

TABLE III—*Continued.*

1902, October 5; 12-inch. Wedge V; seeing good.

C_a	12.0	13.1	11.8	12.3	13.0	12.7	12.49	**6.86**
B_a	17.8	18.5	18.2	21.0	20.5	19.9	19.32	**8.06**
A_a	9.2	9.1	9.1	10.0	9.1	8.8	9.22	**6.46**
g	35.0	34.4	35.7	33.0	33.0	33.3	34.07	11.07
q	41.2	42.8	42.4	43.3	42.0	41.8	42.25	11.97
k	40.7	40.0	40.9	38.5	38.3	38.9	39.55	11.68
n	43.1	44.9	44.0	41.3	40.5	40.3	42.35	11.98
b	10.6	10.3	11.0	11.2	11.8	11.2	11.02	8.54
a	8.8	9.8	8.3	8.8	8.7	7.8	8.70	8.28
v	49.9	50.2	49.8	49.1	48.7	48.0	49.29	12.75

Wedge constant, 0.110 mag.

RESULTING MAGNITUDES FROM MEASURES WITH 6- AND 12-INCH.

Star	1900 Aug. 4	1900 Oct. 13	1902 Oct. 5	Mean
a	8.21	8.39	8.28	8.29
b	8.39	8.70	8.54	8.54
o	9.26	9.43	.....	9.35
m	9.70	9.91	.....	9.81
c	9.96	10.13	.....	10.05
f	....	11.18	.....	11.18
g	....	11.36	11.07	11.22

1900, October 17; 12-inch. Wedge II; seeing fair.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
b	20.4	19.6	19.9	20.9	20.5	19.5	20.14	**8.54**
m	31.7	29.3	29.0	32.7	30.0	30.6	30.55	**9.81**
c	33.1	33.9	32.2	35.0	34.1	35.0	33.98	**10.05**
o	25.0	25.1	24.7	24.8	23.8	25.0	24.73	**9.35**
n	48.9	49.6	47.8	51.3	51.0	49.8	49.90	12.37
k	45.5	46.7	46.3	46.8	46.2	43.8	45.89	11.83
u	61.5	59.8	62.1	....	....	....	61.13	13.83
v	35.7	35.4	35.1	....	....	....	35.40	10.48

Wedge constant, 0.130 mag.

TABLE III—*Continued.*

1902, May 14; 12-inch. Wedge V; seeing good.

a	14.5	15.2	15.1	15.0	15.2	15.7	15.12	8.29
b	18.8	18.8	18.9	17.5	17.7	17.7	18.23	8.54
m	29.9	30.1	30.1	29.0	29.2	29.9	29.70	9.81
c	35.3	35.4	36.3	35.0	35.0	34.2	35.20	10.05
n	52.8	53.4	52.8	51.9	52.5	52.3	52.62	12.26
k	50.7	51.2	50.1	49.2	49.5	50.1	50.14	11.98

Wedge constant, 0.110 mag.

RESULTING MAGNITUDES FROM MEASURES WITH 6- AND 12-INCH.

Star	1900 Oct. 13	1902 Oct. 5	1900 Oct. 17	1902 May 14	Mean
n	.....	11.98	12.37	12.26	12.20
k	12.13	11.68	11.83	11.98	11.91

1900, July 20; 40-inch. Wedge II; seeing very fine.

Star	Scale Readings								Mean Scale Reading	Magnitude
	First				Second					
n	24.1	24.5	23.0	23.0	22.0	24.1	24.0	23.0	23.47	12.20
g	18.5	19.0	20.7	19.8	....	....	....	....	19.50	11.22
k	26.9	24.3	24.0	25.0	....	....	....	....	25.05	11.91
u	31.1	32.2	33.2	35.0	....	....	....	....	32.88	13.11
z	46.0	46.7	47.5	47.9	....	....	....	....	47.05	14.95
δ	57.1	52.0	52.2	51.2	54.5	....	....	....	53.40	15.69
η	36.4	34.0	35.1	34.0	....	....	....	....	34.88	13.37

Wedge constant, 0.130 mag.

1900, October 17, 40-inch. Wedge II; seeing poor.

n	32.5	32.9	34.0	31.4	28.1	30.9	31.63	12.20
k	28.0	30.0	30.9	31.0	30.0	30.9	30.13	11.91
η	41.6	41.8	40.5	39.7	39.6	40.8	41.30	13.32
δ	53.0	54.8	55.2	61.0	59.6	58.5	57.02	15.51
z	51.3	51.4	51.5	47.5	48.9	50.9	50.25	14.57
u	38.1	39.2	38.9	38.5	37.3	39.6	38.60	13.05
v	11.0	15.0	15.0	....	....	....	13.67±	9.8 ±

Wedge constant, 0.130 mag.

TABLE III—*Continued.*

RESULTING MAGNITUDES. FROM MEASURES WITH 40-INCH.

STAR	1900 Oct. 17	1900 July 20	Mean
u	13.05	13.11	13.08
η	13.32	13.37	13.35
z	14.57	14.95	14.76
δ	15.51	15.69	15.60

No correction for differing zenith distance has been applied to these measures, since even in case of the most distant comparison stars, *A*, *B*, and *C*, the distance would differ by less than half a degree, giving a correction of less than 0.01 mag. at the altitude at which the measures were made. For the fainter standards the correction would be still smaller.

MAGNITUDE CURVE.

Fig. I is platted with the photometric magnitudes as abscissæ and the positions in the light-scale as ordinates, giving the

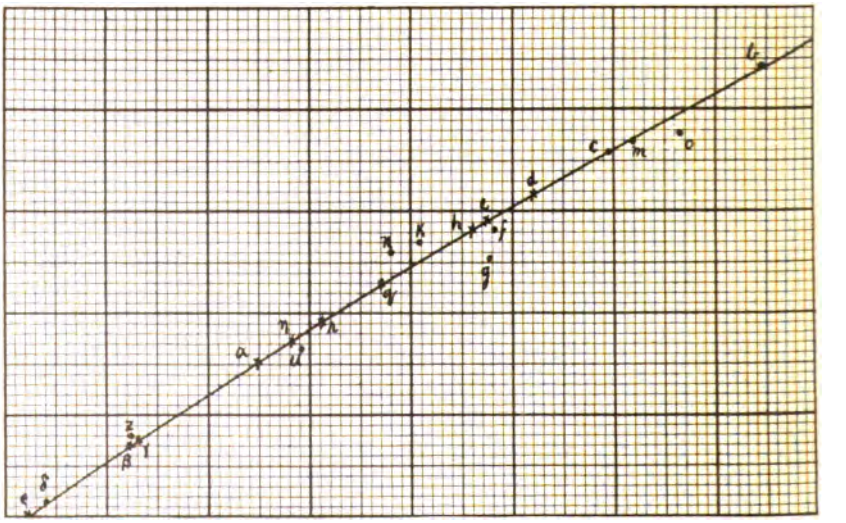


FIG. I.

points indicated by the round dots. Through these the smooth curve was drawn and on this curve the stars not measured with the photometer were entered from their positions in the light scale, the places being indicated by crosses. The average distance between the platted points and the curve is 0.11 magnitude, the greatest residual being for the star *g*, 0.30. This depends on the measures of only two nights. Omitting this star, the mean residual is reduced to 0.09. The mean value of one step is 0.16 magnitude, and is essentially the same with the different apertures.

VISUAL OBSERVATIONS OF THE VARIABLE.

The visual comparisons by Argelander's method, and reductions to magnitudes, are given in Table IV, which should be printed as a double-page table, the comparisons on the left and the corresponding reductions on the right, For convenience the current number is repeated on the right-hand page. When the variable was invisible, the limit of vision was estimated from the faintest comparison star seen, and placed in the mean-step column, preceded by the inequality sign, which is read "fainter than" the quantity following. The limit of vision from these observations is 13.24 for the 6-inch, and 14.76 (perhaps, the record is "1900, April 4, 12-inch, power 275, *z* glimpsed) for the 12-inch.

THE LIGHT-CURVE.

In Fig. 2 the observed magnitudes are platted, showing the star's variations from 1896 to 1903. The magnitudes at maximum vary from 9.0 to 10.5, while only one minimum, that of June 4, 1900, is well enough observed to determine the limit, 15.4 magnitude. A single observation near the following minimum, July 9, 1901, at 15.0 magnitude, confirms the first, as far as it goes.

TABLE IV.

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Comparisons of the Variable by Argelander's Method.

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1896		C. S. T.	G. M. T.			
1	Sept.	22		3825	80	6	*c* 2 *v*, *v* 4 *d*
2		30	7	3833.5	40	6	*c* 1 *v*, *v* 4 *d*
3	Oct.	6	7	3839.5	40	6	*c* 2 *v*, *v* 3-4 *d*
4		13	7	3846.5	40	6	*c* 0-1 *v*, *v* 6 *d*, *m* 2 *v*
5		21	8	3854.6	40	6	*v* 2 *c*, *m* 1 *v*, *o* 3 *v*
6	Nov.	4	7	3868.5	40	6	*m* 4-5 *v*, *c* 2 *v*, *v* 4 *d*
7		12	7	3876.5	40 150	6 6	*v d* *c* 4 *v*, *v* 2 *d*, *v* 6 *g*
8		26	7	3890.5	150 150	6 6	*d* 1 *v*, *v* 5 *e*, *v* 1 *h*, *v* 2 *g* *k* 2 *n*, *n* 3 *q*, *q* 2 *r*
9	Dec.	9		3903	150	6	*e* 1 *v*, *v g*, *v* 3-4 *n*, *h* 1 *v*
10		23	6	3917.5	150	6	*v* 1-2 *e*, *v g*, *v* 3 *n*
	1897						
11	Jan.	9	6	3934.5	80	6	*v* not seen
12	May	25	9	4070.6	150	6	*v* not seen
13	July	1	9	4107.6	150	6	*v* not seen
14		26	9	4132.6	150	6	*v* not seen
15	Aug.	27	8	4164.6	150	6	*q* 2 *v*, *v* 0-1 *r*
16	Sept	16	7	4184.5	150	6	*g* 1 *v*, *v e*, *v* 6 *k*, *k* 3 *q*
17		20		4188	150	6	*d* 1-2 *v*, *v* 2-3 *g*, *v* 6-8 *q*
18		25		4193	40 150	6 6	*v h*, *v* 0-1 *g*, *f* 1 *v* *v* 2 *g*, *v* 1 *f*, *d v*
19	Oct.	1	7	4199.5	40 150	6 6	*c* 3-4 *v*, *d* 1 *v*, *v* 4 *g* *c* 4 *v*, *d* 1 *v*, *v* 4 *g*, *v* 3 *f*
20		7	7	4205.5	40 150	6 6	*v f*, *d* 1-2 *v* *v* 3 *f*, *v* 1-2 *d*, *c* 6 *v*
21		14	6	4212.5	150	6	*v* 4-5 *f*, *v* 2 *d*, *c* 4 *v*
22		23	7	4221.5	150	6	*v* 4 *f*, *v d*, *c* 5 *v*
23		29	6	4227.5	40	6	*v* 1-2 *d*, *c* 3 *v*
24	Nov.	3	7	4232.5	40	6	*v* 2 *d*, *c* 4 *v*
25		17	7	4246.5	150	6	*c* 4-5 *v*, *v* 1 *d*
26		20	6	4249.5	40 150	6	*d* 4 *v*, *v* 1 *g*, *v f*
	1898						
27	June	25	10	4466.7	80	12	*v* not seen
28	July	7	10	4478.7		12	*v* not seen
29		20	10	4491.7		12	*v* not seen
30	Aug.	8	10	4510.7	80	12	*v* not seen
31		18	11	4520.8	80 175	12 12	*v* 1 *u*, *q* 10 *v* *v u*, *r* 2 *v*
32	Sept.	2	8	4535.6	150	6	*v* 3-4 *f*, *v* 3 *e*, *d* 3 *v*
33	Sept.	20	8	4553.6	40	6	*v* 1 *c*, *m* 0-1 *v*
34	Oct.	5	7	4568.5	40	6	*v* 2 *m*, *v* 4 *c*, *b* 6-7 *v*
35		28	7	4591.5	150	6	*v* 1-2 *c*, *v m*
36		31	7	4594.5	40	6	*v* 3 *c*, *v* 0-1 *m*
37	Nov.	7	6	4601.5	40	6	*m* 1 *v*, *v c*, *v* 8 *e*

TABLE IV.

6871 V Lyrae.

Reduction of Observations.

No.	Details in Steps	Means		Seeing	Remarks
		Steps	Mag.		
1	33.9, 35.8	34.9	10.24	Moon, good	
2	34.9, 35.8	35.4	10.17	Moon, good	
3	33.9, 35.3	34.6	10.15	good	
4	34.4, 37.8, 35.0	35.7	10.10	Moon, fair	
5	37.9, 36.0, 34.6	36.2	10.02	good	
6	32.5, 33.9, 35.8	34.1	10.39	fair	
7	31.8 31.9, 33.8, 31.4	32.2	10.75	good, Moon	passing clouds
8	27.4 30.8, 34.2, 29.4	30.5	11.01	good	limit r
9	28.2, 25.4, 29.5, 27.4	27.6	11.52	fair, Moon	
10	30.7, 25.4, 29.0	25.8	11.81	good	
11		<27.0	<11.6	low, Moon	limit k
12		<19.0	<12.9	good	k, n, q, and r seen
13		<19.5	<12.9	good to fair	limit 3-4 $< q$
14		<19.5	<12.9	good to fair	limit 3-4 $< q$
15	21.0, 19.5	20.3	12.74	good	
16	24.4, 29.2, 33.0	28.8	11.32	fair	
17	30.3, 27.9, 30.0	29.1	11.29	good	
18	28.4, 25.9, 27.1 27.4, 29.1, 31.8	28.3	11.40	good	
19	32.4, 30.8, 29.4 31.9, 30.8, 29.4, 31.1	30.8	10.97	fair	
20	28.1, 29.3 31.1, 33.3, 29.9	30.3	11.07	Moon	
21	32.6, 32.8, 31.9	32.4	10.70	good	
22	32.1, 31.8, 30.9	31.6	10.83	good	
23	33.3, 32.9	33.1	10.57	good	
24	33.8, 31.9	33.4	10.52	good, Moon	
25	31.4, 32.8	32.1	10.76	fair to good	
26	27.8, 26.4, 28.1	27.4	11.55	good	
27		<15	<13.6		limit $8 < q$
28		<16	<13.4	fine	limit t
29		<16	<13.4		limit $3 < s$ and t
30			<14	good	limit 2 mag. $< q$
31	17.4, 13.0 16.4, 17.0	16.1	13.38	good	
32	31.6, 32.2, 28.8	30.9	10.95	Moon, good	
33	36.9, 36.5	36.7	9.95	fair	
34	39.0, 39.9, 37.8	38.9	9.50	good	
35	37.4, 37.0	37.2	9.81	Moon, poor	
36	38.9, 37.5	38.2	9.62	Moon, good	
37	36.0, 35.9, 37.2	37.4	9.79	good	

TABLE IV — *Continued.*

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1898		C. S. T.	G. M. T.			
38	Nov.	12	6	4606.5	40	6	*m v, v* 1 *c*
39		19	6	4613.5	40	6	*m* 3 *v, c* 2 *v, v* 1 *d*
					150	6	*c* 4 *v, v* 1 *d*
40		30	6	4624.5	40	6	*m* 5 *v, c* 5 *v, v d*
					150	6	*c* 4 *v, v* 1 *d*
41	Dec.	7	7	4631.5	150	6	*c* 4 *v, v d, v* 3 *e, v g, v* 6 *k*
42		13	6	4637.5	150	6	*d* 5 *v, v* 1 *e, v* 4 *g*
43		30	6	4654.5	150	6	*g* 2-3 *v, v* 1-2 *q*
	1899						
44	Jan.	8	6	4663.5	150	6	*e* 4 *v, v f, v* 2 *g*
45		14	6	4669.5	150	6	*f* 2 *v* ±
46	Feb.	15	17	4702.0	200	6	*q* 4 *v, v* 1-2 *r*
47	Mar.	22	16	4736.9		6	*v* not seen
48	Apr.	16	15	4761.9	150	6	*v* or *u* suspected
49	May	1	9	4776.6	200	6	*v* not seen
50		29	9	4804.6	200	6	*v* not seen
51	June	7	9	4813.6	150	6	*v* not seen
52		24	9	4830.6	150	6	*v* not seen
53		26	9	4832.6	150	6	*v* not seen
54	July	6	10	4842.7	150	6	*v* not seen
55		11	9	4847.6	200	6	*v* not seen
56		29	9	4865.6	150	6	*v* not seen
57	Aug.	9	9	4876.6	150	6	*r* 1-2 *v, v* 1-2 *u*
58		12	9	4879.6	150	6	*r* 2 *v, v* 1 *u*
59		22	8	4889.6	150	6	*q* 2-3 *v, v* 0-1 *r*
60		30	9	4897.6	150	6	*q* 0-1 *v, v* 4 *r*
61	Sept.	4	8	4902.6		6	*g* 2 *v, v* 3 *q*
62		11	8	4909.6	150	6	*v* 2 *g*, d 2-3 *v*
63		26	8	4924.6	40	6	*v* 2 *c, v* 3-4 *m, o* 4 *v*
64	Oct.	6	7	4934.5	40	6	*b* 4 *v, v* 4 *m, v* 3-4 *c, v* 1 *x, o* 1 *v*
65		21	7	4949.5	40	6	*b* 1-2 *v, o* 1 *v, v* 5 *m, v* 4 *x*
66		28	7	4956.5	40	6	*b* 4-5 *v, o* 0-1 *v, v* 2 *m*
67	Nov.	4	6	4963.5	40	6	*o* 2-3 *v, v* 2 *m*
68		15	6	4974.5	40	6	*v m, v* 1 *c, o* 4-5 *v*
69		22	6	4981.5	40	6	*o* 5 *v, m* 3 *v, c* 1-2 *v, v* 3 *d*
70		27	7	4986.5	40	6	*o* 6-8 *v, c* 3-4 *v, m* 3 *v, v d*
					150	6	*c* 2-3 *v, v* 2-3 *d*
71		5	6	4994.5	40	6	*c* 3 *v, v* 2 *d*
					150	6	*c v, v* 3 *d*
72	Dec.	19	6	5008.5	150	6	*d* 2 *v, v* 4 *e, v* 2 *g*
	1900						
73	Jan.	1	6	5021.5	150	6	*g* 1 *v, v* 1 *k, v* 3 *q*
74	Feb.	22	17	5074.0	350	40	*q* 5-6 *v, v* 3 *u*
75	Mar.	22	15	5101.9	350	40	*v u*
76	Apr.	4	15	5114.9	275	12	*v* 4 *u* ±
77		6	16	5116.9	350	40	*u* 6 *v, v* 3-4 *z, zx*
78	May	2	15	5142.6	350	40	*x* 4 *v, z* 3 *v, v* 2 *δ, v* 4 *e*
79		11	13	5151.8	350	40	*z* 5 *v, v* 3 *δ*
80		24	10	5164.7	275	12	*v* not seen

TABLE IV — *Continued.*

No.	Details in Steps	Means		Seeing	Remarks
		Steps	Mag.		
38	37.0, 36.9	37.0	9.85	good	
39	34.0, 33.9, 32.8 33.9, 32.8	33.5	10.50	Moon, good	
40	32.0, 30.9, 31.8 31.9, 32.8	30.2	11.09	good	
41	31.9, 31.8, 32.2, 25.4, 33.0	30.9	10.96	good	
42	26.8, 30.2, 29.4	28.8	11.30	good	
43	22.9, 24.5	23.7	12.19	good	
44	25.2, 28.1, 27.4	26.9	11.62	good	
45		26	11.8	difficult	
46	19.0, 20.5	19.8	12.81	good	
47		<19	13.0		limit *r*
48		<23	12.3		limit *q*
49		<23	12.3	low, good	
50		<19	13.0	good, low	
51		<19	13.0	good, low	
52		<19	13.0	fair	
53		<19	13.0	good	
54		<19	13.0	good	
55		<16	13.4	good	
56		<19	13.0	good	
57	17.5, 17.9	17.7	13.15	good	
58	17.0, 17.4	17.2	13.03	good	limit *u*
59	20.5, 19.5	20.0	12.80	fair, Moon	limit *v*
60	22.5, 23.0	22.8	12.31	fair	limit *r*
61	23.4, 26.0	24.7	12.00		limit 1 < *u*
62	27.4, 28.3	27.9	11.48	Moon, good	
63	37.9, 40.5, 33.6	37.7	9.73	good	
64	40.3, 41.0, 39.4, 36.6, 40.1	39.5	9.37	good	
65	42.8, 36.6, 42.0, 43.1	41.1	9.07	good	
66	39.8, 37.1, 39.0	38.6	9.55	good	
67	35.1, 39.0, 40.9	38.3	9.61	good	
68	37.0, 36.9, 33.1	35.7	10.12	good	
69	32.6, 34.0, 34.4, 34.8	33.9	10.42	good	
70	30.6, 32.4, 34.0, 31.8 33.4, 34.3	32.8	10.61		
71	32.9, 33.8 35.9, 34.8	34.4	10.33	good	
72	29.8, 33.2, 27.4	30.1	11.08	good	
73	24.4, 28.0, 26.0	26.1	11.78	good	
74	17.5, 19.4	18.5	13.00	Moon, poor	
75		16.4	13.34	Moon, fair	
76		12.4	13.97	good	*z* glimpsed
77	10.4, 11.5	11.1	14.18	haze	
78	3.7, 5.0, 3.5, 4.0	4.1	15.18	good	
79	3.0, 4.5	3.8	15.21	fair, Moon	
80		<11	14.2	fair to good	limit 5 < *u*

TABLE IV — *Continued.*

No.	Date				Ocular	Aperture	Comparisions
	Month	Day	Hour	Julian Day			
	1900		C. S. T.	G. M. T.			
81	May	29	12	5169.8	237	40	*u* 5–6 *z*, *z* 5 *v*, *v* δ, *v* 4 ε
82	June	6	10	5177.7	350	40	*v* not seen
83		28	10	5199.7	237	40	*z* 4–5 *v*, *v* 5 ε, *v* 3 δ
84	July	20	10	5221.7	237	40	*z* 1 *v*, *x* 2–3 *v*, *v* 5 δ
85	Aug.	2	9	5234.6	460	40	*v* not seen
86		16	9	5248.6	350	40	*z* 1 *v*, *v* 1 *x*, *v* 4 ε
					350	40	*z v*, *v* 4 δ, *v* 6–8 ε
87		29	12	5261.8	460	40	*u* 6 *v*, *a* 4 *v*, *v* 4 *z*, *v* 3 *x*
88	Oct.	4	9	5297.6	237	40	*v* 5–6 *k*, *v* 3 *g*, *v* 3 *e*, *d* 1 *v*
89		10	7	5303.5	150	6	*v* 4 *e*, *c* 4 *v*
90		13	7	5306.5		6	
91		17	8			12	
92		26	6	5319.5	150	6	*v* 6 *k*, *v* 3 *g*, *v* 4 *e*, *c* 2 *v*
93	Nov.	13	6	5337.5	40	6	*m* 4–5 *v*, *c* 3 *v*, *v* 5 *f*, *v* 6 *g*
94		29	7	5353.5	40	6	*m* 5 *v*, *c* 5 *v*, *v d*, *v* 2 *g*
95	Dec.	11	6	5365.5	40	6	*m* 8 *v*, *d* 4 *v*, *v* 2 *g*
					150	6	*v* 1 *g*, *v* 8–10 *k*
96		29	8	5383.6	150	6	*g* 3 *v*, *v* 2–3 *k*
	1901						
97	July	9	9	5575.6	350	40	*z* 5 *v*, *v x*
98	Nov.	12		5701	350	40	*v* > *n* or *k*
	1902						
99	Mar.	5	14	5814.8	237	40	*u* 2 *v*, *v a*, *v* 6–7 *z*, *g* 4 *e*
100		28	15	5837.9	237	40	*z* 2–3 *v*
101	May	14	13	5884.8		12	*v* < *n* or *k*
102	Oct.	5	10	6028.7		12	
						12	*v* 4–5 *u*
103	Nov.	30	7	6084.5		24	
104	Dec.	26	7	6110.5	237	40	*v* d ±
	1903						
105	Mar.	27	16	6201.9	237	40	*n* 6–8 *v*, *v* 5 η, *v* 3–4 *u*
106	Apr.	3	16	6208.9	237	40	*u* 1–2 *v*

In Chandler's *Third Catalogue* the provisional elements of maximum from observations of 1893–1895, are given as—

$$\text{Maximum} = 1893 \text{ August } 24 \text{ (J. D. } 2412700) + 377\ E.$$

These elements fairly represent the above maxima, but the agreement is somewhat improved by placing the zero epoch thirteen days earlier, giving the elements—

$$\text{Maximum} = 1893 \text{ August } 11 \text{ (J. D. } 2412687) + 377\ E.$$

The agreement with these elements is shown by the following table:

TABLE IV — *Continued*

	Details in Steps	Means		Seeing	Remarks
		Steps	Mag.		
81	3.0, 1.5, 4.0	2.8	15.37	fair to good	
82		<16	13.4	haze	limit *u*
83	3.5, 4.0, 4.5	4.0	15.20	good	
84	7.0, 5.2, 6.5	6.2	14.91	fine	
85					
86	7.0, 8.7, 4.0 8.0, 5.5, 7.0	6.7	14.86	fair better	
87	10.4, 11.3, 12.0, 10.7	11.1	14.17	good	
88	32.5, 28.4, 32.2, 30.8	31.0	10.94		*x* and *z* seen
89	33.2, 31.9	32.6	10.68	good	
90			10.45	good	photom., 6-in.
91			10.31	good	photom., 12-in.
92	33.0, 28.4, 33.2, 33.9	32.1	10.75	good	
93	32.5, 32.9, 33.1, 31.4	32.5	10.69	good	
94	32.0, 30.9, 31.8, 27.4	30.5	11.00	Moon	
95	29.0, 27.9, 27.4 26.4 (36.0)	27.7	11.52	good	
96	22.4, 29.0	25.7	11.83	Moon	
97	3.0, 7.7	5.4	15.00	fair	*v* & *x* near limit
98		<26	<11.8		
99	14.4, 14.5	14.5	13.68	fair	
100		5.5	15.00	Moon	
101					*v* is < *n* or *k*
102		20.9	<12 12.76		photometer
103			10.9		photograph
104		31.8	10.08	good	
105	19.0, 19.9, 19.9	19.6	12.87	good	
106		14.9	13.6	clouds	

Maxima					Minima				
Epoch	Cal.	Obs.		O-C.	Epoch	Cal.	Obs.		O-C.
	J. D.	J. D.	Cal.			J. D.	J. D.	Cal.	
3	3818	3845	1896, Oct. 12	−27	3	3677	....	.....	....
4	4195	4220	1897, Oct. 22	−25	4	4044	4070	1897, May 25	−26
5	4572	4575	1898, Oct. 12	− 3	5	4421	4433	1898, May 23	−12
6	4949	4940	1899, Oct. 12	− 9	6	4798	4808	1899, June 2	−10
7	5326	5315	1900, Oct. 22	− 9	7	5175	5175	1900, June 4	0
8	5703	....	.....	....	8	5552	5545	1901, June 9	− 7
9	6080	6080	1902, Nov. 26	0	9	5929	....	.....	....

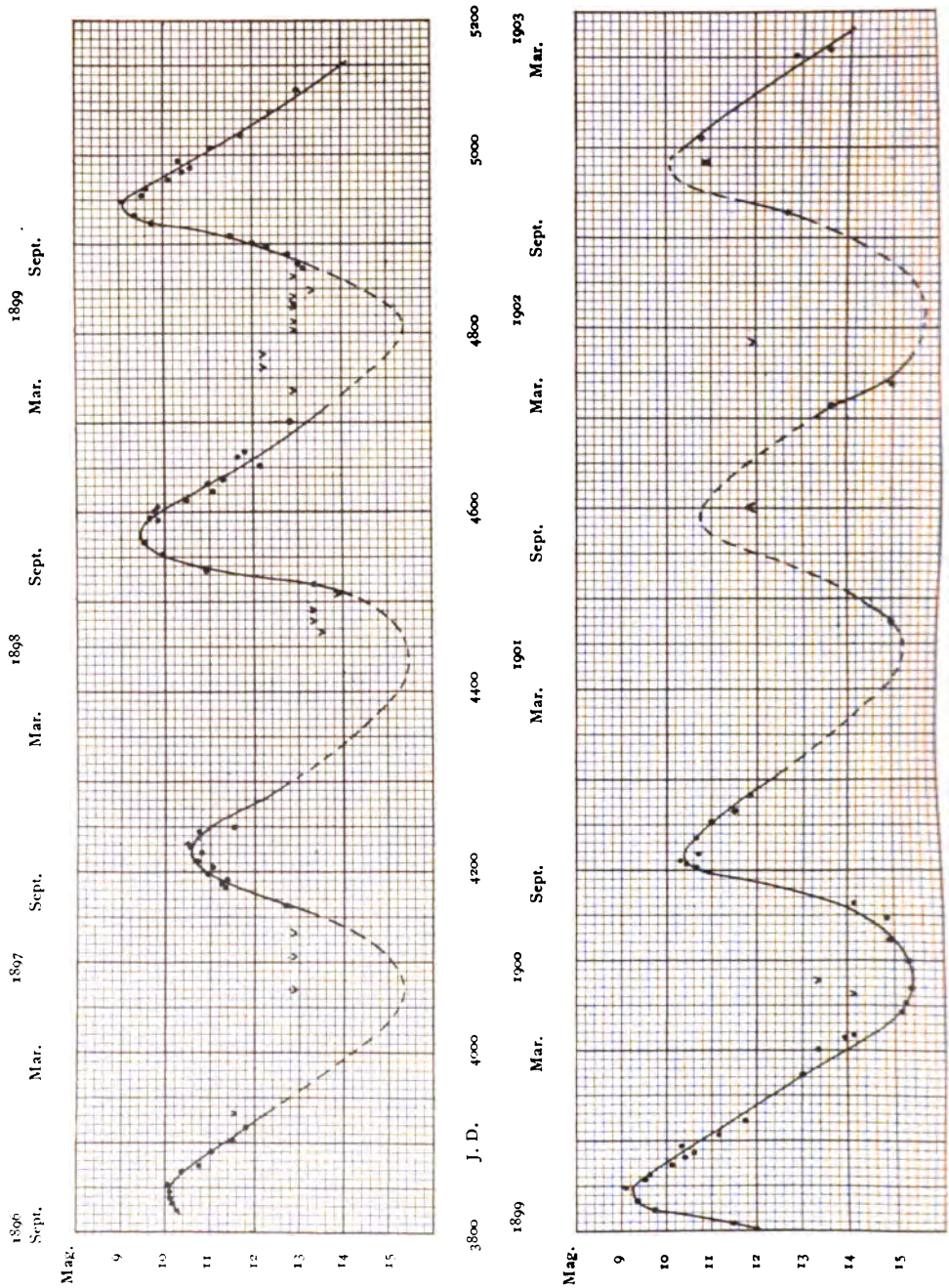
1896 Sept.
1897 Mar.
Sept.
1898 Mar.
Sept.
1899 Mar.
Sept.
Mag.
9
10
11
12
13
14
15
3800
4000
4200
4400
4600
4800
5000
5200
J. D.
1899 Mar.
1900 Sept.
1901 Mar.
Sept.
1902 Mar.
Sept.
1903 Mar.

FIG. 2.

The large residuals of epochs 3 and 4 require mention. The observations began too late to fix maximum 3 with precision. It doubtless took place about twenty days earlier; in fact, the calculated date, J. D. 3818, is the mean of the dates given by Yendell and H. M. Parkhurst (Max. A). For epoch 4 the curve at maximum is not of the usual shape, the difference accounting for something like twenty days, and in the right direction to improve the agreement with the ephemeris. The residuals for the remaining maxima are within the errors of observations.

I wish to acknowledge the assistance rendered, as in the work on *X Cephei*, to Miss Bloodgood, and Messrs. Ellerman and Sullivan.

YERKES OBSERVATORY,
April 1903

PECULIARITIES AND CHANGES OF FRAUNHOFER LINES INTERPRETED AS CONSEQUENCES OF ANOMALOUS DISPERSION OF SUNLIGHT IN THE CORONA.[1]

By W. H. JULIUS.

ATTENTION has been drawn to several variable peculiarities of Fraunhofer lines, mainly by Jewell's investigations on the coincidence of solar and metallic lines.[2] Here we do not mean the irregularities occurring in the spectrum of spots or of faculæ, which relate to disturbances in comparatively small parts of the Sun, but abnormalities shown by average sunlight, as observed when the slit is illuminated by a long strip of an imperfectly focused solar image. In that case, according to Doppler's principle, we may, of course, expect displacements of the lines in consequence of the Sun's rotation, the rotation of the Earth, and the change in the distance between Sun and Earth caused by the eccentricity of the Earth's orbit. But even when all these influences have been allowed for, some irregularities still remain.

Indeed, Jewell has observed that while some Fraunhofer lines exactly coincide with the emission lines in the arc spectrum of elements, others do not, and that the displacements are unequal both for lines of different elements and for the various lines of one and the same element. Moreover, the shifting of certain lines on one set of photographic plates was sometimes found to be different from that on a set of plates taken at another time. With several lines the intensity also appeared to be variable.

Jewell explains these phenomena on certain hypotheses as to

[1] Communicated to the Royal Academy of Sciences, Amsterdam, at the meeting of February 28, 1903.

[2] L. E. JEWELL, "The Coincidence of Solar and Metallic Lines: A Study of the Appearance of Lines in the Spectra of the Electric Arc and the Sun," ASTROPHYSICAL JOURNAL, 3, 89–113, 1896; "Spectroscopic Notes: Absolute Wave-Lengths, Spectroscopic Determinations of Motions in the Line of Sight, and Other Related Subjects," *ibid.*, 11, 234–240, 1900.

density, pressure, and temperature of the absorbing and emitting gases in the different layers of the solar atmosphere, and by variable ascending and descending velocities of matter.

HALE'S ABNORMAL SOLAR SPECTRUM.

Much greater irregularities than those mentioned are found in an "abnormal" solar spectrum, lately described by G. E. Hale.[1]

This highly remarkable spectrum was accidentally photographed as long ago as February, 1894, in a series of exposures, made with the sole intention of investigating the peculiarities of the grating. Only a few months later it was discovered that a very extraordinary phenomenon had been photographed. Hale hesitated to publish this accidental discovery. Copies of the plate were sent to several spectroscopists for examination, with the request that an explanation referring the phenomenon to some origin other than solar be supplied, if possible. As no such explanation was forthcoming, the spectra were very carefully measured and described.

On one and the same plate twelve exposures had been successfully made in the third-order spectrum of a plane grating. A solar image 51 mm. in diameter was so adjusted that the image of a spot fell exactly on the slit. The length of the slit (6.5 mm) corresponded to about one-eighth of the Sun's diameter.

The first exposures show the normal spectrum without any considerable changes. Then came the disturbance which culminated in the eighth spectrum, and in the following four decreased rapidly. Hale gives reproductions of four spectra, each of them extending from $\lambda 3812$ to $\lambda 4132$. No. 1 was taken before the disturbance occurred; No. 2 is the most abnormal spectrum; No. 3 is called by Hale the "intermediate" spectrum; it was obtained a few moments after the abnormal one; No. 4 shows once more the normal solar spectrum, as it was photographed at another time on another plate. Nos. 1, 2, and 3

[1] "Solar Research at the Yerkes Observatory," ASTROPHYSICAL JOURNAL, 16, 211–233, 1902.

show a dark band throughout the whole spectrum, corresponding to the Sun-spot which had been focused on the slit.

The most prominent features of the abnormal spectrum are:

1. The band due to the spot appears much fainter than in the spectra photographed before and after the disturbance.

2. In the case of several Fraunhofer lines the intensity or the width is *greatly diminished.* This is most conspicuous with the broad, dark calcium bands H and K and with the hydrogen line $H\delta$, these being almost totally absent in the abnormal spectrum.

3. Other Fraunhofer lines, on the contrary, appear *uncommonly strengthened.*

4. Many lines are more or less displaced.

The same peculiarities are noticed, though generally in a lesser degree, in the intermediate spectrum, so that the latter, in fact, forms a link between the abnormal and the normal spectrum.

This marvelously complicated disturbance was not confined to light coming from a comparatively small part of the solar disk, for instance from the immediate surroundings of a spot; on the contrary, it extended almost equally over the whole width of the spectrum and was therefore nearly the same for all the light which came from a very great area of the Sun.

The moments of the twelve exposures and the exact date had not been recorded; but there was sufficient evidence that the whole process of the disturbance lasted only a very short time. Hale calls the phenomenon "a remarkable disturbance of the reversing layer." But is it not almost impossible to imagine a rather thin layer in the solar atmosphere undergoing suddenly and simultaneously over a great part of the Sun a change so great as to make its absorbing and radiating power in some parts of the spectrum for a while nearly unrecognizable?

It occurred to me, therefore, that the origin of the phenomenon should be looked for somewhere on the path of the light between the Sun and the Earth. If on this path there be media causing anomalous dispersion, the beam must show an altered composition.

As I formerly indicated,[1] the properties of the chromospheric light may be derived from the supposition that this light has been separated from the photospheric light by anomalous dispersion. According to this hypothesis the spectrum of the chromosphere informs us which are the kinds of light that may follow rather strongly curved paths in the solar atmosphere. So the idea suggested itself that the same waves might play a striking part in Hale's abnormal spectrum.

In order to investigate the question as impartially as possible, I marked (before consulting Hale's table or a table of chromospheric lines) on the reproductions of the spectra in the ASTROPHYSICAL JOURNAL a number of lines which struck me as being *weakened* in the abnormal spectrum. By means of George Higgs' photographic atlas of the normal solar spectrum the wave-lengths of the selected lines were easily read. They are to be found in the first column of Table I.

The second, third, and fourth columns show the intensities of these lines in the normal, the intermediate, and the abnormal spectrum as given by Hale (for the normal spectrum from Rowland's tables, for the other two from estimates by Mr. Adams). Hale remarks that the intensities of the lines were estimated independently for the two disturbed spectra.[2] The fifth column indicates the intensities of corresponding chromospheric lines as found by Lockyer in the spectrum secured at Visiadrug[3] during the 1898 eclipse; the sixth column shows the absorbing substances.

Table II has been prepared in a similar way; here we find the lines, which on the reproduction appeared to be *strengthened* in the abnormal spectrum.

[1] ASTROPHYSICAL JOURNAL, 12, 185–200; 15, 28–37; *Physikalische Zeitschrift*, 4, 85–90; 132–136.

[2] In selecting the lines that appeared weakened in the abnormal spectrum, I of course compared the three spectra together. That is why in my table some lines occur whose intensities as estimated by Mr. Adams are not comparatively low in the abnormal spectrum.

[3] LOCKYER, CHRISHOLM-BATTEN, AND PEDLER, "Total Eclipse of the Sun, January 22, 1898.—Observations at Visiadrug," *Phil. Trans.*, A, 197, 151–227, 1901.

TABLE I.

LINES WHOSE INTENSITY IS LESS IN THE ABNORMAL THAN IN THE NORMAL SPECTRUM.

	Wave-Length	Intensity				Elements	Remarks
		Normal (Rowland)	Intermediate (Hale)	Abnormal (Hale)	Chromosphere (Lockyer)		
	3871.4				4	*C*	Not mentioned in Hale's list, but distinctly weakened in the abnormal spectrum on reproduction.
	3872.6				4	*Fe*	
	3874.09	4	9	..	2(?)	*Fe*	
	3878.47	22	25	..	3-3	*Fe, Fe*	$\lambda = 3878.15$ and $\lambda = 3878.72$. Hale mentions *Fe, Mn*.
H ζ	3889.05	?	15	..	8	*H*	
	3895.80	7	12	..	3	*Fe*	
	3899.30	5	4	..	2	*V*?	
	3903.09	10	12	..	2-3	*Fe*	
	3905.66	12	20	..	2	*Cr, Si*	
	3906.70	14	..	4	2	*Fe*	
	3913.63	9	7	..	6	*Ti*	
	3914.49	7	8	5*		*Ti*	
	3916.54	3	..	4*	3	*V*	* These intensities are very probably estimated too high when compared with the numbers in the second column. *Cf.* note 2 on p. 53.
	3920.41	10	10*	10*	3	*Fe*	
	3923.05	12	12*	12*	3	*Fe*	
K	3933.82				10	*Ca*	
	3944.16	15	15*	12*	5	*Al*	
	3948.91	13	15	..	3	*Fe*	
	3950.10	5	..	2	3	*Fe*	
	3953.02	17	15	..		*Fe*, etc.	
	3958.35	5	8	..	4	*Ti*	
	3961.67	20	20	..	6	*Al*	
H	3968.63	(700)	7	7	10	*Ca*	
H ε	3970.18	7	8	..	10	*H*	
	3977.89	6	8	..	2	*Fe*	
	3986.90	6	8	..		....	
	3998.78	4	4	4*	4	*Ti*	
	4012.50	5	4	5*	5-6	*Ti*, etc.	
	4033.22	7	12	3	3-4	*Mn, Fe*	
	4034.64	6	10	..	3-4	*Mn, Fe*	
	4045.98	30	30	5	7	*Fe*	
	4063.76	20	20	..	6-7	*Fe*	
	4071.91	15	15	15*	6	*Fe*	
	4077.88	8	10	7*	10	*Sr*	
H δ	4102.00	40	7	..	10	*H*	

The result is very striking. *Weakened lines correspond to chromospheric lines almost without exception; most of the strengthened lines, on the other hand, are not to be found in the spectrum of the chromosphere.*

Lockyer gives the strength of the chromospheric lines on a scale such that 10 indicates the strongest and 1 the faintest lines. If we take into account that in his list the greater part of

TABLE II.

LINES WHOSE INTENSITY IS GREATER IN THE ABNORMAL THAN IN THE NORMAL SPECTRUM.

Wave-Length	Intensity				Elements	Remarks
	Normal (Rowland)	Intermediate (Hale)	Abnormal (Hale)	Chromosphere (Lockyer)		
3921.86	4	..	20		*Zr, Mn*	
3927.77	..	..	25		*?*	
3930.45	8	15	28	3-4	*Fe*	
3937.39	..	..	10		*?*	
3940.25	..	7	12		*?*	
3950.50	2	..	13		*Y*	
3962.29	3	..	11		*Fe?*	
3973.77	6	..	15	2 (?)	*NiZrFeCa*	
3981.92	4	13	30	6*	*Ti, Fe*	*In Humphreys' table of chromospheric lines (1901 eclipse) this line does *not* occur.
3992.97	3	4	10		*V, Cr*	
3996.80	..	..	9		*?*	
4013.90	8	12	15		*Ti, Fe*	
4014.67	5	9	20		*Fe*	
4023.38	..	..	10		*?*	
4033.77	2	3	15		*Mn*	
4040.79	3	6	20	4	*Fe*	
4044.09	5	20	15		*Fe*	

the lines bear the numbers 1 and 2, our table shows us that by merely observing the abnormal solar spectrum we have been able to pick out *strong* chromospheric lines. This cannot be chance. Undoubtedly both phenomena—the weakening of Fraunhofer lines in the abnormal spectrum and the origin of the chromospheric spectrum—are to be explained in close relationship with each other.

The *strengthening* of lines in the abnormal spectrum does not, on the contrary, seem to be so directly connected with the composition of the chromospheric spectrum.

If our view be correct that the chromospheric light has been separated by strong ray-curving from the "white" light emitted by deeper layers, those special radiations must, as a rule, show reduced intensity in the spectrum of the Sun's disk.[1] Fraun-

[1] It might be thought that the rays forming the chromospheric light need to be absent only from the spectrum of the *edge*, but not from that of the central portions of the Sun's disk. By a simple consideration following from a glance at Fig. 4 of my paper read in February, 1900 (ASTROPHYSICAL JOURNAL, 12. 191), we see, however, that the chromospheric light visible to us may very well, in part, have its origin even

hofer lines corresponding to chromospheric lines will therefore have a more or less darkened background in the ordinary solar spectrum. The rate of darkening at various distances from the center of an absorption line is, of course, connected with the shape of the dispersion curve near that line; whereas the average shading depends also (1) on the quantity of matter causing anomalous dispersion and (2) on the slopes and directions of the density gradients in the gases through which the light is transmitted, viz., on the Sun's "activity."[1]

We distinguish, therefore, a twofold origin of the dark lines in the solar spectrum, viz.: real *absorption* of those waves which exactly correspond to the periods of the media and *dispersion* of the strongly deviated[2] neighboring light.

The dispersion will be especially evident where extraordinary differences in the density of the medium occur; in this way the widening of most of the Fraunhofer lines in the spectra of spots may be accounted for.

Dispersed light has not, of course, vanished; the absence of certain rays in the spectrum of a spot is counterbalanced by the increased intensity of the same radiations in the light coming from the neighboring faculæ. Thus the distribution of the density in the solar gases may locally be such that a limited part of the disk seems to emit a considerable amount of rays with abnormally high or abnormally low refractive indices. In the spectrum of this part not only will the Fraunhofer lines appear

in points of the Sun which lie opposite to the Earth's direction. The chromospheric light reaching the Earth may proceed from *any* point of Schmidt's "critical sphere." For the greater part it is likely to come from the back half of the Sun. But then the half facing us furnishes the chromospheric light which travels to other regions of the universe, and this light, of course, is wanting in the spectrum of the disk. There is some reason for supposing that, on an average, more chromospheric light is sent forth in directions making great angles with the Sun's equator than to the equatorial regions, including the Earth's orbit.

[1] The possible influence of the general or regular ray-curving (on Schmidt's principle) on the appearance of the spectral lines has in the present paper been left out of consideration. If we were able to observe or to calculate the radii of the "critical spheres" for radiations undergoing anomalous refraction, it would be possible to estimate that influence; but as yet sufficient data are wanting.

[2] ASTROPHYSICAL JOURNAL, 12, 191, 1900.

narrower and fainter than usually, but we may even meet with lines contrasting *brightly* with their surroundings. These bright lines will not coincide with the corresponding absorption lines; their average wave-length will in general be greater or smaller than that of the absorbed light, for, according to the accidental distribution of the density, we shall find either the rays with high or those with low refractive indices most prominent in the beam.

The above considerations suggest an explanation of Hale's abnormal spectrum. In fact, the lines showing especially faint in this spectrum were exactly those which cause strong anomalous dispersion—witness the chromospheric spectrum. With H, K, $H\delta$, and some iron lines it is very evident that the abnormal faintness refers mainly to the broad dark shadings of the lines, *i. e.*, those parts whose darkness in the normal spectrum we attributed, not to absorption, but to dispersion. Moreover, the dark band due to the spot has nearly disappeared. This means that waves, which in normal circumstances are wanting in the spot spectrum on account of their strong dispersion, at the time of the disturbance had been gathered again into the beam, reaching the instrument. How all this may happen will become evident as soon as we shall be able to establish a plausible cause by which, within an angular space great enough to include a considerable part of the solar disk, the *strongly dispersed rays might be gathered again.*

It is not necessary to introduce a new hypothesis for the purpose. The same idea about the Sun's constitution which enabled us to explain the properties of the chromosphere and the prominences[1] furnishes us once more with the required data. Indeed, if (according to Schmidt's theory) the Sun is an unlimited mass of gas, surfaces of discontinuity must exist similar to those whose general feature has been determined by Emden[2] for a sharply outlined radiating and rotating Sun. These surfaces must extend to the remotest parts of the gaseous body—a conclusion in excellent harmony with the visible structure of the corona. For along the surfaces of discontinuity waves and whirls are formed; the core-lines of the vortices nearly coincide

[1] *Physikalische Zeitschrift*, 4, 85–90. [2] ASTROPHYSICAL JOURNAL, 15, 38–59, 1901.

with the generatrices of the surfaces of revolution, and in these cores the density is a minimum. This may account for the streaky appearance shown more or less distinctly in all good photographs and drawings of the corona.

This particular appearance may have another cause, however, but for what follows this is immaterial. We only assume that the density of the coronal matter varies in such a way as to correspond to the striped structure visible at the time of a total eclipse of the Sun.

A coronal streamer which at a given moment runs exactly in the direction of the Earth may be very roughly compared, then, to a bundle of glass tubes through which we are looking lengthwise. Such a structure will gather and conduct rays of various directions, entering it at one end. This takes place also if the parts with the greater and those with the smaller optical density do not alternate abruptly, but gradually.

In Fig. 1 the optical density of the matter may be represented by the compactness of the streaking. A ray for which the medium has a large positive refraction constant would, for instance, follow the path AA', curving round the denser parts of the structure; a ray BB', for which the medium possesses a large negative refraction constant, would move in a similar way through the more rarefied regions. On the other hand, the light CC' for which the constant exactly equals zero is not influenced by the fluctuations of the density; and if for some kind of light the refraction constant is very nearly zero, the ray would have to travel a long way almost parallel to the structure before its curving would be perceptible.

Now the corona sometimes shows exceedingly long, pointed streamers. We only have to suppose *that the Earth was exactly in the direction of such a streamer at the moment the abnormal spectrum was photographed;* then all the irregularities observed in this spectrum become clear. Light under normal circumstances absent from the solar spectrum through strong dispersion has been collected by the coronal streamer; hence the *weakening* of the Fraunhofer lines, especially, also, of those in the spectrum of the spot. As the abnormalities were caused by a peculiar

distribution of matter in the vast regions of the corona lying between the source of light and the Earth (and not by disturbances in a relatively thin "reversing layer"), they could appear in the same way over a great part of the Sun's disk. The *rarity* of the phenomenon is the result of the slight chance we have to take a photograph at the very moment on which an uncommonly long coronal streamer is projected exactly on the part of the Sun's disk illuminating the slit; finally, the *short duration* is a consequence of the Earth's orbital motion, and probably of the rotation of the corona.

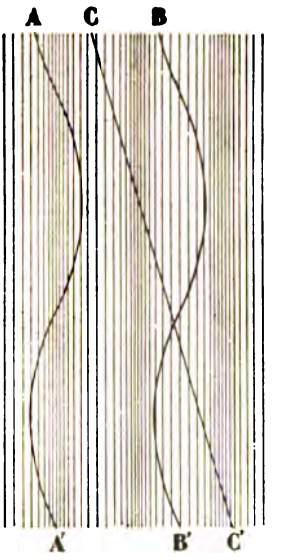

FIG. 1.

As we have mentioned before, *no* chromospheric lines correspond, in general, to those lines appearing extraordinarily *strong* in the abnormal spectrum. How are we to account for the strengthening of these lines?

We might be tempted to think of absorption in the corona; for if it be true that a streamer was turned toward the Earth, the rays had to go an uncommonly long way through an absorbing medium. But on closer examination this idea is less probable.

The particles of the extremely rarefied coronal gases will hardly influence each other; their periods will, therefore, be almost absolutely constant, so as to cause very sharp, narrow absorption lines. Thus it is difficult to understand how an absorption line already present in the normal solar spectrum might be strengthened by the absorbing power of the corona. Further, in studying Hale's table, we observe that many lines which are strong in the abnormal spectrum show a much smaller intensity in the intermediate spectrum (taken only a few moments later); while the reverse happens as well, viz., that lines are strong in the intermediate and very weak in the abnormal spectrum. This hardly fits in with the absorption hypothesis. Some lines showing this peculiarity are given in Table III:

TABLE III.

LINES WHOSE INTENSITY IS VERY DIFFERENT IN THE INTERMEDIATE AND THE ABNORMAL SPECTRUM.

Wave-Length	Intensity				Elements	Remarks
	Normal (Rowland)	Intermediate (Hale)	Abnormal (Hale)	Chromosphere (Lockyer)		
3905.66	12	20	..	2	*Cr, Si*	
3905.81	21	..	20		*Si*	
3921.71	9	14	..		*Ti, La, Zr, Mn*	
3921.87	4	..	20		*Zr, Mn*	
3950 33	..	10	..		*?*	
3950.51	2	..	13		*Y*	
3972.30	2	12	..		*Ni*	
3972.61	2	..	12		*?*	
4005.86	3	25	5		*?*	
4057.39	4	..	15	1–2	*Co, Fe*	
4957.66	7	10	..		*?*	

In the chromospheric spectrum corresponding lines seem to be wanting (at λ 3905.66 and λ 4057.39 the faint chromospheric line may possibly belong to another element than the abnormally strengthened absorption line).

To arrive at a more satisfactory explanation of the strengthening phenomenon, we suppose that these absorption lines do indeed cause anomalous dispersion of neighboring waves, but in a very slight degree. Then, the refractive indices of the neighboring waves differing but little from unity, the direction of those rays will be perceptibly changed only after they have traveled a very long way through the corona and almost parallel to its structure lines. Whereas the strongly refracted rays, entering the coronal streamer in various directions, were obliged to follow the structure lines, curving about them, and so in a sense were concentrated on the Earth, it may happen with the very slightly curved rays we are now considering that they have been bent, for instance, only once over the whole length of the streamer and continue their way in a direction not meeting the observing station. The divergence of a beam consisting of these rays will have increased, the intensity diminished. Thus the resultant spreading of neighboring light causes the absorption

line to appear somewbat widened, and therefore strengthened. But obviously it must be possible, too, that after a short time, under the influence of another part of the corona, circumstances assist that slightly curved light to reach the observer. In that case the absorption line is weak again. (Similar alternations, of course, also occur with the more strongly refracted rays, and that in quicker succession; but this does not alter the fact of their *average* intensity appearing increased as long as the structure lines of the coronal streamer are turned toward the spectroscope. For a detailed discussion of this case see the note at the end of this paper.)

In both abnormal spectra a number of absorption lines are more or less displaced. Perhaps this is partly due to motion in the line of sight, but after the foregoing it will not be necessary to explain in detail that anomalous dispersion also can account for this phenomenon. Dissymmetric form of the dispersion curve as well as a peculiar distribution of the density of the coronal matter may unequally affect the intensity of the light on both sides of the absorption line, and thus bring about a seeming displacement of the line.

CERTAIN PECULIARITIES OF LINES IN THE NORMAL SOLAR SPECTRUM.

If we have been right in connecting the uncommonly great abnormalities in Hale's spectrum with a very particular position of the Earth with respect to the corona, it is to be expected that similar irregularities, though in less degree, will continually be found, as the sunlight always reaches us through the corona.

According to Jewell's above-mentioned investigations, this supposition proves to be well founded. Many solar lines have varying intensities and positions, so that Jewell deems them unfit for standards for very accurate determinations of wave-lengths. And these are for the greater part the most prominent lines of the spectrum, especially the shaded ones.[1]

Jewell emphasizes the fact that all distinctly shaded lines in the solar spectrum show to a greater or less degree the following

[1] ASTROPHYSICAL JOURNAL, 11, 236, 1900.

typical feature:[1] With a broad, shaded, moderately dark background a much darker central absorption line contrasts rather sharply (Fig. 2). Besides, the absorption curve often shows depressions close to the central line, as in Fig. 3, sometimes symmetrical, sometimes dissymmetrical. Jewell affirms that this is not an optical delusion, due to contrast, but a real phenomenon. He assumes, therefore, that the broad absorption band is produced in the lower portions of the solar atmosphere and under a great range of pressure; that in higher levels radiation prevails again, producing a rather wide emission line; and that finally in the highest parts, where the pressure is very much less, the sharp absorption line is produced. The position of this central absorption line with respect to the emission line is usually unsymmetrical, which is conspicuous in the case of H and K. The central line itself also varies somewhat in width upon different plates, and its maximum of intensity is not always in the middle of the line. The displacement of this central line in H and K varies in magnitude, but, so far as has been observed, always toward the red with respect to the emission line and the corresponding metallic line (in the arc). Jewell concludes that the absorbing calcium vapor descends all over the solar surface with a velocity sometimes amounting to about seventy-five miles per minute.

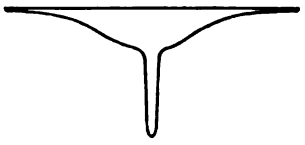

Fig. 2

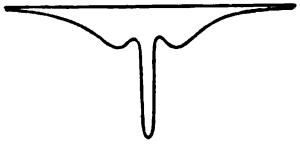

Fig. 3.

Upon the same plates showing strong dissymmetry in H and K the shaded lines of other elements (*Fe*, *Al*, *Mg*, *Si*) have been examined. The strongest iron lines and one aluminum line showed displacements of the same character as that observed in the case of H and K, but to a much smaller degree, and sometimes toward the violet, sometimes toward the red. Certain

[1] "Certain Peculiarities in the Appearance of Lines in the Solar Spectrum and Their Interpretation," *Ibid.*, 3, 1896.

shaded lines of *Mg* and *Si*, on the contrary, showed no evidence of a displacement, nor did the iron lines without considerable shading, the faint calcium line at λ 3949,056, and many other lines.

If we admit no other explanation of line-shifting and widening besides those based on Doppler's principle and on the effect of pressure and temperature, we arrive at very strange conclusions with regard to the condition of the elements in the solar atmosphere. Not less surprising, as noticed by Jewell,[1] is the small amount of the absorption in the shaded parts of the lines, when we consider the enormous depth of the solar atmosphere and the high pressure which must exist in the absorbing layers for them to produce a broad absorption band.

By making various suppositions concerning the condition of the gases in the solar atmosphere, Jewell succeeds in finding an interpretation of most of these astonishing facts. But it must be granted that his explanations include a greater number of arbitrary and mutually independent hypotheses than is the case with our explanations, founded as they are on selective ray-curving and readily deduced from that principle for each separate phenomenon, without introducing new suppositions.

Only the dark central lines of the Fraunhofer lines are to be ascribed, in our theory, to real absorption. Their shaded background of varying intensity we consider as an effect of anomalous dispersion of the not absorbed neighboring waves. This selective scattering will be strongest in those places where the density-gradients are relatively steep, viz., in whirls in the deeper regions of the gaseous body. But some of the widely dispersed rays may be gathered by the corona owing to its "tubular" structure, and be conducted along its greater or smaller streamers. This will especially apply to the most strongly refracted waves, whose position in the spectrum is very close to the real absorption lines; thus pseudo-emission lines are produced in about the middle of the pseudo-absorption bands.[2]

[1] ASTROPHYSICAL JOURNAL, 3, 106, 1896.

[2] A most remarkable fact is that the shading of H, K, the iron line λ 3720.086 and of some other strong shaded lines is sometimes partially broken up in a series of faint

Probably Hale's abnormal spectrum has shown us a case where these seeming emission bands acquired an uncommon extent. We may therefore expect that a systematical investigation of solar spectra, photographed at different times, will afford all kinds of intermediate cases.

It would be desirable, for the moments when the photographs are taken, to know the form and position of the coronal streamers directed toward the Earth. At all events the actual phase of the Sun-spot period, with which the shape of the corona seems to be connected, should be taken into consideration ; and perhaps the simultaneous observation of the photospheric reticulation discovered by Janssen may procure some evidence concerning the position of coronal streamers, and thus contribute to our knowledge of their influence on the Fraunhofer spectrum.

UTRECHT, April 7, 1903.

nebulous lines, symmetrically situated about the central line (JEWELL, ASTROPHYSICAL Journal, 3, 108, 1896). It might have been predicted by our theory that we should meet with this phenomenon. Let us consider a beam of light of an exactly defined wave-length belonging to the shaded background of an absorption line. This beam leaves the deeper layers of the Sun with a certain divergence. As it passes along a "tube" of the corona, its divergence will alternately diminish and increase, and on reaching the Earth it shows in the spectrum an intensity depending on the divergence (or, perhaps, convergence) with which it has left the last traces of the corona. For a beam of light whose wave-length is only slightly nearer to that of the absorption line, the medium will have a considerably greater refraction-constant, so that the rays of this beam on their way through the corona may make part of a bend more than the former ones. The beam may therefore arrive with a quite different degree of divergence, and, consequently, of intensity. Thus, proceeding toward the absorption line from either side, we easily see that we must meet with a periodically changing intensity. Rays corresponding to the middle of one of the fringes so formed will have made one full bend more or less than the rays belonging to the middle of the next fringes.

If this interpretation be correct, the width and the number of fringes visible must prove to be variable. So far as I know, the observations made on this point are not numerous. I trust that the proposed views may serve to further the investigation of this interesting phenomenon.

THE WAVE-LENGTHS OF THE SILICON LINES λ4128 AND λ4131 AND OF THE CARBON LINE λ4267.

By J. HARTMANN.

SUPPLEMENTING my recent determination[1] of the wave-length of the magnesium line λ4481, I should like to communicate my measurements on three other lines which, like the magnesium line, occur as well-measurable lines in stellar spectra of the first type, although hitherto their very diffuse character has not permitted an accurate laboratory determination of their wave lengths.

The lines at λ4128 and λ4131 are among the strongest in the spark spectrum of silicon. I will here mention only the following of the earlier determinations of their wave-lengths:

Eder and Valenta (*Denkschriften d. Wiener Akad.*,[2] **60**, 260, 1893) - - - - - - - - - -	4128.5	4131.5
Lockyer (*Proc. R. S.*, **61**, 443, 1897) - - - - -	4128.6	4131.4
Exner and Haschek (ASTROPHYSICAL JOURNAL, **12**, 49, 1900) - - - - - - - - - - -	4128.1	4131.0

It may readily be seen from the great difference in the above values how uncertain the wave-lengths of these lines have been hitherto on account of their diffuseness. Like the magnesium line, however, they may be obtained as perfectly sharp lines if the pressure of the luminous vapor is greatly reduced. In collaboration with Dr. Eberhard I have accordingly made with Spectrograph III a number of photographs of the spectrum of Geissler tubes containing silicon tetrafluoride under low pressure. Basing the determinations upon Kayser's wave-length of the three iron lines at λ4118.709, 4132.217,[3] and 4144.033, I obtained the following wave-length of the silicon lines:

Plate III: 528, 530 - - - -	4128.205	4131.038
531 - - - -	.201	.036
532 - - - -	.207	.042
533 - - - -	.203	.042
Mean - - -	4128.204	4131.040

[1] *Physikalische Zeitschrift*, **4**, 427, 1903.

[2] In the original the wave-length is given by mistake as 4126.5.

[3] Communicated by Professor Kayser in a letter.

The line at 4267 appears very strong and diffuse in the spark spectrum of carbon. The most important earlier determinations of its wave-length are as follows:

Angström and Thalén (*Nova Act. Soc. Upsala*, 1875)	4266.0
Hartley and Adeney (*Phil. Trans.*, 1884)	4266.3
Eder and Valenta (*Denkschriften d. Wiener Akad.*, 1893)	4267.5
Deslandres (*Comptes rendus*, 1895)	4267.0
Exner and Haschek (*Sitzungsberichte d. Wiener Akad.*, 1897)	4267.10

This wave-length was accordingly hitherto quite inaccurate.

The line may be obtained sharp, if powerful spark discharges (with condenser and spark-gap) are passed through a Geissler tube containing a carbon compound at low pressure. In this way, using a cyanogen tube, I made the following two determinations of the wave-length of this line some time ago.

Plate III:	495	4267.304
	496	.298
	Mean	4267.301

This value is also referred to Kayser's standards.

Even in a vacuum the line always remains rather broad, and it gives the impression of duplicity, the components not being resolvable with the dispersion employed. The determination of the wave-length is therefore somewhat uncertain.

POTSDAM,
April 13, 1903.

SOME MISCELLANEOUS RADIAL VELOCITY DETERMINATIONS WITH THE BRUCE SPECTROGRAPH.

By WALTER S. ADAMS.

THE RADIAL VELOCITIES OF THE TWO COMPONENTS OF 61 *CYGNI*.

THE recent adaptation of the Bruce spectrograph for use with one prism has made possible a determination of the radial velocities of the two stars composing 61 *Cygni* which is of interest because of its important bearing upon the question of the physical connection and relative parallax of this celebrated pair. The spectrograms obtained are as follows:

61 [1] *CYGNI*.

Plate	Date	No. of Lines	Velocity
B 387	1902, August 11	17	−63 km
DB[1] 25	1903, May 17	13	61
DB[1] 36	June 6	17	63
		Mean	−62 km

62 [2] *CYGNI*.

Plate	Date	No. of Lines	Velocity
C 13	1903, January 9	9	−63 km
DB[1] 10	May 7	13	65
DB[1] 32	June 5	16	61
		Mean	−63 km

Plate B 387 was taken with the regular three-prism train and the 24-inch camera; plate C 13, with the same prism train and a 10.5-inch camera; the remainder of the plates, with a single prism and the 24-inch camera.

The degree of accuracy attained is probably somewhat higher in the case of 61[1]. The error of the mean should, however, in neither case exceed about 3 km. A value of −54 km was obtained for 61[1] *Cygni* by Bélopolsky at Pulkowa in 1895, based

upon the measurement of five lines on each of two plates. This constitutes the only previously published determination for either of the stars.

The agreement of the radial velocities of the two components, when considered in connection with their similar proper motions, would seem to indicate that the stars are unquestionably physically connected. Their real motion in reference to the Sun, under the assumption of a parallax of 0".4 and a proper motion of 5".2, would be about 80 km, or in space, when corrected for the solar motion, about 64 km.

THE RADIAL VELOCITY OF ϵ *URSAE MAJORIS*.

The bright star ϵ *Ursae Majoris* was included in the list of stars whose radial velocities were determined by Vogel and Scheiner at Potsdam in 1889–90. The mean of their values is −30.4 km for the epoch 1889.39. A plate obtained with the Bruce spectrograph in April 1902 gave a value of −8 km. While the spectrum of this star is of the Ia2 type of Vogel's classification with faint and broad metallic lines whose accurate measurement is difficult, a discrepancy of this amount is too large to be accounted for in such a way, and is not encountered among other stars of the Potsdam list having similar spectra. Accordingly the star was placed upon the observing list, and the following plates were obtained:

Plate	Date	No. of Lines	Velocity
B 339	1902, April 30	10	−8 km
B 344	May 14	8	8
B 364	June 20	9	8
A 357	July 23	7	8
B 479	Dec. 31	12	11
B 496	1903, March 24	16	10
A 422	April 2	13	10
A 431	April 8	13	10
A 440	April 16	12	11
		Mean	−9.4 km

The accordance of these measures is entirely satisfactory for a star with this type of spectrum, and it would appear that no appreciable change has taken place in the star's radial velocity

during the interval of a year covered by the observations. The difference as compared with the Potsdam results consequently is not accounted for, but it seems quite possible that we may here be dealing with a spectroscopic binary of much longer period than any met with hitherto.

THE VARIABLE VELOCITY OF β *SCORPII* IN THE LINE OF SIGHT.

Four spectrograms give the following values of the radial velocity of this bright star:

Plate	Date	No. of Lines	Velocity
B 327	1902, April 16	7	− 9 km
A 439	1903, April 16	4	+19
DB[1] 29	June 5	5	−99
DB[1] 42	June 12	4	−97

The spectrum is of the *Orion* type, but all the lines are exceedingly broad, and the measures are uncertain to the extent of several kilometers. The use of low dispersion in photographing the spectrum of this star has proved of decided advantage, the gain in narrowness of the lines more than counteracting the effect of the reduced scale.

THE VARIABLE VELOCITY OF ϵ *HERCULIS* IN THE LINE OF SIGHT.

The spectrum of this star appears to be composite, and its variations will be made the subject of further investigation. Three spectrograms furnish the following velocity for the star which gives the stronger lines, the spectrum being of the Ia2 type:

Plate	Date	No. of Lines	Velocity
A 449	1903, April 30	4	−58 km
DB[1] 17	May 7	4	−43
DB[1] 33	June 6	8	−22

These measures are to be regarded as preliminary and may be changed considerably in a further discussion of the star's motion.

YERKES OBSERVATORY,
June 15, 1903.

Minor Contributions and Notes.

A PHOTOGRAPHIC MAP OF THE ENTIRE SKY.[1]

THE collection of photographs at the Harvard College Observatory contains, in addition to the plates taken with the larger instruments, numerous photographs taken with two small anastigmatic lenses, each having an aperture of one inch, and a focal length of about thirteen inches. A region of more than thirty degrees square is covered by a single eight by ten inch plate. With exposures of one hour, stars as faint as the twelfth magnitude are, in some cases, obtained. Owing to the scale of the plates, identification of the individual stars would become difficult if, by using longer exposures, the number of stars were increased. One of these lenses is mounted at Cambridge, and is used principally for the northern stars. The other is similarly used for the southern stars at Arequipa. At each station two sets of photographs have been taken, the first having centers in declinations 0°, 30°, 60°, and 90°, and the second in declinations 15°, 45°, and 75°, the centers of the second set coinciding as nearly as possible with the corners of the first. An attempt is made to cover all parts of the sky, not too near the Sun, at least twice each month, once with each set. These photographs have proved unexpectedly useful here for determining the past as well as the present changes in light of variable stars, new stars, and similar objects. (See *Circular* No. 69, and elsewhere.) Of course, the small scale diminishes their value for measuring positions, although the minuteness of the images in part compensates for this difficulty.

The amount of useful material contained in these plates is so great that we are able to extract but a small portion of it, although an appropriation from the Carnegie Institution has, this year, permitted a great increase to be made. Various plans have been considered for placing copies of the photographs thus collected within the reach of astronomers. It was at first proposed to print a series of engravings on the same scale as the charts of the *Durchmusterung*. Numerous difficulties presented themselves, especially if an attempt was made to engrave the parallels and meridians upon the charts. The defects

[1] *Harvard College Observatory Circular* No. 71.

introduced by paper and ink are very troublesome, and are likely to differ in different copies. The variation in intensity of images which can be represented on paper is small compared with that on glass, and, finally, the expense would be large. The advantages of glass negatives are very great, especially to one accustomed to use them. They can be reproduced by contact printing so as to give results but slightly inferior to the original. A single contact print, forming a positive, with bright stars on a dark background, although more nearly resembling the sky itself, does not prove convenient in actual use. It cannot be superposed upon another positive, nor upon a paper map. A double contact print, however, furnishes a negative which is for some purposes nearly as useful as the original. Measurements of position or intensity of the images can be made, the cost is not large, and is nearly proportional to the number of copies furnished. A set of fifty-five of these prints on glass, covering the sky from the north to the south pole, has accordingly been prepared, and is described in Table I. A current number for designating the plate, and the approximate right ascension and declination of the center, are given in the first three columns. The designation of the original negative is given in the fourth column. It consists of the letters indicating the series, and the number of the plate in that series. The plates taken with the anastigmatic lens at Cambridge are indicated by the letters AC, those at Arequipa by AM. The date, the Greenwich Mean Time, and the length of exposure, expressed in minutes, are given in the next three columns. Remarks on some of the plates follow the table. The position of any particular object is indicated by two numbers placed in brackets. The first gives the distance of the object, in millimeters, from the left-hand edge of the exposed portion of the plate. The second number gives the corresponding distance from the lower edge of the exposed portion.

In the use of these plates certain suggestions may be made, especially for the benefit of those unaccustomed to astronomical photographs. To compare them with the sky, they should be examined with the glass side toward the eye. Objects of interest may then be conveniently marked on this side of the plate with a pen. The label is on the southern end of each plate, excepting in the cases of Nos. 1 and 54, which contain the north and south poles, respectively. Conspicuous configurations, like those of *Orion*, *Ursa Major*, and *Scorpius*, are easily recognized by inspection, aided when necessary by a small atlas. For the faint stars, a comparison may be made with

TABLE I.

CATALOGUE OF PLATES.

No.	R. A.	Dec.	Negative	Date	G. M. T.	Ex.
	h. m.	°		y. m. d.	h. m.	m.
1	0 00	+90	AC 2161	1902, January 4	22 43	39
2	0 00	+60	AC 1943	1901, November 2	17 51	63
3	3 00	+60	.....	... .	.. ..	..
4	6 00	+60	.....	.....	.. ..	..
5	9 00	+60	AC 3625	1903, May 13	13 16	59
6	12 00	+60	AC 3466	1903, March 31	18 42	71
7	15 00	+60	AC 3620	1903, May 12	14 51	58
8	18 00	+60	AC 3629	1903, May 13	18 14	70
9	21 00	+60	AC 3630	1903, May 13	19 25	60
10	0 00	+30	AC 2152	1902, January 4	13 06	57
11	2 00	+30	AC 124	1898, December 16	13 09	62
12	4 00	+30	AC 2155	1902, January 4	16 16	59
13	6 00	+30	AC 3469	1903, April 1	12 48	75
14	8 00	+30	AC 3363	1903, February 26	16 16	59
15	10 00	+30	AC 2160	1902, January 4	21 55	56
16	12 00	+30	AC 3332	1903, February 20	21 04	67
17	14 00	+30	AC 1105	1900, December 28	22 04	68
18	16 00	+30	AC 252	1899, March 20	20 19	64
19	18 00	+30	AC 1013	1900, September 24	12 12	56
20	20 00	+30	AC 3353	1903, February 23	22 03	62
21	22 00	+30	AC 1014	1900, September 24	13 16	69
22	0 00	0	AM 1569	1902, September 25	15 32	58
23	2 00	0	AM 151	1899, September 14	18 45	60
24	4 00	0	AC 1093	1900, December 27	13 27	71
25	6 00	0	AC 2156	1902, January 4	17 22	73
26	8 00	0	AC 513	1899, November 9	22 01	64
27	10 00	0	AC 2518	1902, May 2	14 19	65
28	12 00	0	AM 1431	1902, July 8	12 09	65
29	14 00	0	AC 2547	1902, May 13	16 35	73
30	16 00	0	AM 1420	1902, July 3	13 15	60
31	18 00	0	AM 1436	1902, July 9	14 18	60
32	20 00	0	AM 1439	1902, July 9	17 30	60
33	22 00	0	AM 1441	1902, July 9	19 39	60
34	0 00	−30	AM 1427	1902, July 3	20 53	60
35	2 00	−30	AM 1451	1902, July 10	20 56	61
36	4 00	−30	AM 1802	1903, January 16	15 55	68
37	6 00	−30	AM 1785	1903, January 3	17 33	60
38	8 00	−30	AM 1798	1903, January 15	16 35	60
39	10 00	−30	AM 1419	1902, July 3	12 01	61
40	12 00	−30	AM 1443	1902, July 10	12 07	67
41	14 00	−30	AM 1444	1902, July 10	13 15	60
42	16 00	−30	AM 1452	1902, July 11	14 39	60
43	18 00	−30	AM 1461	1902, July 12	14 37	60
44	20 00	−30	AM 1463	1902, July 12	17 26	61
45	22 00	−30	AM 1440	1902, July 9	18 36	61
46	0 00	−60	AM 703	1900, November 1	13 37	61
47	3 00	−60	AM 1782	1903, January 3	14 12	64
48	6 00	−60	AM 712	1900, November 10	18 33	60
49	9 00	−60	AM 1247	1902, May 15	12 43	60
50	12 00	−60	AM 459	1900, May 8	15 10	72
51	15 00	−60	AM 1469	1902, July 14	14 13	60
52	18 00	−60	AM 809	1901, May 27	17 48	61
53	21 00	−60	AM 1464	1902, July 12	18 32	60
54	0 00	−85	AM 626	1900, September 3	17 11	60
55	14 00	−75	AM 1389	1902, June 25	15 13	60

the charts of the *Durchmusterung*, which are on nearly four times the scale. The plates may be examined with a reading glass or a two-inch positive eyepiece. The latter will, in almost all cases, serve to distinguish defects from actual star images. It will be noticed that images of bright stars are surrounded by sixteen rays, due to the iris diaphragm of the lens.

REMARKS.

1. The position of the North Pole is [95, 104]. [97, 111], the Pole Star. The circular form and absence of diffraction rays show that [110, 118] is a defect.

2. See No. 10.

3. Owing to a change in the scheme of work, photographs have not been obtained in the position of this and of the following region. These omissions will be supplied in a few weeks, or as soon as their positions permit photographs to be obtained.

4. See No. 3.

6. [25, 89], *ζ Ursae Majoris;* also on No. 7 [172, 85].

7. See No. 6.

10. [48, 190], Nebula in *Andromeda;* also on No. 2 [47, 11].

12. [117, 69], *Pleiades*, showing also the nebulosity surrounding them. Star l and perhaps m of the sequence given in *Harvard Annals*, XVIII, p. 153, appears on this plate. [131, 185], *Nova Persei*, No. 2. Nos. 54, 58, and 64 of Hagen's Catalogue are readily seen.

13. [80, 58, *Neptune.* [39, 107], *Nova Geminorum.* Stars t and w of *Circular* No. 70 (*Hagen* 73 and 77) appear on this plate. Numerous clusters are also shown where it is crossed by the Milky Way. [82, 71], the cluster *N. G. C.* 2129.

14. [40, 48], *Praesepe.*

17. [115, 92], the cluster *Messier* 3, N. G. C. 5272.

18. [51, 125], *Messier* 13, *N. G. C.* 6205, the cluster in *Hercules.*

19. [46, 163], the double stars ε and 5, *Lyrae;* also on Plate 20 [180, 169]. [31, 124], *Messier* 57, *N. G. C.* 6720. The Ring Nebula in *Lyra;* also on Plate 20 [175, 127].

20. See No. 19.

25. [134, 80], *N. G. C.* 1976, the great Nebula in *Orion*, [126, 91], *ζ Orionis* showing the faint nebulosity, *N. G. C.* 2024, north following it. [106, 105], *α Orionis* The red color of this star is well shown by its faintness on the photograph. [40, 161], the cluster *Messier* 14, *N. G. C.* 2287. [31, 3], *Sirius.* The circle around this, and other very bright stars, is due to light reflected from the back of the plate.

30. [167, 116], the cluster *Messier* 5, *N. G. C.* 5904.

37. [74, 83], *Pallas.*

40. The fogging on the northern edge is due to the Moon.

42. [68, 119], *α Scorpii.* The red color of this star is well shown by its faintness on the photograph.

43. [166, 141], *Uranus.* [126, 89], the cluster *Messier* 7, *N. G. C.* 6475. [109, 75], the cluster *Messier* 6, *N. G. C.* 6405. [99, 144], the Trifid Nebula, N. G. C. 6514. See *Annals* XXVI, p. 204, and Plate III. [96, 136], *N. G. C.* 6523.

44. [127, 136], *Vesta.* [113, 149], *Saturn.*

45. [169, 169], *Jupiter.* Satellite III is seen about half a millimeter to the left of it

46. [87, 25], the cluster 47 *Tucanae*, *N. G. C.* 104. [76, 17], the Small Magellanic Cloud. See *Annals* XXVI, p. 205, and Plate IV. [24, 102], *Achernar* α *Eridani.* Next to *Sirius*, this is the brightest star in the sky; also on No. 47 [167, 102].

48. [115, 40], the Large Magellanic Cloud. See *Annals*, XXVI, p. 206, and Plate IV. [77, 137], *Canopus*, α *Carinae.* [13, 73], the cluster *N. G. C.* 2516; also on No. 49 [145, 86].

49. Several fine clusters appear on this plate, which are better shown on Plate 50.

50. [182, 49], the cluster *N. G. C.* 3114; also on Plate 49 [56, 94]. [153, 64], η *Carinae* or η *Argus*; also on Plate 49 [24, 88]. This is one of the most remarkable regions in the sky. See *Annals* XXVI, p. 206, and Plates V and VII. [138, 74]. *N. G. C.* 3523; also on Plate 49, [6, 86]. [113, 61], *N. G. C.* 3766. The Southern Cross is well shown on this plate. See also *Annals* XXVI, p. 203, and Plate II. [85, 79], δ *Crucis.* [79, 52], α *Crucis.* [72, 88], γ *Crucis.* [62, 71], β *Crucis.* The faintness of γ *Crucis* is due to its red color. This renders the Cross less conspicuous on the photograph than to the eye. [58, 66], the cluster κ *Crucis*, *N. G. C.* 4755. [7, 50], β *Centauri.* [6, 134], the cluster ω *Centauri*, *N. G. C.* 5139, the finest globular cluster in the sky.

51. [144, 90], β *Centauri*; also on No. 55 [99, 193]. [118, 91], α *Centauri;* also on No. 55 [72, 188]. So far as known this is the nearest star in the sky. [42, 124], the cluster *N. G. C.* 6067.

54. As the latitude of the Arequipa station is $-16^\circ\ 22'$, a good photograph could not be obtained if the declination of the center of the plate was at -90°, Accordingly, this plate has been taken in R. A. $0^h.\ 0^m$, December $-8^\circ.36$. Plate 55, taken from the other set of photographs, has been added, to cover the region which would otherwise be omitted. The position of the South Pole is [92, 61]; also on No. 55 [96, 15]. [96, 64], σ *Octantis;* also on No. 55 [91, 14]. [84, 166], 47 *Tucanae.* [73, 157], the Small Magellanic Cloud.

55. See Nos. 51 and 54.

The cost of these plates will prevent the wide, gratuitous distribution of them that is made of the *Annals.* In any case, this would not be advisable, since many would thus be placed where little or no use would be made of them. Copies of this set of photographs, consisting of fifty-five glass negatives, each eight by ten inches, will be supplied for $15; selected sets of ten plates, $3. This is less than the actual cost, the balance being paid from the Advancement of Astronomical Science Fund of the Harvard Observatory. The privilege of increasing the price later is reserved. If the demand justifies it, copies of the second set of plates, whose centers are near the corners of these, will be issued later.

EDWARD C. PICKERING.

MAY 19, 1903.

REVIEWS

The Theory of Optics. By PAUL DRUDE. Translated from the German by C. RIBORG MANN and ROBERT A. MILLIKAN, of the University of Chicago. New York: Longmans, Green & Co., 1902.

IT is a satisfaction to note that there has appeared a translation of this work, which received such instant recognition at the hands of physicists the world over upon its appearance in Germany. The translators deserve much praise for the conscientious and accurate manner in which they have accomplished their task.

As to the subject-matter: Professor Drude has produced a work of the greatest value—distinctly modern and up to date, logical and concise, yet clear. The keynote of the work is sounded by the author in his preface: "My purpose is attained if these pages strengthen the reader in the view that optics is not an old and worn-out branch of physics, but that in it also there pulses a new life whose further nourishing must be inviting to everyone." (The English here is not a fair sample of the work of the translators.)

In detail we may note the following:

The mathematical treatment of the formation of images is very thorough. A brief practical survey of optical instruments (telescope, microscope, photographic lens systems, etc.) is given. The subject of photography in natural colors is merely touched upon.

One of the most valuable and remarkable parts of the book is that which deals with diffraction problems. Starting with Huyghen's elementary light-wave principle, Fresnel's modification of it, and Kirchhoff's further development (simplified by Voigt), the reader is led logically through an exhaustive mathematical discussion of important diffraction phenomena. This is accompanied by a brief treatment of grating, prism, and echelon.

Under polarization phenomena Wiener's stationary light-wave experiment is mentioned and emphasized, and rightly so, for from it the important conclusion follows inevitably that the direction of the light vector is perpendicular to the plane of polarization.

Attention is called to the first brilliant success of the electromagnetic theory of light, that "the velocity of light in ether is equal to the ratio of the electromagnetic to the electrostatic units."

In discussing the subject of dispersion in the magnetic rotation of the plane of polarization, the bearing of the phenomenon known as the

Hall effect is shown, and the Ampère-Weber theory of molecular currents is modified so as to give the correct results. From this discussion we are then led directly to Voigt's theory of the Zeeman effect. The statement is made that the physical significance of Voigt's explanation of anomalous Zeeman effects has not yet been shown.

The "ion hypothesis leads to some new dispersion formulæ for the natural and magnetic rotation of the plane of polarization."

A comprehensive treatment of light problems with reference to bodies in motion leads to the aberration of light and to Michelson's experiment with the refractometer oriented in two positions with respect to the rotation of the earth. This latter is followed by the rather startling explanatory proposition advanced by Lorentz and by Fitzgerald, that the length of a solid body may depend upon its absolute motion in space.

Part III, "Radiation," covering in the original German but fifty-seven pages, and in the translation sixty, is a fine piece of work. The subject treated might well demand an entirely separate book; yet Professor Drude's discussion is clear and logical.

The electromagnetic theory is adopted as fulfilling most completely the test of experiment; and as Professor Michelson says in his preface to the English edition, "No complete development of the electromagnetic theory in all its bearings, and no comprehensive discussion of the relation between the laws of radiation and the principles of theorenodynamics have yet been attempted in any general text in English."

Throughout the whole treatise we discern a strong tendency to view nature as a mechanism. The discussion and interpretation of equations resulting from mathematical reasoning are both able and clear. Descriptively, the book is fully on a par with Preston's *Theory of Light* and mathematically more valuable, as well as more lucid and attractive, than Basset's *Treatise on Physical Optics.*

The general criticism may be made that the book is unbalanced, too little space being given to some subjects, too much to others. This, however, is quite a pardonable fault, especially as the author makes "no claim to such completeness as is aimed at in Mascart's excellent treatise, or in Winkelmann's *Handbuch*," and writes: "For the sake of brevity I have passed over many interesting and important fields of optical investigation."

Professor Drude's *Theory of Optics* should be assigned a most prominent place in the library of every modern physicist—a place which it should hold for many a year despite the rapid progress of the science.

N. A. K.

THE

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ON MEASUREMENTS OF WAVE-LENGTH WITH THE CONCAVE GRATING OBJECTIVE SPECTROSCOPE.

By F. L. O. WADSWORTH.

IN a recent paper[1] I have investigated the question of the aberration which is introduced by the use of the concave grating as a direct objective spectroscope. It was found that for all points not exactly on the axis of the grating the aberration was different in nature, and, in most cases, very much greater in amount than that produced by using the grating in the manner proposed and adopted by Rowland. The question of the effect of this increased unsymmetrical aberration on the size of the field of good definition and on the accuracy of measurement of wave-lengths with the instrument were very briefly discussed in the paper to which reference has been made and in a second paper, "On the Optical Conditions Required to Secure Maximum Accuracy of Measurement in the Use of the Telescope and Spectroscope," published in preceding numbers of this JOURNAL.[2]

[1] "On the Aberration of a Concave Grating when Used as an Objective Spectroscope," *Phil. Mag.*, (6) **6**, 119–156, July 1903.

[2] **16**, 267–299; **17**, 1–19, 100–133, 163; See particularly pp. 285–288 (note errata on p. 287 as corrected in **17**, 163).

The theory and method of using the concave grating as an objective spectroscope was first published by the writer in 1896.[1] Since then it has been used in this way by a number of investigators, among whom may be mentioned Poor, Mitchell, Frost, Mohler and Daniel, and Jewell and Humphreys. It was used by the writer in 1900 at Union Springs, Alabama, in photographing the flash spectrum and coronal rings during the total solar eclipse of May 28.[2] The unsatisfactory results yielded by the preliminary examination of the plates then obtained led me to the more detailed investigation of the aberration of the instrument; and the conclusions reached indicate that both as a spectrograph and as a spectrometer the concave grating objective spectroscope is far below the standard of the objective prism and the plane grating objective spectroscope. In the case of our own plates the errors of displacement of the lines are so large and the definition at the edges of the field so imperfect that I have not considered the results of wave-length measurement that might be obtained from them worthy of comparison with those determined by other parties with the slit, objective prism, and plane grating objective spectrographs.[3] This bad definition and lack of accuracy is in part due, in our case, to the very large angular aperture employed (with the view of securing great rapidity), and in part to the method of mounting adopted (also with a specific object). This, as I have since found by the mathematical investigation, is less favorable than another form would have been.

Other observers who have used the instrument in eclipse work also report unfavorably[4] as to the performance of the instrument, but ascribe the poor results either to the use of improper plates or to defects in the focusing. While these causes may have operated to contribute to the difficulties, they

[1] ASTROPHYSICAL JOURNAL, 3, 54–60, January 1896.

[2] *Report of the Director of the Allegheny Observatory for 1900*, pp. 23–24 (9). Also "Account of Allegheny Observatory Eclipse Expedition," S. M. KINTNER, *Western University Courant*, 15, 260–270.

[3] See also footnote on p. 87.

[4] See for example FROST, ASTROPHYSICAL JOURNAL, 12, 85, 307; MOHLER and DANIEL, *ibid.*, 12, 391 and HUMPHREYS, *ibid.*, 16, 313.

are not, as it now appears, the chief sources of error. The latter are to be found in the inherent optical defects of the instrument itself.

The general expression for the aberration in the primary plane of a concave grating when used as an objective spectroscope is, as has been already found,[1]

$$\begin{aligned} Z = & -\frac{\rho}{2}\sin^3\beta\sin\theta\cos i\frac{(\cos i+\cos\theta)}{\cos^2\theta} \\ & +\frac{\rho}{8}\sin^4\beta\frac{\cos i+\cos\theta}{\cos^2\theta}[\cos^2 i-(\cos\theta+2\cos i)^2\tan g^2\theta] \\ = & -U+S, \end{aligned} \tag{1}$$

where ρ is the radius of curvature of the grating surface, β is the semi-angular aperture measured at the center of curvature, and i and θ are the angles of incidence and diffraction respectively. Both β and θ are measured positively to the right from the normal to the grating (viewed from the center of curvature), while i is measured positively to the left of this axis. The nature and numerical value of the aberration depends both on the dimensions and optical constants of the grating and on the form of mounting adopted for the instrument. As described in a previous paper, there are four general types of mounting that may be employed, and these have been designated respectively as A–B, C–D, E–F, and G–H.

In the solar eclipse work with which we are now dealing only two of these types have been used; *i.e.*, the A–B type by Frost, Mohler and Daniel, and Jewell and Humphreys, and the G–H type by the writer. The dimensions and other optical constants of the four gratings and the positions in which they were used are given in Table I. The aberrations at the centers and the edges of the photographic fields, computed from the data in Table I by the aid of (1), are tabulated in Table II.

In order to determine the effect of a given amount of aberration, Z, on the form and position of the spectral image at different parts of the field we must find how the distribution in intensity in the diffraction pattern at the focal plane is affected by the unsymmetrical constituent U and the symmetrical constit-

[1] *Phil. Mag.*, **6**, 124, July 1903.

TABLE I.

Observer	Grating	Constants of Grating							Position of Use			Field	κ
		ρ	a	$\beta = \frac{a}{2\rho}$	n	$N = nb$	r In Tenth meters	ϵ	Mounting	i	θ_0	λ_1 to λ_2	θ_1 to θ_2
		cm	cm			about	about	about					
Frost, Wadesboro, 1900	F.	150	4.4	0.0147	5684	25,000	0.17 0.20	0.01	A-B	15° 10′	0° 0′	4200 4600 4950	+(1°18′) to −(1° 9′)
Mohler and Daniel, 1900	M.	305	10.0	.0164	5684	57,000	0.08 0.10	0.006	A-B	16	0°0′	4300 to 5880	+(1°45′) to −(3°25′)
Wadsworth, Union Spgs., 1900	W.	175.3	13.2	.0377	1421	18,750	0.10 0.11	0.007	G H	12°50′	12°50′	5880 3d 000 6270 3d	+(28°15′) to −(2°35′)
Jewell and Humphreys, Fort de Kock, 1901	J.*	915	10.0 2.0	$\beta_1 = .0109$ $\beta_2 = .0022$	5684	68,200	0.045 0.075	0.003 0.005	A-B	14°50′	0°0′	3100 4500 5200	+(4°34′) to −(2°17′)

TABLE II. Aberrations.

Grating	$\text{Sin}^3\beta$	$\frac{1}{2}\rho \sin^3$	$\frac{1}{8}\rho \sin^4\beta$	At Center of Field. λ_0 θ_0					At Right Edge. λ_1 κ_1					At Left Edge. λ_2 κ_2				
				Sym. S.		Unsym. U.			Sym. S.		Unsym. U.			Sym. S.		Unsym. U.		
				10^{-6}	λ	10^{-6}	λ	$d\lambda$	10^{-6}	λ	10^{-6}	λ	$d\lambda$	10^{-6}	λ	10^{-6}	λ	$d\lambda$
F	3.18×10^{-6}	238×10^{-6}	0.88×10^{-6}	1.6	.03	....	....	....	1.6	0.03	10.2	0.24	.04	1.6	0.03	9.02	0.18	.04
M	4.41×10^{-6}	672×10^{-6}	2.75×10^{-6}	4.98	.09	....	....	....	4.9	.09	38.6	.90	.07	4.8	.09	75.6	1.3	.13
W	53.6×10^{-6}	4698×10^{-6}	44.3×10^{-6}	44.3	.80	2087	37	?	144	2.5	5180	88.0	?	82.6	1.2	408.0	6.8	?
*β_1	1.29×10^{-6}	500×10^{-6}	1.6×10^{-6}	2.94	.07	....	....	....	2.78	.09	89.6	2.9	.15	2.92	.06	44.7	0.9	.09
J β_2	0.01×10^{-6}	5×10^{-6}	0.00×10^{-6}															

TABLE III.

$\frac{a}{a_0}$	I_0^2	I_M^2 $U=\frac{\lambda}{4}$	$U=\frac{\lambda}{2}$		$U=\frac{3}{4}\lambda$		$U=\lambda$		$U=\frac{7}{4}\lambda$		$U=2\lambda$	
			$\frac{a}{a_0}$	I_M^2	$\frac{a}{a_0}$	I_M^2	$\frac{a}{a_0}$	I_M^2	$\frac{a}{a_0}$	I_M^2	$\frac{a}{a_0}$	I_M^2
−2.0	.0000	.0086	−2.52	.0030	−2.88	.0016	−3.16	.0001	−3.84	.0004	−4.0	.0000
−1.9	.0027	.0081	−2.40	.0026	−2.74	.0009	−3.00	.0000	−3.65	.0001	−3.8	.0001
−1.8	.0108	.0061	−2.27	.0014	−2.59	.0002	−2.84	.0006	−3.46	.0000	−3.6	.0002
−1.7	.0229	.0035	−2.14	.0002	−2.45	.0000	−2.68	.0015	−3.26	.0004	−3.4	.0002
−1.6	.0358	.0012	−2.02	.0001	−2.30	.0005	−2.53	.0020	−3.07	.0008	−3.2	.0000
−1.5	.0450	.0000	−1.89	.0016	−2.16	.0010	−2.37	.0019	−2.88	.0010	−3.0	.0001
−1.4	.0468	.0004	−1.76	.0045	−2.02	.0010	−2.21	.0012	−2.69	.0008	−2.8	.0006
−1.3	.0392	.0018	−1.64	.0081	−1.87	.0004	−2.06	.0005	−2.50	.0004	−2.6	.0014
−1.2	.0243	.0032	−1.51	.0110	−1.73	.0000	−1.90	.0001	−2.30	.0001	−2.4	.0018
−1.1	.0080	.0033	−1.38	.0123	−1.58	.0011	−1.74	.0000	−2.11	.0000	−2.2	.0018
−1.0	.0000	.0018	−1.26	.0125	−1.44	.0049	−1.58	.0001	−1.92	.0002	−2.0	.0015
−0.9	.0119	.0001	−1.14	.0112	−1.30	.0119	−1.42	.0006	−1.73	.0013	−1.8	.0014
−0.8	.0547	.0017	−1.01	.0101	−1.15	.0212	−1.26	.0034	−1.54	.0042	−1.6	.0020
−0.7	.1353	.0125	−0.88	.0099	−1.01	.0313	−1.10	.0112	−1.34	.0101	−1.4	.0041
−0.6	.2545	.0399	−0.76	.0120	−0.86	.0408	−0.95	.0234	−1.15	.0164	−1.2	.0092
−0.5	.4053	.0911	−0.63	.0181	−0.72	.0493	−0.79	.0428	−0.96	.0287	−1.0	.0189
−0.4	.5754	.1714	−0.51	.0323	−0.58	.0579	−0.63	.0672	−0.77	.0388	−0.8	.0337
−0.3	.7368	.2813	−0.38	.0605	−0.43	.0688	−0.48	.0934	−0.58	.0489	−0.6	.0522
−0.2	.8751	.4159	−0.25	.1103	−0.29	.0856	−0.32	.1194	−0.38	.0605	−0.4	.0719
−0.1	.9675	.5640	−0.12	.1886	−0.14	.1128	−0.16	.1440	−0.19	.0764	−0.2	.0910
−0.0	1.0000	.7093	−0.00	.2976	−0.00	.1553	−0.00	.1688	−0.00	.1002	−0.0	.1097
+0.1	.9675	.8326	+0.12	.4312	+0.14	.2178	+0.16	.1966	+0.19	.1348	+0.2	.1302
+0.2	.8751	.9158	+0.25	.5731	+0.29	.3013	+0.32	.2304	+0.38	.1804	+0.4	.1551
+0.3	.7368	.9450	+0.38	.6983	+0.43	.3998	+0.48	.2722	+0.58	.2320	+0.6	.1860
+0.4	.5754	.9137	+0.51	.7790	+0.58	.4984	+0.63	.3205	+0.77	.2784	+0.8	.2208
+0.5	.4053	.8246	+0.63	.7919	+0.72	.5733	+0.79	.3682	+0.96	.3056	+1.0	.2519
+0.6	.2545	.6901	+0.76	.7278	+0.86	.5994	+0.95	.4029	+1.15	.3022	+1.2	.2679
+0.7	.1353	.5287	+0.88	.5951	+1.01	.5592	+1.10	.4077	+1.34	.2653	+1.4	.2569
+0.8	.0547	.3629	+1.01	.4206	+1.15	.4528	+1.26	.3695	+1.54	.2032	+1.6	.2147
+0.9	.0119	.2144	+1.14	.2422	+1.30	.3027	+1.42	.2863	+1.73	.1316	+1.8	.1490
+1.0	.0000	.0999	+1.26	.0983	+1.44	.1498	+1.58	.1748	+1.92	.0673	+2.0	.0785
+1.1	.0080	.0287	+1.38	.0161	+1.58	.0386	+1.74	.0686	+2.11	.0222	+2.2	.0244
+1.2	.0243	.0009	+1.51	.0029	+1.73	.0000	+1.90	.0061	+2.30	.0015	+2.4	.0007
+1.3	.0392	.0089	+1.64	.0457	+1.87	.0379	+2.06	.0135	+2.50	.0000	+2.6	.0090
+1.4	.0468	.0395	+1.76	.1159	+2.02	.1262	+2.21	.0864	+2.69	.0281	+2.8	.0305
+1.5	.0450	.0776	+1.89	.1807	+2.16	.2198	+2.37	.1913	+2.88	.0656	+3.0	.0776
+1.6	.0358	.1095	+2.02	.2138	+2.30	.2749	+2.53	.2764	+3.07	.1080	+3.2	.1095
+1.7	.0229	.1261	+2.14	.2046	+2.45	.2674	+2.68	.3088	+3.26	.1424	+3.4	.1258
+1.8	.0108	.1239	+2.27	.1592	+2.59	.2030	+2.84	.2467	+3.46	.1538	+3.6	.1215
+1.9	.0027	.1052	+2.40	.0968	+2.74	.1128	+3.00	.1473	+3.65	.1323	+3.8	.0968
+2.0	.0000	.0762	+2.52	.0402	+2.88	.0358	+3.16	.0505	+3.84	.0822	+4.0	.0580

uent S respectively. The problem has been investigated in part by Lord Rayleigh for small values of U and S., *i.e.*, values not exceeding 0.5λ.[1] The aberrations with which we have to deal in the above cases range from 0.03λ for S with the F grating, to

[1] *Phil. Mag.*, (5) **8**, 404.

88λ for U with the W grating. Disregarding the very large values for this latter instrument, we have still to consider the effect of unsymmetrical aberrations amounting to as much as 2.9λ for the J grating. I have therefore extended Rayleigh's results for the effect of an unsymmetrical aberration to values as large as $U=2\lambda$. The values of I_u^2 for $U\cong \frac{1}{4}\lambda, \frac{1}{2}\lambda, \frac{3}{4}\lambda, 1\lambda, 1\frac{1}{4}\lambda$, and 2λ for a rectangular aperture are tabulated in columns 3, 5, 7, 9, 11, and 13, respectively, of Table III. For the sake of comparison the value of I_0^2 for $U=0$ (no aberration), is given in column 2 of the same table. The abscissae a for each curve are expressed as usual in terms of a_0, the angular distance from the center, o, of the geometrical image to the first minimum, m_0, of the normal diffraction pattern I_0^2 (column 2). Owing to the manner in which the integral involved was originally evaluated by Airy,[1] the tabulated values of I_u^2 correspond to different values of a for each value of U.[2]

The curves expressing the relation between I_u^2 and $\frac{a}{a_0}$ for these different values of U (0 to 2λ) are plotted in Fig. 1.

From an examination of the values of the table, or better still, of the above curves, we see that the relation between U, the extreme unsymmetrical wave front aberration, and the angular displacement $o-o'$ of the point of maximum intensity of the resulting distorted image is almost linear, and that the amount of such displacement is

$$o-o'=\Delta\gamma=0.3\ a_0\frac{U}{\frac{\lambda}{4}}, \tag{2}$$

or, expressed in linear measure at the focal plane,

$$\Delta\xi=1.2\frac{u}{b}U. \tag{3}$$

Hence, substituting the value of U from (1), we obtain

$$\Delta\xi=0.6\frac{\rho}{\beta}u\sin^3\beta\frac{\sin\theta}{\cos^2\theta}(\cos i+\cos\theta)\cos i, \tag{4}$$

where u is the focal length of the grating.

[1] See *Cambridge Phil. Trans.*, 6, 402, 1838.

[2] See RAYLEIGH, *Phil. Mag.*, (5) 8, 405, 406.

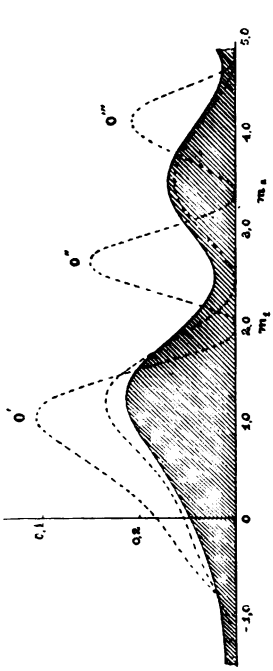


Fig. 1

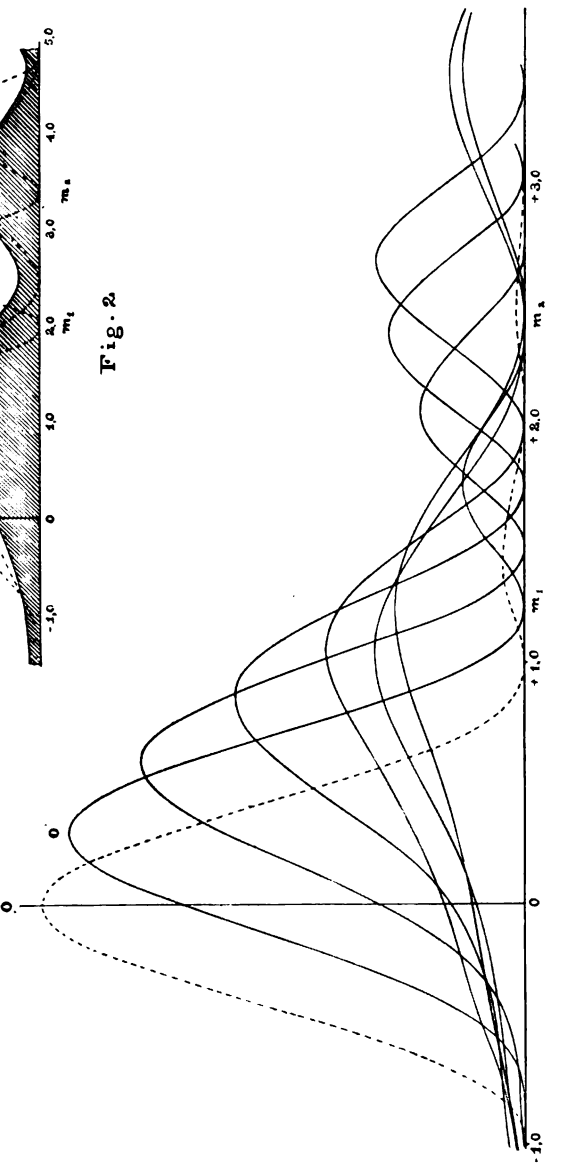


Fig. 2

For the purpose of determining the effect of this displacement on the measurement of wave-lengths it is convenient to express $\Delta\xi$ in terms of $\Delta\lambda$. From the general equation of the grating, we have

$$N\lambda = b(\sin i - \sin\theta) \tag{5}$$

and

$$\begin{aligned} d\lambda &= -\frac{b}{N}\cos\theta\, d\theta \\ &= -\frac{b}{N}\cos\theta\frac{dx}{u} . \end{aligned} \tag{6}$$

If the cross-wires of the measuring micrometer are always set on the point of maximum intensity, o', of the unsymmetrical image, the error in the wave-length measurement due to the aberration of the concave grating spectroscope will therefore be

$$d\lambda = -1.2\frac{U}{N}\cos\theta . \tag{7}$$

These values have been computed for the values of U given by (1) for the four instruments already considered and are found in the columns headed $d\lambda$ of Table II.

The maximum errors of displacement due to aberration are therefore 0.04 tenth-meters in the case of Frost's plates; 0.13 tenth-meters in the plates obtained by Mohler and Daniel; 0.15 tenth-meters in the case of those obtained by Jewell and Humphreys; and so large as to be indeterminate at both the ends and center of the plates obtained by the writer. Comparing these errors of displacement with the quantities ϵ of Table I, which represent the limiting metrological power of the instruments under the best conditions of use, we see that the effect of aberration is to displace the spectral images by amounts which, in the case of gratings F, M, and J, are respectively, about four, fourteen to nineteen, and twenty to one hundred and fifty times as great as the metrological errors of setting on the lines. In the case of grating W, $d\lambda$ is, as already stated, so large as to be indeterminate.

The above considerations are based on the assumption that the setting of the micrometer wire in the measurements for determination of wave-length is affected only by the displace-

ment of the maximum of intensity in the central band of the diffraction pattern. For small values of U this assumption will be very nearly correct, but for large values the setting will also be affected by the depression of the principal maximum, o', and the very decided augmentation of the lateral bands, o'' and o'''. The effect of the symmetrical aberration S will also tend to depress still further the principal maximum, o', and to obliterate the minima, m_1, m_2, m_3, by a re-distribution of the relative intensi ties between the points, m_1o', m_2o'', m_3o''', etc.

In the case of the M and J gratings the extreme values of U are 1.3λ and 2.9λ, respectively. At the same points in the field the corresponding values of S are about 0.1λ. The effect of this amount of symmetrical aberration is to depress the central maximum about 5 per cent. and correspondingly augment the lateral bands. Considering this, and taking into account also the effect of mechanical vibrations, small errors of focusing, etc. all of which tend in the same direction, we shall obtain for these large values of U a distribution in intensity in the spectral image similar to that represented in the shaded curve in Fig. 2. In setting the micrometer wire on such an image as this, the tendency would undoubtedly be to place it considerably farther to the right than the point o', which marks the location of the principal maximum. The error of setting in these cases would therefore considerably exceed the quantity $\Delta\xi$ as given by (3).

In cases like that of the W grating, where the aberration, both symmetrical and unsymmetrical, amounts to a very much larger number of wave-lengths, the central and lateral bands are all blended into a broad indefinite blur on which it is impossible to set the micrometer wire with any pretensions to accuracy. At the points very near the axis of this grating ($\theta = 0$) there is a narrow strip over which the definition is about equal to that at the extreme edges of the field of the M and J gratings; but even here, as we have already seen, the aberrations are from five to twelve times greater than the limit imposed by considerations of "good definition," and from twenty to one hundred and fifty times the limit imposed by the conditions of maximum accuracy. It is for these reasons that the plates taken at Union Springs

have been, as already stated, rejected as unsuitable for accurate measurement.[1]

On account of the small angular and linear apertures of F and J[2] the definition of these gratings is good not only at the center of the plate, but over a fair extent of field. The plates obtained with these instruments have been measured and the wave-lengths of the lines so determined published.[3] Mohler and Daniel have also published a short list of wave-length measurements obtained with grating M.[4]

I have been interested in comparing the apparent accuracy attained in these measurements with that to be expected from the preceding considerations. The probable uncertainties in the wave-length determinations as assigned by the observers themselves are about 0.05 tenth-meters at the center and about 0.2

[1] If the mathematical investigation of the aberration of the concave grating objective spectroscope had been undertaken before, instead of after, these eclipse observations had been made, much disappointment on my own part at least would have been avoided. No excuse can be given for such lack of preliminary examination save that of an unavoidable delay that occurred in organizing and financing the expedition, and the lack of sufficient assistance. The grating had been hastily examined beforehand to test its definition near the axis at the principal focus, and this had been found to be very good. In these tests, however, I had no collimator of sufficiently large aperture to cover the entire surface of the grating, and the semi-angular aperture β was in consequence of this only about six-tenths as large as that used in the actual eclipse work. The aberration on the axis of the grating, was, under these circumstances, only about one-eighth as great as with full aperture, and therefore considerably less than would injuriously affect definition. The effect of increasing the *angular aperture* and the *field* was not considered as carefully as it should have been, but this was partly in consequence of the results which Mitchell had published a short time previously (ASTROPHYSICAL JOURNAL, **10**, 29). As I have already pointed out in my previous paper (*Philosophical Magazine*, July 1903) these results, although obtained with a grating of even larger angular aperture than that here used, should not have been accepted as conclusive. The whole experience is a good illustration of the danger of adopting a new type of instrument without sufficient preliminary examination of its optical defects and limitations in the line of work which it is desired to carry out.

[2] In the case of J the small angular aperture β_2 was due to a part of the ruled surface being defective. In the light of subsequent developments this must be regarded as a rather fortunate accident. Had it not been for this, the aberration at the edges of the field would have been even larger than it was.

[3] FROST, ASTROPHYSICAL JOURNAL, **12**, 342–344; HUMPHREYS, *ibid.*, **15**, 318–325, Table II.

[4] MOHLER and DANIEL, *ibid.*, **12**, 363.

tenth-meters at the edges of the plates by Frost (pp. 325, 326);[1] and about 1 tenth-meter by Mohler and Daniel (p. 363). In both cases these errors or uncertainties, whichever they may be called, are considerably greater than the theoretical errors of displacement $d\lambda$. At first sight this would indicate that the effect of bad definition due to both the terms U and S is more important than that of the actual displacement $d\lambda$ produced by U alone; and this would probably be the case were it not for the fact that the errors of measurement due to the first cause are likely to be either positive or negative, while those due to the second cause are negative alone. To detect any effect of the latter nature we must examine the residuals, $\lambda_{O.G} - \lambda_R$, for systematic and periodic changes in sign and magnitude. In doing this we are met at the outset with a difficulty which is introduced by the method of reduction and interpolation employed. The values of $\lambda_{O.G}$ from the objective grating plates have been determined not as absolute values, but as relative values, by identifying certain lines on the *O.-G.* plates with lines on the Rowland map, and assuming that the average residuals, $\lambda_{O.G} - \lambda_R$, are zero for these lines and that the spectrum is *normal* between them. This process is a perfectly legitimate and accurate one on plates taken with long focus concave gratings mounted in the manner originally designated by Rowland, and for which there is *no* unsymmetrical aberration. It cannot, however, be adopted without question for the *O.-G.* plates, where conditions both as to curvature of field and aberration are so different.

From equation (5) we have for any point θ in the field of a grating

$$\lambda = \lambda_0 - \frac{b}{N} \sin \theta \ , \qquad (8)$$

where λ_0 is the wave-length of the line which falls on the axis of the grating, $\theta = 0$. When there is an aberrational displacement, $d\lambda$, of the position of the geometrical image, the line of wave-

[1] In this connection it seems desirable to call attention to the fact that Professor Frost himself recognized the inferiority of the wave-length measurements made with the objective grating to those made with the objective prism train, and by far the greater part of his wave-lengths was determined from plates obtained with the latter instrument.

length λ will not fall at the point θ indicated by (8) but at a point

$$\theta_m = \theta + \Delta\gamma \ ,$$

and therefore

$$\theta = \theta_m - 1.2\frac{U}{b} \ , \tag{9}$$

as indicated in (2) and (3). From (1) we obtain for U in terms of θ_m (neglecting powers higher than the fourth) the value

$$U = \frac{\rho}{2}\sin^3\beta\left[a_1 + \left(\frac{5}{6}a_1 - \frac{1}{2}\cos i\right)\theta_m^2\right]\theta_m \ , \tag{10}$$

where

$$a_1 = (1 + \cos i)\cos i \ , \tag{11}$$

as defined and tabulated (for different values of i) in my previous paper (Table I).[1]

From (8), (9), and (10) we finally obtain

$$\lambda - \lambda_0 = \Delta\lambda = -\frac{b}{N}\sin\left\{1 - 0.3\sin^2\beta\left[a_1 + \left(\frac{5}{6}a_1 - \frac{1}{2}\cos i\right)\theta_m^2\right]\right\}\theta_m \ . \tag{12}$$

Let us put for convenience

$$\begin{aligned} 0.3\,a_1 &= a_2 \\ 0.3\left(\frac{5}{6}a_1 - \frac{1}{2}\cos i\right) &= a_3 \end{aligned} \ . \tag{14}$$

Then expressing (12) in terms of θ we have to the same degree of approximation as before

$$\Delta\lambda = -\frac{b}{N}\left\{(1 - a_2\sin^2\beta)\,\theta_m - \left[\frac{1}{6} - \sin^2\beta\left(\frac{a_2}{2} - a_3\right)\right]\theta_m^3\right\} . \tag{15}$$

The values of a_2 and a_3 which appear in (14) and (15) have, for convenience, been computed for the same values of i used in the table for a and a_1 above referred to. The results are tabulated in Table IV.

TABLE IV.

i	a_1	a_2 $(=0.3\,a_1)$	a_3	i	a_1	a_2 $(=0.3\,a_1)$	a_3
0	2.000	0.600	0.350	50°	1.056	0.317	0.168
5°	1.988	.596	.348	55°	.902	.271	.139
10°	1.954	.586	.340	60°	.750	.225	.113
15	1.899	.570	.330	65°	.601	.180	.087
20	1.823	.547	.315	70°	.459	.138	.064
25°	1.728	.518	.296	75°	.326	.098	.043
30	1.616	.485	.274	80°	.203	.061	.025
35	1.490	.447	.249	85°	.095	.029	.011
40°	1.353	.406	.223	90°	.000	.000	.000
45	1.207	.362	.196				

[1] *Phil. Mag.*, (6) **6**, 130.

When the plate is bent to the focal curve

$$u = \frac{\rho}{1 + \cos i} \cos \theta \tag{17}$$

it has a curvature of $\frac{4}{\rho}$. If it is then straightened and measured on a linear dividing engine we have for the relation between the linear run ΔS of a screw and the angle θ_m

$$\theta_m = \frac{\Delta S}{u_0} = \Delta S \cdot \frac{1 + \cos i}{\rho} . \tag{18}$$

From (8), (15) and (18) we finally obtain

$$\Delta\lambda = -\frac{b}{N}(1 - a_2 \sin^2\beta)\frac{\Delta S}{u_0} + \left[\frac{1}{6} - \sin^2\beta\left(\frac{a_2}{2} - a_3\right)\right]\frac{N^2}{b^2}(\Delta\lambda)^3 \tag{19}$$
$$= A\Delta S + x_c.$$

If the spectra are both photographed and measured on a flat plate the relation between the run of the micrometer screw and the angle θ_m will be

$$\theta_m = \tang^{-1}\frac{\Delta S}{u_0}$$
$$= \frac{\Delta S}{u_0} - \frac{1}{3}\theta_m^3 .$$

Substituting this value in (15) and reducing as before, we obtain

$$\Delta\lambda = -\frac{b}{N}(1 - a_2 \sin^2\beta)\frac{\Delta S}{u_0} + \left[\frac{1}{2} - \sin^2\beta\left(\frac{5}{6}a_2 - a_3\right)\right]\frac{N^2}{b^2}(\Delta\lambda)^3 \tag{20}$$
$$= A\Delta S + x_f.$$

The amount from which the spectrum departs from normal is therefore about three times as great when photographed on flat plates as when photographed on curved plates.

In the case of Frost's measurements, in which the plates used were flat, the extreme value of $\Delta\lambda$ (see Table I) was about 400 tenth-meters. The value of $\frac{N}{b} = n$ for the F grating is 5684×10^{-8}. Hence the value of x_f, the second term of (20), which expresses the correction required to a "normal" spectrum is about

0.1 tenth-meter.

The corrections at the intermediate points, $\Delta\lambda$ 100, 200 and 300 tenth-meters, are respectively

For $\Delta\lambda = 100$ tenth-meters, $x = 0.002$
For $\Delta\lambda = 200$ tenth-meters, $x = 0.013$
For $\Delta\lambda = 300$ tenth-meters, $x = 0.043$
For $\Delta\lambda = 400$ tenth-meters, $x = 0.103$.

Plotting these values as abscissae and ordinates respectively we obtain the curve in Fig. 3. This expresses graphically the corrections that must be applied to the measurements on the assumption that the spectrum is normal over the entire field.

In the actual reduction of the plate the necessary corrections were somewhat reduced by dividing the plate into three sections; the first extending from $\lambda 4200$ to 4383, the second extending from $\lambda 4383$ to 4572, and the third from $\lambda 4572$ to 4950. The spectrum was assumed normal over each of these three sections. As the constant A of the micrometer screw was determined separately for each section, the corrections between these points would be the differences between the ordinates to the curve, *a*, *b*, *c*, *o*, *d*, *e*, and the straight lines, *ab*, *bc*, and *cd*. The maximum correction in section I (*ab*) would therefore be about 0.015 tenth-meters, in section II (*bc*) not more than 0.004 tenth-meters, and in section III (*cd*) about 0.03 tenth-meters.

In section I the correction is just a little larger than the limit of metrological power ϵ (see Table I), and in section II it may be entirely disregarded; in section III, however, it is about three times as large as ϵ. In this last section the correction is sufficiently large to make its neglect felt in the sign of the residual errors. In order to determine whether this were true I formed a table of residuals, $f(\lambda)$ = Rowland − Frost, for all lines whose identification was assumed to be reasonably certain. These residuals are plotted in Fig. 4 and the sums taken for each interval of 100 tenth-meters (from 4200 to 4300, from 4300 to 4400, etc.). The latter quantities are indicated by the circled crosses ⊗ of the figure.

These averaged residual differences correspond in distribution and sign to those which would be produced by neglect of the correction x for sections I and II, but are larger in magnitude than would be expected. For section III they also correspond if the constant of the micrometer run was determined by settings

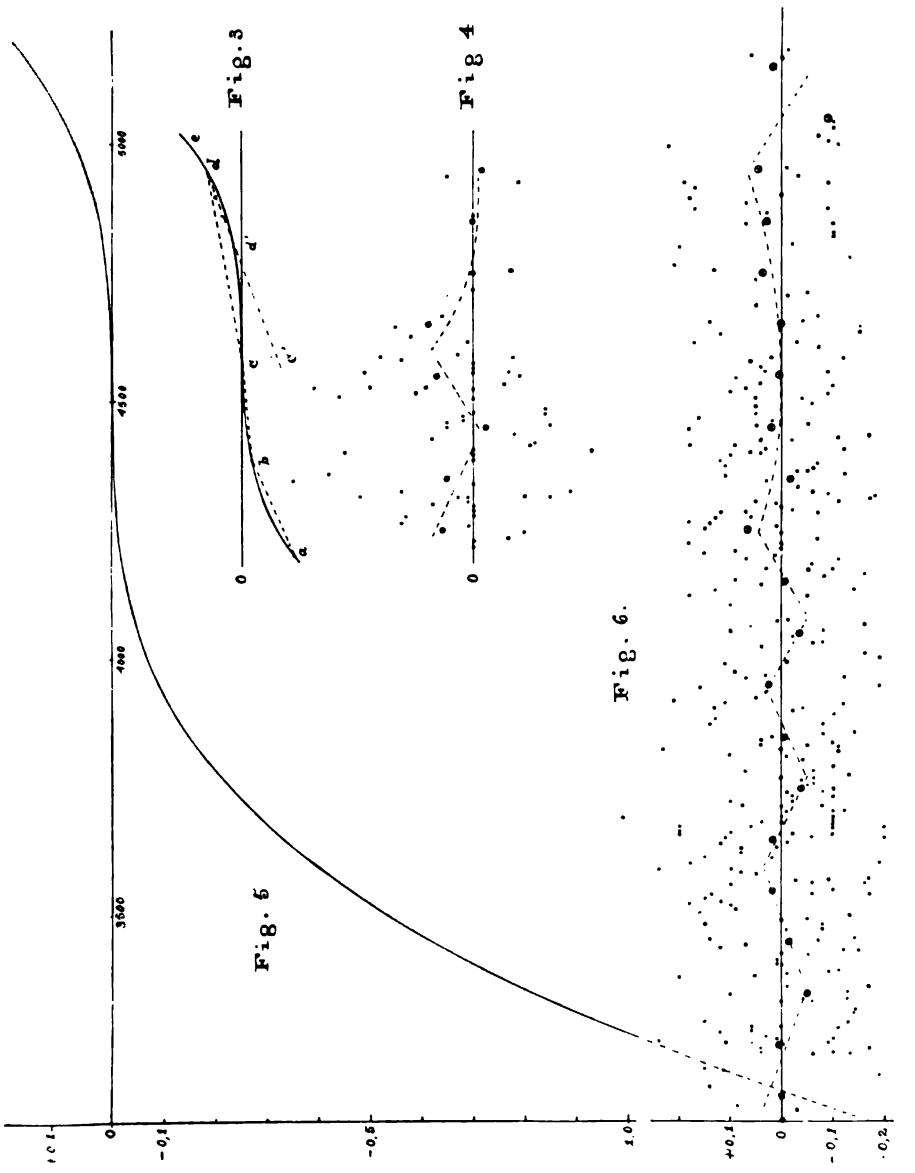


Fig. 3
Fig. 4
Fig. 5
Fig. 6.
3600
4000
4500
5000
0
-0,1
-0,6
1,0
+0,1
-0,2

on lines near d and d', or was assumed to be the same as in section I. In such case the line dc would be parallel or nearly so to the line ab as at dc', and the corrections for x would be the differences of the ordinates to dc', and the curve c, o, d, e. These are nearly the same, not only in sign but also in magnitude, as those actually found. The processes employed in the work of reduction of the plates are not stated in sufficient detail by Professor Frost to enable us to determine whether or not this explanation is the correct one, but the periodic character of the averaged residuals, $\Sigma f(\lambda)$, is quite striking and the great preponderance of residuals of positive sign is an evidence of some systematic error of the kind indicated.

In the case of Humphreys' measurements the plates were curved. The maximum values of $\Delta\lambda$ (see Table I) measured are

$$+\Delta\lambda = 700$$
$$-\Delta\lambda = 1400 .$$

The values of x_c (from 19) are

For $\Delta\lambda = 200$ tenth-meters, $x_c = 0.004$
For $\Delta\lambda = 400$ tenth-meters, $x_c = 0.034$
For $\Delta\lambda = 600$ tenth-meters, $x_c = 0.117$
For $\Delta\lambda = 700$ tenth-meters, $x_c = 0.185$
For $\Delta\lambda = 800$ tenth-meters, $x_c = 0.276$
For $\Delta\lambda = 900$ tenth-meters, $x_c = 0.394$
For $\Delta\lambda = 1000$ tenth-meters, $x_c = 0.538$
For $\Delta\lambda = 1100$ tenth-meters, $x_c = 0.716$
For $\Delta\lambda = 1200$ tenth-meters, $x_c = 0.930$
For $\Delta\lambda = 1300$ tenth-meters, $x_c = 1.182$
For $\Delta\lambda = 1400$ tenth-meters, $x_c = 1.476$

In this case the correction required for curvature at the violet end of the spectrum is nearly five hundred times the limiting metrological power, ϵ.

The relation between $\Delta\lambda$ and x is plotted in Fig. 5. The residuals $h(\lambda) =$ Rowland $-$ Humphreys, and their sums over each interval of 100 tenth-meters, have been found in the same way as for Frost's measurements and are plotted on the same scale in Fig. 6.

In this case also there is a marked correspondence between x and the residuals $\sum_{0}^{100} h(\lambda)$. Near the center of the plate where the spectrum is nearly normal and x is small, the average residuals, $\Sigma h(\lambda)$, are also small and lie very nearly in a straight line parallel to the λ axis. As we go from this point toward the violet the residuals become distinctly periodic in character. This would be explained if the wave-lengths in this part of the spectrum were determined by extrapolation from standard reference lines at about 3250, 3500, 3700, 3850, 4000 and 4150. In Humphreys' paper the method of reduction is not described, and here again it is not possible to satisfactorily compare the actual results of computation and measurement.

EFFECT OF ABERRATION ON THE APPEARANCE OF THE LINES.

For small values of θ, *i. e.*, near the center of the plate, the value of U is small and the value of S is relatively large. In this portion of the field the lines will therefore be simply broadened without being sensibly displaced or rendered assymmetrical. As we go away from the center the value of S decreases and the value of U increases. The effect of this is not only to displace the apparent center of intensity of the central image but to increase very greatly the relative intensities of the lateral diffraction fringes on the right hand, *i. e.*, the violet side, of the central image.

An examination of Fig. 2 shows that for large values of U the first lateral fringe on the right becomes very nearly as bright as the central image, and is much narrower and sharper. This image was actually selected by Humphreys as the one corresponding most nearly to the true geometrical image. The multiplication of lateral images at the edges of the field is not, as Humphreys assumes, an evidence of imperfect focusing, but of *good* focusing. Under favorable conditions it is possible to observe not only the principal (central) component of the diffraction image and the first two right hand fringes (forming an apparent tripling of the line) but several more lateral fringes still further to the right.

ALLEGHENY OBSERVATORY,
March 1903.

THE FLUORESCENCE AND ABSORPTION SPECTRA OF SODIUM VAPOR.

By R. W. Wood and J. H. Moore.

THE FLUORESCENCE OF SODIUM VAPOR.

The green fluorescence which sodium vapor exhibits when illuminated with an intense beam of white light, was first observed and studied by Wiedemann and Schmidt.[1] The method which they employed was to heat the metal in an exhausted glass bulb, concentrating a beam of sunlight on the vapor by means of a lens. A cone of green light is seen where the intense beam enters the mass of metallic vapor. Examined with the spectroscope, this light was found to consist of a band in the red, a narrower band nearly in the position of the D lines, and a broad green band which appeared to be broken up into channels or bands. The wave-lengths of some of these bands were roughly determined, but the authors do not appear to have determined exactly the relation which they bore to the absorption bands which appear in the same part of the spectrum.

Inasmuch as we have at the present time no very satisfactory theory of fluorescence, and as practically all quantitative work has been done with solutions, it seemed worth while to make a careful study of the relation between the fluorescent light emitted by sodium vapor and the light absorbed by the vapor under the same conditions. The chief points of interest which have been brought out by these investigations are the establishment of the fact that the fluorescent spectrum coincides exactly with the absorption spectrum, band for band and line for line, and a determination of the relation existing between the wave-length of the light which provokes the fluorescence and the nature of the fluorescent spectrum. The somewhat remarkable fact has been ascertained that the D line absorption

[1] *Wied. Ann.*, 57, 447, 1896.

is in no way connected with the production of the fluorescence, though the absorption at this point of the spectrum is much more intense than at any other.

By means of improved apparatus we have not only been able to photograph the fluorescent spectrum by means of a concave grating, but have been able to observe by means of a grating the fluorescent spectrum when provoked by approximately monochromatic light obtained with the Fuess monochromatic illuminator. The results of the work throw a great deal of light on the mechanics of the sodium molecule, and will doubtles prove of considerable value in the development of the theory of fluorescence.

APPARATUS EMPLOYED.

The fluorescence as observed in exhausted glass bulbs is never very intense, and the experiments can be continued only for a few moments owing to the speedy corrosion of the glass surface. Moreover, it is not possible to make use of very dense vapor, the generation of which requires a high temperature, owing to its action on glass. The form of tube which was employed by one of us in a previous investigation on the subject[1] enables vapor of great density to be obtained, but owing to the rapid distillation to colder parts of the tube, the experiment cannot be continued long enough for satisfactory photographic records. It is important not only to have a very dense vapor, but also to have the vapor confined within a small region and sharply bounded, in order that the light may not be weakened by absorption before it reaches the denser portions. To meet these requirements a new form of tube was designed and constructed which gave admirable results. With it a fluorescence ten or fifteen times as bright as anything that can be secured with glass bulbs was obtained and maintained continuously for five or six hours without recharging the tube. This tube we have had in action for fully forty hours, and it is only just beginning to show signs of leakage around the brazed joints, due to the action of sodium at a red heat on the silver, with which the joints were brazed. The tube can be very easily constructed, and when

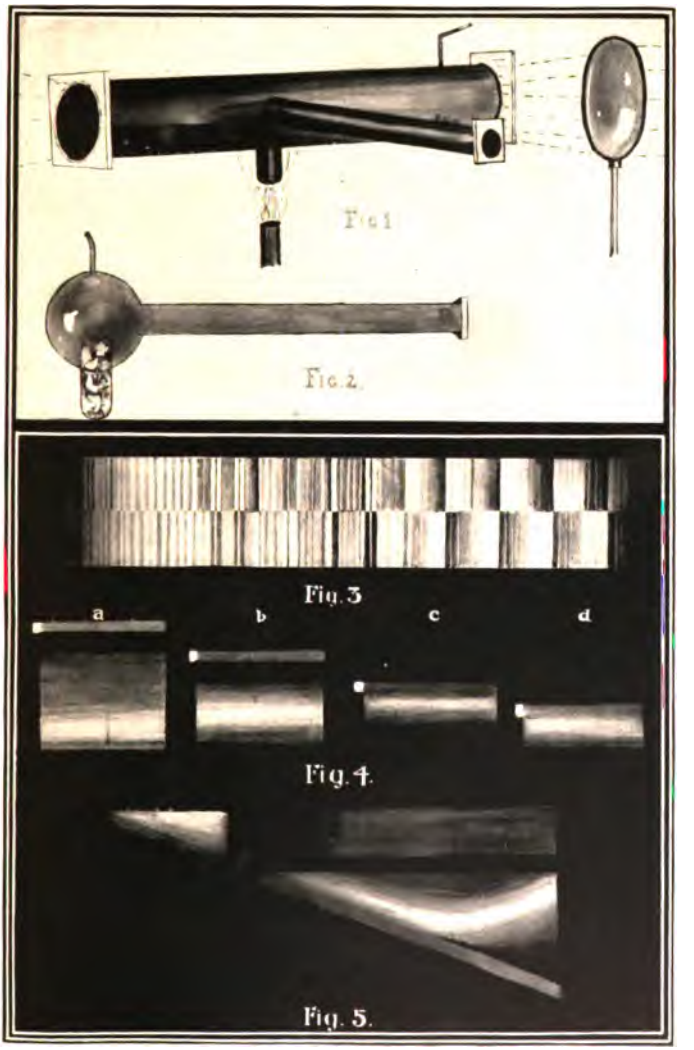
[1] R. W. Wood, *Proc. R. S.*, **69**, 157; also *Phil. Mag.*, (6) **3**, 141, 1902.

once charged can be used over and over again without any preparation whatever, for the exhibition of this most beautiful example of fluorescence. Its general form is shown in Plate II, Fig. 1. The large tube was a piece of three-inch (sixteen gauge) Shelby seamless steel tubing, two feet in length. A ten-inch piece of thin three-fourths inch steel tubing was brazed with silver into the large tube at the center, making a right angle with it. (Fig. 2, is a cross section of tube.)

Immediately below this tube, and at a right angles to both, a small iron crucible was brazed into the larger tube, projecting into its interior to such a distance that its rim was visible through the side tube. This crucible was made by boring out a three-fourths-inch iron cylinder. A small piece of brass tubing was also brazed into the large tube near one end, through which the whole could be exhausted. The crucible was filled with lumps of sodium, and the ends of the large tube closed with plate glass, carefully cemented on with sealing wax. The side tube was then closed in the same manner, and the whole exhausted to a pressure of about a millimeter, by means of a mercurial pump. The tube was then sealed off from the pump, a small piece of glass tubing having been cemented into the brass tube and drawn down into a capillary.

The tube was now mounted in a horizontal position and a large Bunsen burner placed beneath the crucible, which was soon raised to a red heat. The dense sodium vapor poured out of the mouth of the crucible and gradually condensed on the cooler portions of the tube. Sunlight from a heliostat was sent down the large tube, a lens placed close to the glass window forming an image of the Sun immediately above the neck of the crucible. On looking down the side tube an intensely brilliant green cone of light was seen, many times brighter than anything that has ever been obtained with glass bulbs. The floating specks of oxide which appear when the tube is first heated, and which shine with a dazzling white light, soon disappear and leave the fluorescence entirely uncontaminated. It is apparent that with this arrangement light enters at once into the densest vapor without suffering previous loss by absorption in vapor of less density.

PLATE II.



Red
Orange
Yellow
Green
Gr.-Blue
Blue

Red
Yellow
D_1 D_2

Green blue

Blue

Moreover, the fluorescent light passes down the observation tube without having to traverse more than a very thin layer of the vapor, a matter of considerable importance, as we wish to examine the fluorescent light unmodified by subsequent absorption. The large amount of sodium which could be stored in the crucible enables us to deliver a dense stream of vapor in front of the observation tube for a very long time, which is absolutely essential if photographic records are to be obtained.

SPECTRUM OF THE FLUORESCENT LIGHT.

The spectrum of the fluorescent light was first examined with a two-prism Steinheil spectroscope. The spectrum consisted of a red band and a green band, the latter appearing distinctly fluted. No trace of any bright line or band at or near the position of the D lines has ever been seen in any of our experiments. Its presence in the spectrum described by Wiedemann and Schmidt, and also by one of us, may possible have been due to the fact that in both of these cases the vapor was contained in a glass bulb heated by a Bunsen burner. This flame, colored by the sodium of the glass, may have been responsible for the appearance of a bright line in the place mentioned, a matter which can be very easily settled by repeating the experiments with the bulbs.

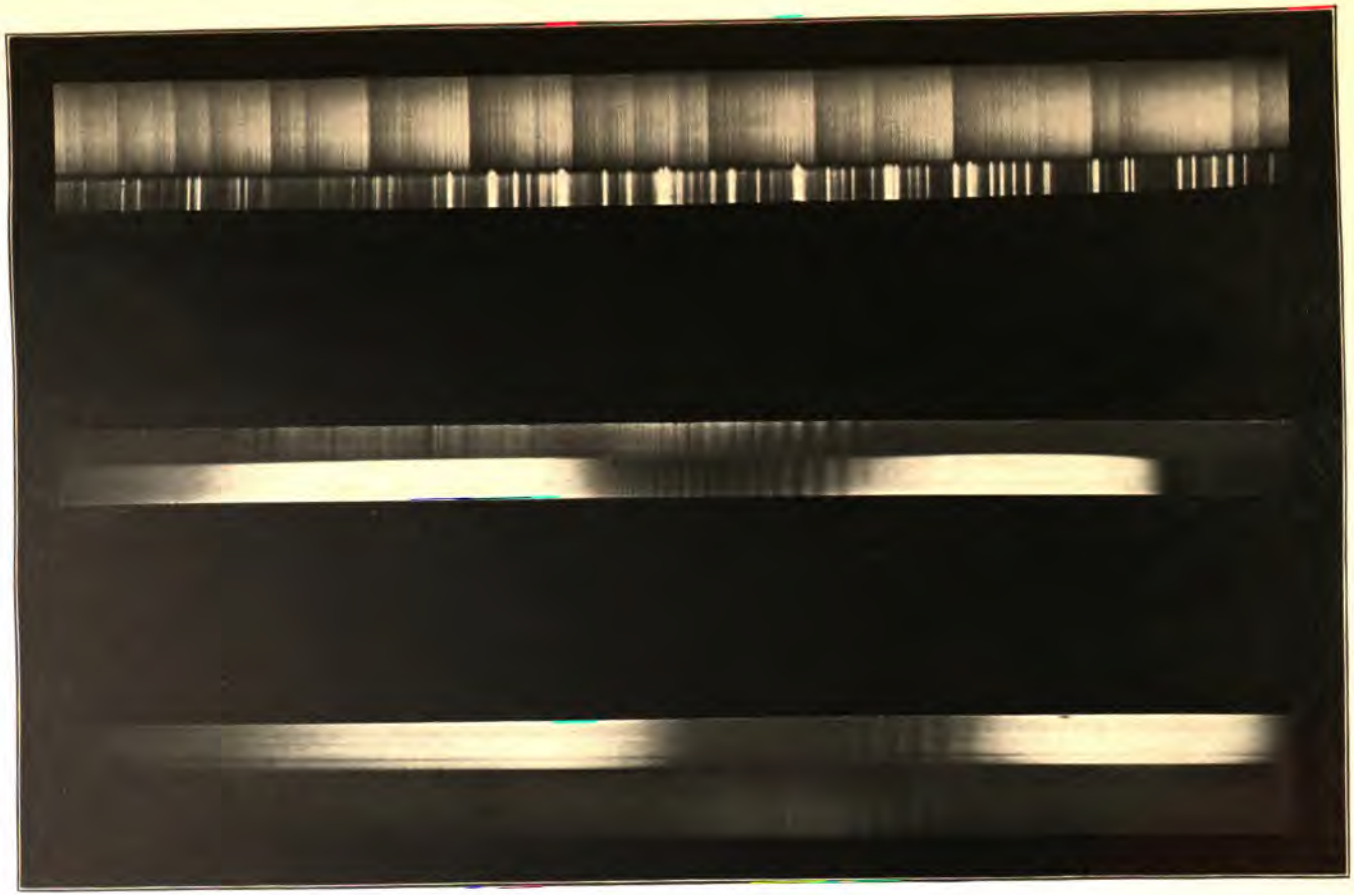
The marked resemblance which the fluted spectrum bears to the absorption at once suggests that it may be due to the fact that the fluorescent light has to pass through a certain amount of vapor before reaching the spectroscope; in other words, that it does not belong to the fluorescent spectrum at all, but is the result of absorption. To determine whether or not this was the case, an absorption comparison spectrum was formed by throwing some of the light which had passed through the tube into the instrument by means of a couple of mirrors and a small right-angled prism. It was at once apparent that the bright lines and bands of the fluorescent spectrum were exactly out of step with those of the absorption spectrum. To secure a fixed record of this fact, a camera was attached to the spectroscope and the two spectra photographed. The spectrogram confirmed the

visual observations in every respect, but the dispersion was too small to make the pictures very satisfactory.

A Rowland concave grating with 15,000 lines to the inch, of one meter radius, was then used in place of the spectroscope, and some excellent photographs were obtained with an exposure of less than an hour. The fluorescent spectrum was found to extend from wave-length 5340 to wave-length 4600 in the green and blue region. All of the photographs show in addition a faint solar spectrum extending from the end of the fluorescent spectrum down to the H and K lines. This is due to a small amount of white light which is scattered by occasional specks of oxide, or perhaps reflected from the wall of the tube. So far as we have been able to determine, the fluorescent spectrum is not contaminated with solar lines, since it is located in a less actinic region, and the scattered light is not of sufficient intensity to leave any appreciable record in this region.

These photographs brought out the remarkable fact that the fluorescent spectrum is the exact complement of the absorption spectrum. The two spectra were photographed in contact on the same film, and either one might have been a contact print taken from the other. In the upper spectrum, for example, there were two broad light bands with a fine dark line down the center, while in the lower spectrum occurred two broad dark bands with bright lines down the center. This same thing was true for all of the irregularities of shading in the very complicated fluted spectrum. A number of these photographs are reproduced in Plate III, Figs. 2 and 3. As most of the fine detail will doubtless be lost in the process of reproduction, a very careful drawing of the two spectra has been prepared from the original negative, which is reproduced with the direct records. (Plate II, Fig. 3. Absorption spectrum above, fluorescent spectrum below.)

The fluorescent spectrum in the green-blue region may be divided into three groups of bands: (1) those from λ 5340 to λ 5080, consisting of six broad hazy bands, which correspond in position to the fluted bands of the absorption spectrum; higher dispersion would doubtless resolve them into lines; (2) those



I (*a*). Absorption spectrum of sodium vapor. (Region from λ 4750 to λ 5100.)
(*b*). Corresponding comparison spectrum of iron.
II (*a*). Fluorescent spectrum. (The solar spectrum showing at the left half of the plate is due to diffuse light.)
(*b*). Absorption spectrum, taken with same grating.
III (*a*). Absorption spectrum, (*b*) fluorescent spectrum, showing their complementary character.

from λ 5080 to λ 4865, a group of irregularly fluted bands, the heads of which point toward the red; *i. e.*, they are strongest on the less refrangible side and shade off on the side of the shorter wave-lengths; (3) those from λ 4865 to λ 4600 which appear, under the dispersion used, as uniform narrow bands. The spectrum is the exact complement of the absorption spectrum taken with the same instrument, and further work with the concave grating of fourteen feet radius will show whether all of the very minute and narrow lines in the flutings of the latter spectrum are present in the former. We feel sure that the spectrum can be photographed with the large grating without difficulty.

Visual observations have shown that the red fluorescence is also fluted, though the flutings are only to be seen when the vapor is very dense and the illumination very intense. It has not yet been definitely proven whether this fluting belongs properly to the fluorescent spectrum or is the result of absorption. A new form of tube has been designed which, it is believed, will give a much denser vapor and make the settlement of this question possible.

The complementary nature of the photographs of the fluorescent and absorption spectra might lead one to suppose that the wave-length absorbed was re-emitted without change of wave-length. To test whether or not this was the case, it was necessary to illuminate the vapor with monochromatic light.

The Fuess monochromatic illuminator, which enables one to cut out a region of any width from a very intense solar spectrum and focus this light at a definite point in space, was arranged so as to send a cone of violet light down the tube, the rays meeting just above the mouth of the crucible. No fluorescence was detected. On gradually increasing the wave-length of the light, by turning the graduated screw which rotated the prisms of the instrument, it was found that the fluorescence appeared when wave-length 4600 was reached. On looking into the end of the large tube a very beautiful phenomenon was seen. The cone of deep-blue monochromatic light was distinctly visible, owing to

traces of oxide floating about, while at the point where the rays met in the dense sodium vapor there appeared a brilliant spot of *green* fluorescent light. As the wave-length was further increased, this spot increased in brilliancy, still remaining green, passed through a maximum, and then gradually faded away, disappearing entirely when the illuminating light became yellow. The vapor remained dark until the wave-length of the light exceeded that of the D lines, when the red fluorescence gradually developed, passing through a maximum in the same manner and then fading away.

The fluorescence of the vapor produced in this way was much less brilliant than in the former experiment, since the total radiant energy thrown into the vapor was very much less than when direct sunlight was employed; still the light sent down the lateral tube was intense enough to give a spectrum when the Steinheil spectroscope was directed down the tube. This spectrum was very feeble, however, and it was only with great difficulty that the changes could be followed which took place when the wave-length of the illuminating beam was changed. When blue light was employed, the complete green fluorescent spectrum seemed to be present with a maximum intensity at the yellow end. As the wave-length of the light was increased, the point of maximum intensity in the fluorescent spectrum moved toward the blue.

The important bearing which the phenomenon has on the theory of fluorescence made a more careful study of the relation between the wave-lengths of the absorbed and emitted light very desirable. To accomplish this, the arrangement of the apparatus was changed in the following manner: the monochromatic illuminator was rotated through a right angle, so that its emitting slit was horizontal, instead of vertical. The dense sodium vapor above the mouth of the crucible was now illuminated with a thin horizontal sheet of monochromatic light (the image of the horizontal slit of the illuminator projected in space by a lens). This arrangement, of course, limits the fluorescence to a thin horizontal layer of vapor, which, when viewed through the lateral tube, appears as a narrow line of bright green light. By viewing the

sheet of vapor edgewise we get a much more intense light, and since its width is small, we can dispense with a spectroscope entirely, simply viewing it through a prism or grating.

A small transmission grating of 14,480 lines to the inch, which gives a first-order spectrum nearly as bright as a 60° flint prism, was mounted in front of the lateral tube, and the line of fluorescent light viewed through it, the head being covered with a black cloth to cut out all extraneous light. It was found that by slightly inclining the large tube to the illuminating beam, it was possible to illuminate a minute projection on the inside edge of the lateral tube with the monochromatic light which caused the fluorescence; in other words, the near end of the horizontal image of the slit was brought upon this projection. The line of fluorescent light was thus tipped at one end with a small point of light similar to the light which produced the fluorescence. The spectrum of this small illuminated spot, which was of course a narrow band, fell alongside of the fluorescent spectrum, enabling a comparison to be made at once. Any exception to Stokes's law would make itself evident as an extension of the fluorescent spectrum on the more refrangible side of the position occupied by the small spot of illuminating light.

Some very remarkable and significant results were obtained with this disposition of the apparatus. Starting with violet illumination, there appeared through the grating only the small comparison spot of light which moved along as the wave-length was increased. As soon as wave-length 4600 was reached, the fluorescent spectrum appeared. Its appearance is indicated in Plate II, Fig. 4*a*. A strip of blue fluoresent light continuous with the reference spot showed that light of the same wave-length as the absorbed light was being emitted by the vapor. A region of darkness occurred on the less refrangible side (usual sense of the term), and then came a broad green band with a maximum of intensity on the end toward the yellow. Then came another gap extending considerably above the D lines, followed by a *very faint* trace of the red fluorescent band (not shown). On gradually increasing the wave-length of the light the following changes were noted. The spot of reference light, accompanied

by its fluorescent prolongation, moved up the spectrum, pushing the dark region before it, while the point of maximum fluorescence in the wide green band moved down the spectrum to meet the advancing spot. These changes are represented in Fig. 4 *b*, *c*, *d*, the point of maximum fluorescence nearly coinciding in position with the reference spot in *d*. On still further increasing the wave-length, the fluorescence became very faint, and appeared to extend a little farther down the spectrum than the position occupied by the small spot of light. This means an exception to Stokes's law when the wave-length of the illuminating light (green) has the maximum value which still yields fluorescence.

In Plate II, Fig. 5, an attempt has been made to represent these changes in the conventional way.[1] Everything shown in this diagram is, however, due to fluorescence, the deviated continuous spectrum of the light exciting fluorescence having been omitted. It will be seen that there is an emission of fluorescent light of the same wave-length as that of the exciting light, which gives the diagram an appearance not unlike the diagrams where the deviated continuous spectrum is shown. To make this diagram exhibit the changes just described, one has only to move a narrow vertical slit across it from right to left.

It was impossible to tell whether flutings were present in any portion of the fluorescent spectrum or not, owing to the comparatively wide source, and small dispersion employed. There is no reason for believing that they are not, but it does not seem safe to assume that the spectra are identical when the fluorescence is produced by white light and monochromatic blue light. The extent of the spectrum, and the distribution of the intensity in it, have been shown to be different in the two cases, and the flutings, say in the green, which are present when white light is used, may be absent when the fluorescence is produced by light of much shorter wave-length. To test this the spectrum will have to be photographed when the tube is illuminated with monochromatic blue light. To accomplish this with the present apparatus would probably require an exposure of six or eight

[1] MÜLLER-POUILLET, *Lehrbuch der Physik*, II, 1, 368.

hours. More favorable conditions are expected with a new form of tube now under construction, and it seems probable that we shall eventually learn a good deal about the dynamics of the sodium molecule. The results already obtained apparently prove that the light of longer wave-length emitted when the vapor is fluorescing is not the result of damping of the vibration, as assumed in Lommel's theory, but an emission resulting from the fact that the ions of longer free period are set in vibration either by the waves of shorter period or by the ions of short period which are thrown into vibration by these waves. It is not worth while to speculate about this, however, until the fluorescent spectrum produced by monochromatic blue light has been photographed.

In general, the fluorescence of a substance has its maximum intensity when the wave-length of the light is that of the light most strongly absorbed. Sodium vapor is an exception to this rule, for the D line absorption, which is far more intense than the fluted absorption, has nothing to do with the fluorescence. On Lommel's theory of fluorescence, the absence of any lateral emission of light by an absorbing medium is explained in one of two ways: either the absorption is of waves of different period (an octave below, for example) from the free period of the ions, or else the damping factor is so large that the emitted light lies in the infra-red region. In the case of sodium vapor, neither explanation is sufficient to account for the absence of fluorescence when the wave-length of the exciting light is that of the D lines, for, since Kirchhoff's law is obeyed in this case, the absorbed waves and emitted waves have the same period, and the absorption is a resonance phenomenon; moreover, the damping factor must be very small, since we have interference with large path-difference in the case of sodium light. The non-luminous vapor employed in these experiments may, of course, be in a different state from the vapor in a sodium flame, but even if this be the case, it does not seem justifiable to assume a large amount of damping, for this should cause a displacement of the absorption lines with reference to the position which they occupy in the case of absorption by a sodium flame.

It is worthy of note that lines corresponding in position to the fluted bands are absent in the emission spectrum of sodium vapor, except perhaps in the case of the temperature emission studied by Evershed, which does not appear to have been studied under high dispersion.

There seems to be no way of explaining the emission of the green light when the vapor is illuminated with blue light. It cannot be the result of the damping of the ions, whose free period is that of the blue light, for it is in all probability a discontinuous spectrum complementary to the fluted absorption spectrum.

It will be interesting to see whether the absorption of the vapor is directly affected by the circumstance that it is fluorescing at the same time. This was found to be the case in some experiments made by Burke upon uranium glass. It is also important to determine in what way the absorption and fluorescence are influenced by pressure. These matters will be investigated in the near future.

The statement that, when the wave-length of the exciting light is that of the D lines, no fluorescence is produced, requires modification. Strictly speaking, this is not true, though it is almost certain that the D line absorption is in no way responsible. This is due to the fact that the beam from the monochromatic illuminator is not strictly monochromatic, being in fact a band varying from ten to twenty Ångström units in width. As we shall show in the part of this paper dealing with the fluted absorption of the vapor, the fine lines can be traced up to the very edges of the broad band produced by the widening of the D lines when the vapor is very dense. It is unquestionably the absorption at these lines which gives rise to the very feeble reddish fluorescence which can be seen when the light furnished by the monchromatic illuminator is symmetrical about the D lines. Strictly monochromatic light of the wave-lengths of D_1 and D_2, no matter how intense, we feel sure would produce no fluorescence, unless the flutings actually cross this region, which is very likely the case. The only light which produces no fluorescence is green light in the vicinity of $\lambda 5530$ and the violet

below λ 4600, which, as we have shown, is all that is transmitted by the vapor when it is very dense.

THE ABSORPTION SPECTRUM OF SODIUM VAPOR.

The fluted absorption spectrum of sodium vapor was first observed and studied by Roscoe and Schuster in 1874. Subsequent investigations were made by Liveing and Dewar in connection with their work on the reversal of the lines of metallic vapors, and also by one of the writers.

Previous experiments by one of us having shown that it was impossible to secure photographs of the fluted spectrum with the concave grating, using the arc as a source of light, that were not contaminated by bright lines from the vapor of the lamp, it was necessary to find a source of light of great intensity and having a continuous spectrum. After experimenting with various sources of light, we finally adopted the Nernst lamp, which was found to fulfil the conditions specified.

In the first series of experiments the sodium was vaporized in an atmosphere of hydrogen, generated by electrolysis, and dried by passage over calcium chloride and phosphorus pentoxide. In order to remove traces of oxygen which caused the tube to smoke, the gas was finally passed over red-hot copper gauze. The metal was heated in tubes of thin steel, the ends of which were closed with plate glass. In the later experiments the tubes were exhausted with a mercury pump, and the metal volatilized in a vacuum. The latter method was found to be most satisfactory. The spectra were found to be identical in the two cases.

The tubes were either heated by means of Bunsen burners, or by a coil of No. 20 iron wire, insulated from the tube by a thin layer of asbestos board. The vapor is more uniform when the tube is heated electrically, for if the upper side of the tube is colder than the lower, a non-homogeneous medium results, the density being greatest along the floor of the tube. A Bunsen burner is better, however, for some experiments, where a very dense vapor is required. With it the metal does not distil so rapidly to the colder parts of the tube, and, with careful reg-

ulation of the flow of gas to the burner, it was used very successfully in many of the experiments.

The absorption spectrum was photographed with a fourteen-foot concave grating in the first order, the time of exposure varying from twenty minutes to an hour, according to the density of the vapor. The second-order spectrum was observed from time to time during the exposure, in order to keep the density of the vapor properly regulated.

It was found that the best results were obtained with a slit-width of 0.065 mm. The width of the slit for the iron comparison spectrum was 0.030 mm. In order to eliminate any errors due to changing the width of the slit, photographs were taken of both spectra with a slit-width of 0.030 mm, and the measurements made with the two sets of plates compared. The plates were measured on the dividing engine of the laboratory, and the measurements can be considered accurate to within 0.05 of an Ångström unit. The ultra-violet region was explored in the same manner, using a tube closed with quartz plates. No traces of any flutings were found, but the lines of the principal series were strongly reversed.

The fluted absorption spectrum makes its first appearance when the D lines are two or three Ångström units in width. It begins as nine bands, the heads of which point toward the violet end of the spectrum, *i. e.*, in each band the absorption decreases from the head toward the longer wave-lengths. The heads of the bands as they first appear are:

I.	4783.35	VI.	4932.97
II.	4809.92	VII.	4962.96
III.	4837.72	VIII.	5001.94
IV.	4865.60	IX.	5040.71
V.	4894.94		

There is also evidence of absorption lines in the first seven bands. These lines, however, do not come out strong at this stage, but appear as a slight shading on the bright background. In Band I can be seen the line $\lambda 4793.10$; in Band II, line $\lambda 4820.72$; in Band IX, $\lambda 5053.50$, and in Band VII, $\lambda 4979.28$, which appear as fainter heads of small bands into which the

others break up. Bands V and VI are about the center of the region of absorption.

A very slight increase in the density of the vapor is sufficient to bring out the heads of the bands as strong dark lines, as well as the fainter heads in the bands themselves. The bands are now seen to be made up of an immense number of lines, some of which are broad and some exceedingly fine, and in some of the bands there is an appearance as if two series were superposed.

At this time the region of absorption extends from about $\lambda 4600$ to $\lambda 5200$, a series of three bands, the more refrangible edges of which are broken up into smaller bands, appearing between $\lambda 5079$ and $\lambda 5200$, and a series of twelve bands, each of which is divided into smaller bands coming into view between $\lambda 4780$ and $\lambda 4600$. The wave-lengths of the principal lines in the fluted spectrum have been carefully measured by comparing them with standard lines in the iron spectrum.

A fluted absorption spectrum makes its appearance in the red and orange portion, when the density of the vapor is such as to give the green-blue flutings at their best. This spectrum has not been photographed and measured at the present, owing to the difficulty of getting plates sufficiently sensitive to the red. The Erythro plates, which were used by one of us in securing photographs of this region with a fourteen-foot grating some years ago, are no longer on the market, and the few experiments which we have made in sensitizing our own plates have been only partially successful. This region will be studied as soon as suitable plates can be obtained.

As the density of the vapor is further increased, the greenish-blue region disappears entirely, and the fine lines can be traced nearly up to wave-length 5400 from the blue end of the spectrum, and from the red end they are seen to fill the spectrum quite up to the broad D line absorption, which is now forty Ångström units in width. On crossing this broad black band the fine lines make their appearance again, which makes it seem probable that the red fluted spectrum crosses the region occupied by the D lines. A further increase in the density results in blotting

out the red, orange, and yellow completely, leaving only a rather narrow green region, the center of which is at wave-length 5530, and the violet below λ4600. The color of the transmitted light is now a very deep violet. The fine lines can be pushed into the green band from its opposite edges, and traces of them have been found all through it. Finally a very black broad band appears at the center of the green strip, which in a spectroscope of small dispersion appears as a rather narrow line. The center of this band is at wave-length 5530, approximately. We have not been able to photograph it with the grating, owing to the feebleness of the light, and the difficulty of keeping the vapor at this great density for a sufficient length of time; but its position was determined visually with a grating of about eight feet radius. This band has been seen by previous observers (Liveing and Dewar) and is described as persisting until the last traces of the fluted spectrum disappeared, as the vapor cooled off. We cannot understand this, for we have found it only when the vapor had the maximum density attainable, namely, when the blast-lamp played directly against the bottom of the tube, raising it to a bright red heat.

In order to see if an increase in the length of the absorbing column of the vapor produced the same effect as increasing the density of a short column, a steel tube five feet in length was used. In this were placed eight lumps of sodium of the same size, at intervals of about six inches. Eight Bunsen burners were regulated so that when one of them was placed under a lump of sodium the D lines were strongly reversed, but no trace of the fluted spectrum appeared. On adding burner after burner, exactly the same sequence of events were observed as in the previous experiments, where the density of the vapor was increased by raising the temperature of the tube. Though this was to be expected, it seemed worth while to try the experiment.

We have measured the wave-lengths of about 460 of the strongest lines between wave-lengths 4616 and 5738, which are given in the following tables :

WAVE-LENGTHS OF THE PRINCIPAL LINES AND BANDS IN THE ABSORPTION SPECTRUM.

GROUP A. λ 4600–5200.

Wave-Length	Character	Wave-Length	Character	Wave-Length	Character	Wave-Length	Character
4616.42	f	4815.84	8 b	4858.05	7 fine	4889.84	8
52.04	f	16.49	6 fine	58.75	8	90.75	7
52.18	b.h.	17.14	7 fine	59.67	7	91.52	9
59.60	b.h.	17.64	8 fine	60.41	8	92.43	7 fine
84.69	fine	18.57	8 b	60.99	fine	93.28	10 b
92.89	7	19.64	8	61.52	fine	94.13	6 fine
4704.36	1	20.72	7	61.78	fine	95.00	H 0.7
07.09	7	21.88	8	62.21	9		wide
09.72	1	23.15	7 fine	63.02	fine	96.06	4
18.29	7	23.82	8	63.78	6 b.h.	97.02	8
21.33	8	24.46	8 fine	64.45	fine	97.92	6
24.42	8	25.81	7 fine	64.88	7 fine	98.96	7 b.h.
28.82	8	26.58	8 fine	65.67	H broad	99.99	6
32.77	6	27.27	7 fine		0.4 wide	4900.94	9
35.78	6 b	27.99	8 fine	66.64	7	01.86	8
36.67	8	28.66	9 fine	67.44	7	02.64	4 fine
40.95	8 fine	30.28	8 b.h.	68.53	7	04.96	6
47.56	8 fine	32.83	8	69.54	8	05.79	7 fine
58.55	7 b.h.	33.47	9 b	70.01	10 b.h. Hs	06.43	4
51.96	8.5	35.36	fine	70.44	7 fine	07.16	7 b.h.
53.83	8.5	36.10	fine	71.33	8 b	08.22	9
58.68	9	36.51	7	72.05	fine	08.84	fine
60.85	7 fine	36.93	7	72.59	fine	09.47	9
61.86	7 fine	37.71	H *broad head* 0.5 wide	73.01	fine	10.12	fine
63.05	8 fine			73.50	fine	10.73	8
64.98	9 b	38.92	9 b.h.	73.93	fine	11.41	6
66.88	9 fine Hs	39.88	8 b.h.	74.46	fine	12.88	7
66.99	6 fine Hs	40.86	9	74.93	8	13.57	7
70.45	7 b	41.60	9	75.55	fine	14.26	8
71.47	7	42.40	8 b	75.95	fine	15.09	fine
77.63	8 fine Hs	43.14	9 fine	76.55	7	15.90	9
81.43	10	44.01	9 fine	77.11	8	16.62	6 fine
82.32	8 b	44.92	9	77.77	7 fine	17.52	7 fine
83.35	H 10 b	45.83	8 fine	78.31	8	18.27	7
89.83	9 fine	46.84	10	78.93	8 fine	19.10	4
90.84	9 fine	47.98	7 fine	79.52	9	19.99	8 fine
93.00	10 fine Hs	49.12	7	80.16	8.5	21.66	6 b.h.
95.30	10 b	49.75	fine	80.80	9.5	22.52	6
99.30	10 fine	50.39	9	81.51	7 fine	23.40	2
4800.74	8 fine	51.00	fine	82.22	10	24.33	7
02.80	8 fine	51.61	9	82.93	7 fine	25.28	7
04.62	10	52.27	fine	83.63	10	27.11	faint
05.77	9	52.89	9	84.33	7 fine	28.14	7
06.46	6 fine	53.65	fine	85.17	8	29.13	7
07.20	9 b	54.35	9	85.88	8	31.22	7 fine
07.90	7 b	55.06	fine	86.59	8	32.20	7 fine
09.93	H 8 b.h.	55.74	9	87.42	7 fine	33.03	H 0.7
10.68	10	56.54	7 fine	88.18	9		wide
13.68	7 fine	57.32	7 b.h.	89.06	7	33.70	fine

WAVE-LENGTHS OF THE PRINCIPAL LINES AND BANDS IN THE ABSORPTION SPECTRUM — *Continued.*

GROUP A. λ 4600–5200.

Wave-Length	Character	Wave-Length	Character	Wave-Length	Character	Wave-Length	Character
4934.27	7	4970.74	8 fine	5002.55	7 fine	5034.36	fine
35.22	9	71.21	fine	06.31	fine	35.38	7 b
36.04	7	71.68	8 fine	06.90	7 fine	36.33	5 b
37.00	fine	72.09	fine	07.62	7 fine	37.34	8 fine
37.61	7 b.h.	72.61	8 fine	08.38	fine	38.17	fine
38.80	6	73.16	fine	08.78	7 fine	39.52	fine
39.54	7	73.70	7 fine	09.26	fine	40.02	fine
40.26	fine	74.28	fine	09.76	7 fine	40.94	H. b. 0.9
40.74	fine	74.40	8	10.24	7 fine		wide
41.29	fine	75.41	fine	10.68	7 fine	42.58	7
42.23	6 fine	75.98	8	11.26	7 fine	43.44	fine
43.16	7	76.61	7	11.77	7 fine	44.93	fine
43.82	fine	77.17	9	12.36	7 fine	44.59	fine
44.37	8	77.85	fine	12.83	7 fine	48.81	6
44.97	7	78.45	8	13.38	7 fine	53.39	6 Hs
45.53	fine	79.20	4	13.99	7 fine	53.49	10 Hs
46.10	7	79.91	6	14.50	7 fine	54.46	8 b.h.
46.72	7	80.56	7	15.13	7 fine	56.06	8 b.h.
47.49	7 fine	81.26	8	15.77	7 fine	58.34	7
48.07	7 fine	82.00	6	16.47	7 fine	58.97	7
49.24	8	82.65	7	17.06	7 fine	61.07	7
50.08	8	84.08	8	17.77	7 fine	62.79	7
50.83	fine	86.52	6	18.38	7 fine	63.38	8
51.50	8	87.18	6 fine	19.05	7 fine	64.11	fine
52.23	2 fine	88.19	6	19.68	6 fine	64.84	h
52.97	2 fine	88.98	10 b	20.42	8	66.69	7
54.30	10 b	90.90	5	21.14	6 fine	67.37	7
56.86	2	91.52	fine	21.89	8	68.29	b
57.70	2 fine	92.56	9 b	22.61	6	69.08	6 b
59.43	8	93.34	fine	23.37	7	69.82	7 b
60.15	fine	94.18	5 b.h.	24.18	6 fine	80.38	H 10 fine
61.06	6	95.15	4 fine	24.99	7	87.37	H 7 b
62.13	fine	96.11	fine	25.77	5	95.86	9 H
63.02	H	96.53	fine	26.33	7	5118.78	Hs b
63.80	f.h.	97.29	3 b	27.37	fine	20.84	h
64.88	10 b.h.	98.05	8 b	28.21	8	33.80	Hs b
66.81	fine	99.25	8	28.98	7 fine	41.27	Hs b
67.37	fine	5000.09	6 b	29.99	8		
68.79	8 b.h.	00.86	6	30.73	7 fine		
69.31	fine	01.84	H 0.7 wide	31.76	8		
69.77	7 fine		wide	32.53	6		
70.23	fine	02.17	fine	33.47	5		

GROUP C. λ5300–5700.

The lines of this group are all of about the same intensity and very faint. Only the strong lines of intensity 8 or greater are marked.

Wave-Length	Character	Wave-Length	Character	Wave-Length	Character	Wave-Length	Character
5269.38		5409.78		5572.53		5651.94	broad and
75.34		14.96		74.82			strong 8
79.58		16.52		75.90		52.80	strong
5305.68		18.90		76.81		54.27	strong
11.47		25.42		81.86		55.83	
17.65		26.20		83.07		59.97	strong
25.64		32.68		84.59		64.12	
30.61		33.63		93.45	strong 8	65.63	broad
34.85		39.25		94.87	strong 8	71.25	
37.70		40.33		5602.04	broad	72.68	strong 8
43.66		41.36		03.34	broad	79.03	strong 8
46.47		48.20		07.60		83.01	strong 8
49.67		52.94		12.31		84.30	
52.44		58.09		13.78		89.70	
58.36		63.11		19.09	strong 8	97.87	strong and
65.31		82.21		20.56	strong 8		broad 9
66.09		96.99		26.31	strong 8	5706.08	strong and
70.28		5502.48		28.13	strong 8		broad 9
71.23		23.48		32.79		10.84	strong 8
78.19		28.58		36.90		12.48	
82.12		30.44		38.02		13.56	
83.38		33.16		41.18		14.72	
84.33		35.89		42.26		17.23	
90.46		40.03		43.55	broad	18.05	
91.41		59.52		45.37	broad	20.82	
97.77		62.85		46.92	broad	34.78	
98.46		64.28		48.78		36.89	
5402.61		65.70		49.60		38.32	
03.90							

JOHNS HOPKINS UNIVERSITY,
Baltimore, June 1903.

THE SPECTRUM OF HYDROGEN.

By LOUIS A. PARSONS.

ÅNGSTRÖM[1] was the first to observe the hydrogen spectrum. In 1853, upon examining the light from an electric spark in hydrogen at atmospheric pressure, he noticed "an intense line in C not clearly limited, and two maxima in F and G; and a third maximum in *h* was observed later." Plücker and Hittorf[2] were the first to employ glass tubes with metallic electrodes and provided with capillary portions, and to study the spectra of gases at low pressure which were rendered luminous by an electric discharge. They found that the spectrum of some gases they examined was different when they employed a condenser in the discharge circuit of their induction coil, from that produced by the induction coil alone, and concluded that the former was due to higher temperature resulting from the condenser discharge. They obtained by means of the condenser discharge in hydrogen the three bright lines which they designated as *Hα*, *Hβ*, and *Hγ*. These lines they found expanded as the pressure of the gas was increased, and at high pressures the spectrum was transformed into a continuous one with *Hα* as a broad band rising from it. Without the condenser they observed another spectrum of hydrogen, "consisting of very many bright fine lines, especially in the neighborhood of the sodium line. This spectrum," they observed, "may be seen simultaneously with the three characteristic lines *Hα*, *Hβ*, and *Hγ*; but at increased temperature, when these lines begin to expand, it entirely disappears." With water vapor alone they obtained the lines *Hα*, *Hβ*, and *Hγ*.

There are now generally recognized two spectra of hydrogen: (1) the "elementary" or "four-line" spectrum, and (2) the many-line spectrum. The former includes the lines *Hα*, *Hβ*, *Hγ*, and *Hδ* in the visible portion, the last of which was soon added to the three given by Plücker and Hittorf, and nine lines in the ultra-violet. The many-line spectrum has also been designated

[1] *Phil. Mag.*, (4) **42**, 395, 1871. [2] *Phil. Trans.*, **155**, 1, 1865.

as the "compound spectrum" or the "secondary spectrum." The work of various observers has shown that the two spectra may exist together, and that in general as the lines of the elementary spectrum broaden the compound spectrum disappears. The question was early raised as to what caused the broadening of the lines, and what determined the presence of one spectrum or the other. Plücker and Hittorf, Wüllner,[1] and others[2] believed that the broadening of the lines and the change in the four-line spectrum was due to increased temperature. Others believed the effect was due to pressure,[3] and still others that the intensity of the electric discharge was the controlling factor.[4]

Trowbridge and Richards in 1897[5] found that a continuous discharge from their high potential storage battery of 5,000 cells, obtained by attaching the terminals directly to the electrodes of the vacuum tube with a high resistance in series, always gave a white glow in the capillary and the many-line spectrum of hydrogen, while the discharge from a condenser of considerable capacity gave the four-line spectrum, the color of the discharge being deep red. The introduction of self-induction, or of resistance to damp the oscillations, changed it back to the many-line spectrum. They believed that one spectrum was characteristic of the oscillatory and the other of the non-oscillatory discharge, the character of the electric discharge being the determining factor.

Further work, in which he employed his enlarged battery of 20,000 cells, however, led Trowbridge to abandon this idea[6] and to come to the conclusion that the many-line spectrum is the true spectrum of hydrogen, while the so-called four-line spectrum is due to water vapor; the existence of the four lines in the solar spectrum being, as he believed, due to the presence of water vapor in the Sun. His reasons for the conclusion were: (1) With powerful condenser discharges in apparently dry hydrogen he always obtained the four-line spectrum, and essentially the same

[1] *Phil. Mag.*, (4) **37**, 405, 1869. [2] FIEVEZ, *Comptes Rendus*, **92**, 521, 1881.

[3] SCHUSTER, *Brit. Assoc. Rep.*, **39**, 1873.

[4] STEARN and LEE, *Phil. Mag.*, (4) **46**, 406, 1873.

[5] *Phil. Mag.*, (5) **43**, 135, 1897. [6] *Ibid.*, (5) **50**, 338, 1900.

spectrum whether the tube was filled with nitrogen, air, or hydrogen, notwithstanding the greatest care taken in drying the tubes. He believed it was impossible to so thoroughly dry glass tubes as to remove all water vapor, and that the water on the glass is torn off by the powerful condenser discharges. (2) The four-line spectrum was easily obtained in all cases where water vapor was known to be present. (3) After a partial melting of the aluminium electrodes it was difficult or impossible to get the many-line spectrum (owing to the formation of compounds), but the four-line spectrum was obtained as easily as before with the powerful condenser discharges.

In a later paper Trowbridge[1] described some further experiments on high potential discharges through hydrogen. In one experiment, by using heavy copper electrodes he obtained by means of a continuous current a deposit of bright copper around the kathode and of oxidized copper at the anode, "the dissociation of the water vapor thus showing an electrolytic action closely analogous to that of a voltaic cell." He repeats his conviction "that the so-called line spectrum of hydrogen cannot be considered apart from the spectrum of water vapor," and is also "led to the conclusion that a certain amount of water vapor is essential to all electrical discharges through gases." He says: "Just as aqueous vapor seems to play an important rôle in most chemical reactions, so, it seems to me, its presence in rarified gases enables dissociation to take place which determines the strength and character of the electrical discharge;" and in another place: "The passage of electricity through a gas depends on the dissociation of the hydrogen and oxygen, by means of which change in the distribution of energy the gases are made luminous." While still holding that the four-line spectrum, as distinguished from the many-line one, is that of water vapor, he here advances to the idea that water vapor is essential to all electric conduction, in that it makes possible the passage of the current. During the passage of the current there may be produced the characteristic spectrum of the gas and also the spectrum of water vapor. In a recent article Trow-

[1] *Amer. Jour. Sci.*, **12**, 310, 1901.

bridge speaks of another spectrum of water produced by condenser discharges of great intensity giving a brilliant white light, which spectrum is continuous throughout the visible portion and consists of bands in the ultra-violet. These "spectra of water vapor" he believes are spectra arising from dissociation of water molecules.

J. J. Thomson in 1895[1] observed what appeared to be electrolysis of hydrogen gas in a vacuum tube, or the separation of positive and negative hydrogen ions. Using a tube with a metallic diaphragm, and allowing the current to run in one direction for some time, he obtained the *Hα* line strong on the positive side of the plate and *Hβ* faint, while the reverse was true on the negative side. His interpretation was that on the negative side there was an excess of positively charged hydrogen ions, and on the positive side of negatively charged hydrogen ions, the former having a natural vibration giving *Hβ* and the latter one corresponding to *Hα*. He also obtained by passing the current in one direction for a time through a mixture of hydrogen and chlorine, the hydrogen lines bright near the kathode and the chlorine lines weak or absent, and the reverse at the anode. This experiment was called in question by Morris-Airey[2] working in Kayser's laboratory, who believes that these effects were due merely to temperature differences. He employed two tubes, connected as shown in the figure, running a continuous current through a mixture of chlorine and hydrogen between electrodes *A* and *B* until the chlorine lines practically disappeared and the hydrogen lines became very bright at the kathode, and then

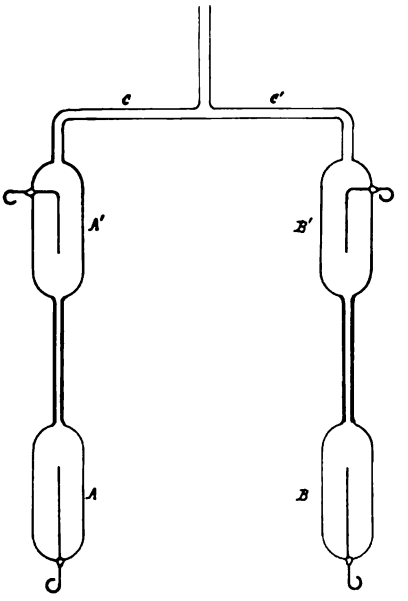


FIG. 1.

[1] *Nature*, **52**, 451, 1895. [2] *Phil. Mag.*, (5) **49**, 307, 1900.

he immediately sealed off the tubes at c and c'. Afterwards when they were used as separate tubes they gave identical spectra.

In a number of the *Philosophical Magazine* about a year ago, Lewis[1] discussed "the rôle of water vapor in gaseous conduction" suggested by Trowbridge's paper. He says: "It seems that the complete removal of water vapor from vacuum tubes is perhaps possible, and still the hydrogen or other gases contained therein may transmit the current and give their characteristic spectra." Deslandres, Wiedemann, and Lewis himself all found that by heating sodium in a tube containing hydrogen, although the hydrogen lines were weakened when the sodium vapor was dense, they came out with their full intensity when the sodium was cool. Using hydrogen with the same treatment, however, Deslandres succeded in getting rid of the water vapor lines. Lewis put pure sodium in a tube filled with hydrogen, and after heating it frequently to drive off impurities, he filled it with pure hydrogen, and then formed a "sodium dew" over the capillary and walls of the tube. Whenever it was heated the hydrogen lines were weak, but came out quite strong when the tube was cooled, and were just as strong after the tube had been used continuously for two weeks. When the tube was broken there were found large patches of sodium whose metallic surface was not tarnished, which led Lewis to observe that whatever other impurities might have been there, it is difficult to see how any free water vapor or oxygen could have been present." He concludes that "it seems at least doubtful whether water vapor is necessary to gaseous conduction."

Wilsing[2] recently made some interesting experiments on the spectrum of hydrogen. Making the usual observation that in general the introduction of a spark gap changed the many-line to the four-line spectrum and broadened the lines, he remarks that it cannot be an electrical resonance effect due to the oscillatory discharge, for the effect was not changed by varying the

[1] *Phil. Mag.*, (6) **3**, 512, 1902.

[2] "Investigations on the Spectrum of *Nova Aurigae*," *Pub. d. Astrophys. Observ. zu Potsdam*, **12**, 77–102, 1900.

period of the oscillation; and he comes to the conclusion that it is due to high temperature resulting from the condenser discharge.

A large number of the ultra-violet lines of the elementary spectra have been found as bright lines in the chromosphere of the Sun, but, on looking for their reversals, among the Fraunhofer lines of the solar spectrum, those of shorter wave-length $H\delta$ (H line) do not appear. The possibility of certain conditions of temperature or pressure in the photosphere being responsible for this suggests itself. Campbell[1] found that in the spectra of certain stars, the ultra-violet hydrogen lines are dark, while those of greater length are bright, there being a regular gradation between the two; and Kayser[2] calls attention to the fact that this is in accordance with the deductions of Kirchhoff's laws, assuming that they apply to this radiation.

Schumann[3] on examining the spectra of hydrogen from an end-on tube found, on varying the pressure, what appeared to be a reversal and a displacement of $H\beta$ or separation into components. Trowbridge[4] recently obtained a reversal of hydrogen lines in employing his intense condenser discharges, which he attributed to a selective reversibility of the silver salts of the photographic plate, and concluded that it was unsafe to draw conclusions from reversals as to the temperature, pressure or any other condition of the source, and considered this to have a very important bearing on astrophysical problems. Wood,[5] however, obtained effects entirely analogous to those of Trowbridge by subjecting a photographic plate to two light radiations, one of short duration and great intensity, followed by one of relatively great duration and feeble intensity (the Clayden effect); and he also found no selective reversibility in the silver salts, and concluded this could not have any bearing on astrophysical problems.

It is because of the bearing on astrophysical problems, the question of the physical condition and constitution of the heavenly bodies, that the determination of the exact conditions governing

[1] ASTROPHYSICAL JOURNAL, **2**, 177, 1895.

[2] *Ibid.*, **4**, 313, 1901.

[3] *Astronomy and Astro-Physics*, **12**, 159, 1893.

[4] *Phil. Mag.*, (6) **4**, 156, 1902.

[5] *Phil. Mag.*, (6) **4**, 606, 1902.

the changes in the hydrogen spectrum become of great importance. There are certain differences in the spectra of hydrogen as obtained in the laboratory and in the spectrum of the Sun and of the stars. Wright and Campbell,[1] from measurements of hydrogen lines in the spectrum of *o Ceti*, came to the conclusion that there was an error of one-tenth of an Ångström unit in the value of the wave-length of $H\delta$ given in Rowland's tables of the solar spectrum. The same conclusion has been reached by Evershed from measurements of the flash spectrum obtained in India during a total eclipse of the Sun. Jewell,[2] after carefully going over Rowland's measurements again, however, concluded that there was no error in the original calculation, the wave-length of $H\delta$ as a dark line in the Sun being 4102.00, while the value obtained by Evershed and by Wright and Campbell for $H\delta$ as a bright line in the Sun and stars (4101.85) agreed with the value obtained in vacuum tubes, which last also agreed with the place as given by Balmer's formula ($H\epsilon$ in the Sun also is not in accord with Balmer's formula).

Not all of the lines of the four-line spectrum of hydrogen appear in the dark-line spectrum of the Sun. Only $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$ are distinct; those of shorter wave-lengths are of a different character—broad, hazy, and indistinct; and those of shortest wave-length are not distinguishable at all. This also is at variance with the four-line spectrum as ordinarily observed in the laboratory.

The elementary, or four-line, spectrum, according to the classification of spectra founded on the numerical relations and character of the lines, is not the principal but the first subordinate series. In 1897 Pickering[3] discovered another series of hydrogen lines in the spectrum of the star *ζ Puppis*, which according to Kayser[4] and Rydberg[5] constitute a second subordinate series following a different law from the first series, but ending at the

[1] W. H. WRIGHT, ASTROPHYSICAL JOURNAL, 9, 50, 1899.

[2] ASTROPHYSICAL JOURNAL, 9, 211, 1899.

[3] *Harvard College Observatory Circular* No. 12; ASTROPHYSICAL JOURNAL, 5, 92, 1897, also 13, 230, 1901.

[4] ASTROPHYSICAL JOURNAL, 5, 95, 243, 1897. [5] *Ibid.*, 6, 233, 1897.

same point in the spectrum. Rydberg also calculated the wave-lengths of the principal series of hydrogen, beginning at the same point in the spectrum as the second subordinate series. The second subordinate series, though observed in some stars, has never been obtained in the laboratory, and the principal series has never been observed at all.

The spectrum of hydrogen, then, is not by any means invariable; there are differences in the spectrum as observed in the Sun, in the stars, and in vacuum tubes in the laboratory; and in the latter there are observed changes due to different conditions of the discharge. Because of the bearing of the latter on the astrophysical problems involved, it becomes of great importance to accurately determine the causes of the changes we are able to bring about in the laboratory so as to have a guide in interpreting the particular variations in the spectra of the heavenly bodies. What, we are led to inquire, is the cause determining the occurrence of the four-line and the many-line spectrum, and of the changes in the spectrum as the conditions are varied? Is it high temperature, pressure, the electrical conditions of the circuit, or water vapor, or what? It was to obtain evidence to aid in answering these questions that the experiments described here were undertaken.

APPARATUS.

The hydrogen used in these experiments was prepared by the electrolysis of dilute sulphuric acid, and was passed through a system of tubes containing substances for drying and purifying to the Geissler tubes, and thence to another series of tubes to the mercury pumps used in exhausting, making a completely closed system about sixty feet long from generator to pump, with only one stop-cock at the generator. Calcium chloride, sulphuric acid, caustic potash, potassa-lime, and phosphorous pentoxide were used for drying, sulphur and gold foil for the removal of the mercury, and heated copper foil to remove oxygen. There was no trouble from hydrocarbon vapors, oxygen, or air after everything had been pumped out thoroughly, and most of the time there was no trouble from mercury vapor, the mercury lines not appearing in the spectrum, or else very faintly.

As a source for the electric current a storage battery of 4000 cells was constructed. For obtaining the continuous discharge the battery was connected to the vacuum tube with a water resistance of about a million ohms in series. A milliammeter was used to give the current. The oscillatory discharge was obtained by connecting the terminals of the battery to a glass plate condenser, equivalent to one or more Leyden jars, which in turn was connected through a spark-gap and coil of self-induction of 0.009 henry (which coil could be short circuited at will) to the vacuum tube.

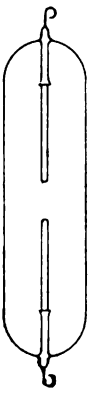


FIG. 2.

A number of tubes were used of different diameters and with different distances between the electrodes, and the spectrum was observed with a Steinheil spectroscope as the pressure of the gas, the temperature, and the electrical conditions of the circuit were varied. Some of the tubes had capillaries, but most of them had none, being of uniform diameter of from 6 to 22 mm. The electrodes in most cases were of aluminium rod 2 mm in diameter. The pressure of the gas was varied from nearly an atmosphere down to a millimeter (and sometimes a fraction of it).

CONTINUOUS DISCHARGE — EFFECT OF PRESSURE.

The continuous current of a few milliamperes from the battery always gave the many-line spectrum and the four lines all sharp and fine even at the highest pressures (500 mm, or even 700 mm). The discharge in wide tubes was characterized by a bright point on the anode (which appeared always to give the many-line spectrum), and a white velvety glow at the kathode, which appeared on the tip of the electrode at high pressures and gradually expanded until it filled that part of the tube at a pressure of a millimeter, and in which $H\beta$ could always be seen. There was no essential difference in the action of the tubes with different electrode distances, as the pressure was lower; the discharge ceased sooner in the tubes with the electrodes close together than in those with electrodes farther apart. In the capillary the spectrum was faint at high pressures, appearing first in

the green and blue, with $H\beta$ not as bright as other fine lines in that region. With decreasing pressure the spectrum first spread toward the red and violet, and then the intensity of the whole increased, and also the relative intensity of the four lines, until the point of maximum resistance was reached, beyond which the process was reversed.

VARIATION OF CURRENT DENSITY.

The same changes that occur in the glow discharge and in the spectrum during a decrease of pressure were produced by increasing the current either by an increase of voltage or a decrease of external resistance, but the changes also took place with decreasing pressure at constant current (though less rapidly than with decreasing pressure and constant external resistance).

The abruptly oscillating discharge, occurring with the condenser and spark-gap, gave the deep red spark discharge between the electrodes, and always tended to broaden the lines and to increase the relative intensity of the four lines. At high pressures, with considerable capacity, the four lines appeared very bright and broad, while the *Al* lines from the electrodes were bright and fine. With decreasing pressure the four lines gradually became narrower, $H\alpha$ first becoming fairly sharp, and then $H\beta$. All decreased in intensity, but $H\delta$ faded away relatively quite rapidly. The electrode lines disappeared in the central portion, and finally extended only a short distance from the electrodes. At a pressure of more than 100 mm the many lines appeared in the red and orange, at first broad and indistinct and massed together, but sharp and distinct at lower pressures. With decreasing pressure the many lines increased in intensity relatively to the four lines. The introduction of self-induction even at pressures as high as an atmosphere made the four lines narrow, and brought out the many lines quite distinct and fine. Excepting at very high pressures, self-induction increased the intensity of the many-line spectrum, and decreased the intensity of the four lines.

A tube 15 mm in diameter, with electrodes 6 mm apart, showed a heavy red spark discharge at high pressure, which,

when the pressure was lowered to about 100 mm, contracted to a cone with its base at the anode and extending almost to the kathode, and was surrounded by a blue sheath broadest at the kathode. The blue glow gave a faint continuous spectrum and, apparently, the many-line spectrum of hydrogen, though faint.

FIG. 3.

The red discharge gave the four lines broad and bright near the electrodes, $H\alpha$ and $H\beta$ extending all the way across and rather fine near the kathode, while $H\gamma$ and $H\delta$ were seen only near the anode. With decreasing pressure the red cone shortened until it became a point on the tip of the anode, and finally disappeared, leaving the discharge entirely blue. Another tube of a larger diameter, but in all other respects identical with this one, gave the deep red discharge throughout this entire range of pressure. It was at first difficult to see how two tubes, both connected to the apparatus and at precisely the same pressure, and differing only in diameter (and in that very slightly), would act so differently. It was easily shown, however, that the resistance of the tube was the determining factor, being high enough in the narrower tube to prevent the abruptly oscillating discharge which took place easily in the larger one. When a high external resistance was put in series with the large tube, it gave the blue discharge similarly to the narrower tube, and when the capacity was increased the red discharge occurred in the narrow tube at comparatively low pressures. In fact, in all the experiments, whenever the electrical conditions of the circuit were such as to give the abruptly oscillating discharge, the red discharge and the four-line spectrum resulted. Tubes of different diameters were employed, but they showed no differences, excepting in so far as the greater resistance of the narrower ones causes the change to the blue spectrum at higher pressures.

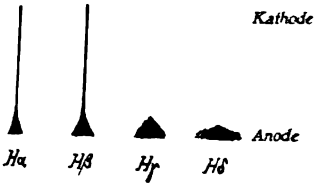


FIG. 4.

EFFECT OF HEATING.

If the four-line spectrum is due to water vapor, one might expect an external heating of the tube sufficient to drive off water vapor, to cause an increase in the red discharge, and a corresponding increase in the intensity and width of the four lines. Or if the four-line spectrum is due to high temperature (of the gas as a whole), we should look for the same effect when the tube is heated. However, heating the tube externally by means of a Bunsen burner, or internally by continuing the discharge for some time, instead of having the effect of increasing the amount of red in the discharge and producing a change in the direction of the four-line spectrum, had the opposite effect, decreasing the red or changing to the blue discharge.

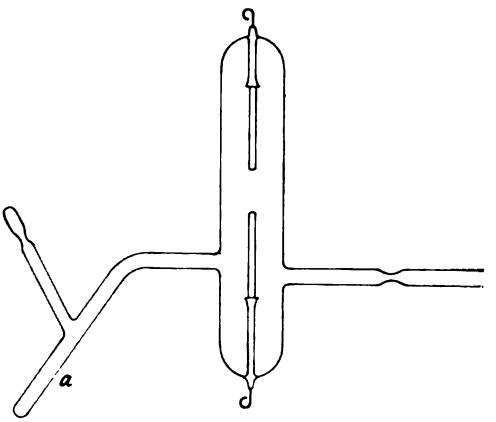


FIG. 5.

EFFECT OF WATER VAPOR.

Two tubes, as nearly identical in all respects as possible and of the form shown in the adjoining figure, were connected, one, *A*, directly to the apparatus, and the other, *B*, through drying tubes of calcium chloride and phosphorous pentoxide. In the bulb of *B* was placed about $\frac{1}{10}$ cu. cm of water, which was kept frozen while the tubes were being exhausted, washed out, and filled with hydrogen. The pressure was lowered to 44 mm, when the discharge was blue, with red extending only a short distance from the tips of the electrodes, and identical in the two tubes. The ice was melted and a short time allowed for the water to evaporate. No effect was observed then or when the water was slightly warmed. But when the water was boiled so that some passed over into the main tube, the discharge in *B* became red all the way across, and *Hα*, *Hβ* and *Hγ* became much brighter.

The discharge remained very decidedly red after running for some minutes, when it was certain that the pressure was the same in the two tubes. Tube *A* gave the blue discharge with the red on the tips of the electrodes, just as before. On cooling the side tube in the freezing mixture, the discharge became decidedly less red. When nearly all of the water was boiled over into the main tube, the discharge became very red, and $H\alpha$ and $H\beta$ came out very bright and broad, $H\gamma$ was broad, and $H\delta$ seemed to be present, faint, and broad. On again placing the side tube in the freezing mixture and passing the current through it, the discharge became decidedly less red, and finally became somewhat blue, though not as much as the other tube.

The experiments appear to show a very marked connection between the four-line spectrum and the presence of water vapor or water. But it is to be noted that the effect was produced only when water or water vapor was present in large quantities, and did not appear in the least with only small traces of the vapor. With large quantities present some evidently was on the electrodes and in the path of discharge, and the vaporization and dissociation of the water by the current would produce a high local increase in the pressure; and the effect observed was exactly that which would have been brought about by an increase in pressure. Or, the sudden ionization of the water might well produce a violent change in the passage of the electric current—causing the abruptly oscillating discharge of which the four-line spectrum was characteristic.

When one of the tubes cracked, the presence of the air leaking in was shown by the oxygen and nitrogen bands. With the discharge without the condenser, or with the condenser and self-induction, most of the current was carried by the nitrogen, scarcely anything else being seen in the spectrum excepting the nitrogen. With capacity without self-induction and with a small spark-gap, the oxygen predominated, while with a large spark-gap giving a very definite disruptive discharge the current was carried almost entirely by the hydrogen, the four lines coming out brilliantly.

DISCHARGE IN AIR.

A system of four tubes varying in diameter from 6mm to 22mm, all with electrodes about 7mm apart, and a bulb 90mm in diameter, with electrodes 10mm apart, were first used, when filled with air, before any hydrogen had been admitted. Excepting at quite low pressures, the walls of the bulb were so far away from the discharge as not to be at all heated or apparently affected by the discharge in any way. In the narrower tubes, where the discharge reached the glass walls, *Hα* and *Hβ* were seen very bright (especially *Hα*) and broadened throughout a great range of pressure, while they were not seen at all under like conditions in the bulb. This pointed very strongly to the conclusion that the four-line spectrum is due to water vapor driven from the walls of the tube. But although *Hα* was bright in the narrower tubes, *Hβ* was usually comparatively faint, *Hγ* was never seen with certainty, and *Hδ* not at all. When the tubes were filled with dry hydrogen, it is not likely that they contained more water vapor than when filled with air from the room, and yet, with the hydrogen, all the tubes, including the bulb, showed with the condenser discharge *Hα*, *Hβ*, *Hγ*, and *Hδ*, all very bright and broad. The four-line spectrum was plainly not the same in air as in hydrogen, and certainly the evidence was not in favor of the assumption that the four-line spectrum is due only to water vapor and not at all to hydrogen as such.

It may be, as Trowbridge claims, that "a certain amount of water vapor is essential to all electrical discharge through gases," and that the four-line spectrum of hydrogen is intimately associated with the dissociation of water vapor. The idea that a perfectly dry gas of any kind not ionized is a non-conductor of electricity is certainly not a new one. In a paper published in the *Philosophical Magazine* in 1893,[1] J. J. Thomson considered the effect upon the formation of drops and said: "In a vacuum tube with electrodes at high potential, the presence of the electric charges would tend to form drops of water, if there should be any water vapor present;" and he concluded that the mole-

[1] (5) 36, 313.

cules might be dissociated into charged ions by condensation on the surface of little spheres of water or of any other liquid of high specific inductive capacity, and these charged ions in turn would facilitate the formation of other drops of water. This action would render easy the passage of a current from one electrode to the other. The presence of water vapor in this way would tend to ionize whatever gas were present in a vacuum tube, and thus render possible the passage of a current and put it into a condition to vibrate in such a way as to give its characteristic spectrum. But it cannot be said that water is absolutely necessary for this, for the same might be done by any liquid of high specific inductive capacity. In fact, anything that could ionize the gas in the tube, as X-rays, Becquerel rays, ultra-violet light, or the *Entladungstrahlen* from a spark, would render the gas conducting, and in a perfectly dry gas, not ionized with high enough potential, we may obtain a disruptive spark discharge. And a gas that from any cause whatever is carrying electricity through it, it seems fair to assume, may be giving off its characteristic spectrum, entirely apart from the presence of water vapor or any other foreign substance.

As stated before, some of the experiments described in this paper, as well as the experiments of Trowbridge, point to a connection between water vapor present in a tube and the four-line spectrum of hydrogen. It does not seem, however, that one is warranted in concluding that the four-line spectrum is the spectrum of water vapor. It may be that this spectrum is due to the particular vibrations produced at the instant of dissociation of water molecules, or of recombination of hydrogen and oxygen ions. But may it not even in that event be that the four-line spectrum is a true spectrum of hydrogen, which is due to the particular vibration of the hydrogen ion occurring at the instant of ionization, which ionization may be that of a hydrogen molecule, or of a molecule of any other compound of hydrogen as well as of water, while the natural vibrations of the free hydrogen ion give rise to the many-line spectrum? The experiments stated in this paper could be explained in this hypothesis, and could not the experiment of Trowbridge as well?

But we are not excluded from the possibility that pressure or the electrical conditions of the circuit are the determining factors. This much seems certain, whenever the discharge is comparatively steady, as with a battery, the many-line spectrum is produced, while the abrupt oscillating discharge always gives the four lines bright. The presence of water vapor in quantity (as in one of these experiments) would in the electric field result in the formation of drops of water, the vaporization and the dissociation of which would, as has been stated, cause great local increase of pressure, which would account for the facts observed, if we consider it a pure pressure effect.

The most natural conclusion from this investigation is that the four lines are increased in intensity and in width whenever anything occurs that renders the electrical discharge discontinuous and very abrupt and violent. As to what the process or mechanism is by which the characteristic mode of vibration is produced, whenever this occurs, we cannot say.

A series of photographs of the hydrogen spectrum were taken with a quartz spectrograph with capacity and self-induction, at pressures ranging from about 700 mm down to 2 mm, which gave results in harmony with the eye observations already noted. A point of interest that attracts one's attention in looking over the plates is that the lines $H\epsilon$, $H\zeta$, $H\eta$, $H\theta$, etc., assigned to the four-line spectrum, are either not present at all or else do not seem to behave in the same way as $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$, which are very intense and broad at high pressures, and gradually decrease in width and intensity with decreasing pressure; while the others are faint and narrow at high pressures and change very slightly if at all with decreasing pressure.

CONCLUSIONS.

1. The "compound spectrum," "secondary spectrum," or "many-line" spectrum of hydrogen never occurs without the four lines. The four-line spectrum may occur at high pressures without the many lines.

2. The four-line spectrum is characteristic of an abruptly oscillating discharge, and the many-line spectrum of the contin-

uous discharge of the battery, the less violent oscillatory discharge which occurs with self-induction in the circuit giving a spectrum midway between the two—the many-line spectrum with the four lines very bright and fine.

3. The production of the four-line spectrum, or the broadening of the lines, may be due to high temperature occurring along the path of discharge, or locally at points where the disruptive discharge takes place, *i. e.*, it may be due to an increased translatory motion of the particles which have this mode of vibration—a point which it seems impossible to determine. But it seems certain that this is not due to high temperature, if we mean the average temperature of the gas in the tube.

4. It is likely that the presence of water vapor in a tube plays an important rôle in the conduction, because of its effect in ionizing the gas, but it seems also likely that other agencies may produce the same result. There seems to be a connection between the presence of a quantity of water vapor and the brightness of the four lines, and it may be that the process of dissociation of water gives rise to vibrations producing the four-line spectrum. But there does not seem to be any evidence for saying that the four-line spectrum is the spectrum of the water vapor. On the contrary, the "four lines" appear to be true lines of hydrogen.

In conclusion I wish to acknowledge my obligation to Professor Ames, at whose suggestion the work was undertaken, for the kindly and helpful interest he has always shown; to Professor Wood, for his suggestions; and to students who have assisted me, particularly Mr. George W. Middlekauff, who was for a considerable time associated with the work.

JOHNS HOPKINS UNIVERSITY,
May 1, 1903.

SOME EFFECTS OF CHANGE OF ATMOSPHERE ON ARC SPECTRA WITH REFERENCE TO SERIES RELATIONS.[1]

By A. S. KING.

SINCE the discovery of series relations in the spectra of a number of elements, those investigators who have studied the changes produced in such spectra by variation of physical conditions have noted in many cases that lines belonging to a series are affected differently from the unrelated lines; and that when two or more series are present, they may be affected in different ways by change of condition. Such changes in physical conditions not only furnish us with a means of separating the lines of an element into groups between the members of which series relations may be looked for, but we may be enabled to draw conclusions as to the character of the vibrations which give rise to such lines by observing what conditions seem to be most favorable for their production. Any points which can be established in regard to these matters will be of great use in astronomical investigations, where the spectra of heavenly bodies show many differences in relative intensities of lines, which, if explained, would give indications as to the condition on these bodies.

The influences upon spark and arc spectra of changes in temperature, pressure, nature of discharge, and by a magnetic field have been observed, and the tendency of such changes to affect certain lines more than others has been frequently noted. Another method of separating lines into groups is the use of different gases around the electric arc or in the tube through which a spark is passed. It has long been known that such changes of atmosphere may affect certain lines in the spectrum of an element more than others, but it is only recently that any notice has been taken of the method as a means of grouping related lines.

[1] Extract from a dissertation presented in partial fulfilment of the requirements for the degree of doctor of philosophy in the University of California.

The work of Professor Crew[1] with metallic spectra influenced by an atmosphere of hydrogen has shown that the lines belonging to Kayser and Runge's series are unaffected by a change from air to hydrogen in the cases of the metals he investigated, and he points out that the use of hydrogen may offer a means of dividing spectra into series and non-series lines. R. A. Porter[2] has supplemented Professor Crew's work by the use of oxygen, nitrogen, and ammonia with several metallic arcs, but his results relate chiefly to the changes in prominent spark lines. The method thus seems to be only in the beginning of its usefulness.

The work of which an account is to be given was undertaken for the purpose of making a systematic study of the arc spectra of several metals whose lines show regularities in their arrangement, with reference to changes produced by the atmosphere surrounding the arc. The attempt has been made to connect the observed changes with the series relations among the lines of the metal, to separate the lines into groups by this means, and to deduce general conclusions as to the nature of the physical conditions which bring about the observed effects.

With this object in view, a detailed study has been made of the spectra of calcium, strontium, barium, and copper, with the arc in atmospheres of air, oxygen, carbon dioxide, nitrogen, ammonia, and the vapors of mercury and sodium. A few observations have been made of the spectra of magnesium and zinc. The use of copper terminals in some cases has given an opportunity to observe the effect of copper vapor mixed with that of the metal under observation.

Carbon terminals were used for most of the work with the metals of the calcium group. The chlorides of calcium, strontium, and barium were used, and powdered magnesium. Carbons made from calcined sugar by the method previously described[3] were used to advantage in some cases by mixing the metallic salt or powder with the powdered charcoal, then molding and baking the compound. This gives a very uniform distribution of the metal through the carbon and freedom from impurities found in the

[1] Astrophysical Journal, 12, 167, 1900. [2] *Ibid.*, 15, 274, 1902.

[3] A. S. King, Astrophysical Journal, 14, 323, 1901.

commercial carbons. Calcined rock-candy has been found to give a very pure carbon for this purpose. In using calcium chloride it is difficult to get enough of the metal in the carbon, as a large amount causes the carbon to disintegrate in the baking. Furthermore, carbons containing any hygroscopic salt must be used at once after baking, as the absorption of water vapor causes them to go rapidly to pieces. When a large amount of metal in the carbon was desired, commercial carbons were used, with their cores bored out and filled with the metallic salt or magnesium powder. These were satisfactory for the most of the work, as the spectrum showed but few impurity lines. In cases where it was desirable to eliminate the carbon spectrum, or to observe the effects of copper vapor, copper rods were used as electrodes, their ends being bored out and filled with the metallic salt.

In order to try the effects of sodium and mercury vapors, the lower carbon was passed through the bottom of a small graphite or clay crucible which was fitted around the carbon a short distance below the end, and filled with metallic sodium or mercury. The arc being started, the heat caused the sodium or mercury to give off large quantities of vapor. The use of sodium vapor with carbon terminals causes the lower carbon to disintegrate rapidly where the liquid sodium comes in contact with it, possibly on account of the presence of water in the interior of the carbon, which is attacked by the sodium. This action on the carbon limits photographic work to short exposures.

During the first part of the work the inclosure used for the arc was a glass bulb made from a round-bottomed flask by introducing a tube into a hole blown in the bottom. Stoppers were placed in this tube and in the neck of the flask, through which carbons were passed, also tubes for the passage of gases. With this chamber, either the vertical or the horizontal arc could be used. Later a heavy graphite crucible 10cm in diameter was used for the chamber. This was made almost gas-tight by inverting it upon an asbestos base through which the lower carbon and the inlet tube for gases were passed. The upper carbon and outlet tube entered through holes in the top, and the light from the arc passed through a hole in the side 1 cm in diameter, over which a glass or quartz window was placed.

The spectrum was formed by a fifteen-foot Rowland concave grating, of which the first order spectrum was usually used. Comparison photographs were made on the same plate, usually four in number, the two outer with the arc in air with different lengths of exposure, the two inner with the arc in the gas or vapor whose effect was being studied, and with exposures corresponding to those for air. An effort was made to avoid overexposure, which would tend to equalize the intensities of lines.

CALCIUM.

The spectrum of calcium has been photographed by the writer, with the arc in atmospheres of oxygen, carbon dioxide, nitrogen, ammonia, and the vapors of sodium, mercury, and copper. The fact that the calcium salts are very volatile in the arc makes it difficult to control the conditions as to the amount of calcium vapor present in the arc. We cannot know that the same amount of calcium was fed into the arc for each of two comparison photographs, so that a general strengthening of the lines under certain conditions of surrounding atmosphere means little. It seems useless, therefore, to make any statements as to the absolute effect of a certain atmosphere on the calcium spectrum, and so I have considered only relative differences, and have looked for the causes which produce a greater intensification of some lines than of others when conditions are changed.

The most striking relative difference appears when we compare the behavior of the ultra-violet triplet of wave-lengths 3644.45, 3630.82, 3624.15 with a pair of sharp lines at λ3737.08 and λ3706.18. The triplet is a member of the first subseries of Kayser and Runge. The pair does not belong to a series, but has the same difference in wave-number (223) as the H and K lines, and almost the same as the pair at λ3179.45 and λ3158.98 (204). The triplet and pair just mentioned are close together, can be easily compared, and are observed to be affected differently by a surrounding atmosphere in nearly every photograph of the large number I have taken. The pair at λ3179.45 and λ3158.98 behaves very similarly to the pair just mentioned.

The action of the H and K lines is similar in a general way, but they seem to change more easily than the other pairs, perhaps on account of greater strength and frequent reversals. The changes in the triplet at λλ3644.45, 3630.82, 3624.15 are a type of the behavior of the more diffuse lines of the spectrum, including the triplets belonging to Kayser and Runge's series and most of the unrelated lines. Triplets belonging to the second subseries show a change in the same direction, but not always to the same degree as those of the first subseries. Lines belonging neither to the series nor to the pairs mentioned above show an effect intermediate between that of the sharp lines of the pairs and that of the triplets of the first subseries.

Taking the pair and the triplet as types of the two kinds of lines into which the calcium spectrum may be divided, let us notice first the effect of varying quantities of calcium in the arc, other conditions remaining the same. With a very little calcium in the arc, the pair and triplet are of about the same intensity and both sharp. Sometimes the pair is stronger than the triplet. On increasing the quantity of calcium in the arc, the pair increases but slowly in intensity and remains sharp. The triplet, on the other hand, rapidly becomes strong and diffuse, the lines broaden toward the violet, and with a large amount of calcium reverse. This behavior is typical of that of all triplets belonging to the two Kayser and Runge series, the action being most pronounced in the case of the more hazy triplets, as those whose strongest lines are at λλ3361, 3487, and 4099.

It appears, therefore, that the increase of vapor density, which should result from an increased supply of calcium, would account for the broadening observed in the case of the more diffuse lines, and that the sharp lines do not show a corresponding increase in intensity. An increase in vapor density is probably accompanied by an increase in the temperature of the arc, since when the arc is started with a large amount of the salt in the carbon, the current is above normal, and other conditions which relatively enhance one set of lines usually give a slight increase in current.

This selective effect of change in density makes it difficult to

form tables of relative intensities for spectra like that of calcium whose lines do not all behave alike. I have often noticed that the relative intensities of lines on my plates do not correspond with those given by Kayser and Runge. For example, the latter give the intensities of the lines forming the triplet λλ3644, 3631, 3624 as 1, 2, 2 respectively, and the lines λ3737 and λ3706 each as 4. The arc in air does not give this relation unless a large amount of calcium is used, as was probably done by Kayser and Runge in order to have all of the calcium lines appear distinctly.

If we now observe the effects of various atmospheres upon the calcium spectrum, we find that the theory of change of relative intensity in certain lines by increased vapor density is the only one which will harmonize the effects of change of atmosphere with the observed effects in air when the amount of metal consumed is changed. Consider first the effect of oxygen. Oxygen greatly increases the consumption of metal in the carbon arc. The carbons waste away rapidly and a large amount of calcium oxide is formed. The arc is very luminous; the current usually increases one or two amperes from its value with air around the arc. The effect on the spectrum is to enhance the diffuse triplets much more than the sharp lines which do not belong to series. This is especially noticeable in the case of the triplets with lines of greatest wave-length at λλ3361, 3487, 3644, 4099, 4586, as compared with the sharp lines λλ3159, 3179, 3706, 3737, and the group from λ4283 to λ4319, which do not broaden readily. The H and K lines are strengthened, but not so much as the *g* line, the reversal of which is widened. There is no essential difference, however, between these effects and those given by the arc in air with a large amount of calcium present. I am not now taking up the question as to whether oxygen really increases the *absolute* intensity of the calcium lines. That could be tested only by an arc formed between terminals of etallic calcium and with a constant current. The greater ount of calcium consumed by the arc in an atmosphere of ygen would account for the general strengthening.

The effect of carbon dioxide on the calcium spectrum is very

similar to that of oxygen, but not so decided. It is probable that large quantities of carbon dioxide are generated by the use of oxygen around the carbon arc, as the oxidation rapidly reduces the diameter of the carbons.

Turning now to the effects of some atmospheres which exclude oxygen to a greater or less degree, we shall consider first the atmosphere of pure nitrogen. This was prepared from ammonium sulphate and sodium nitrite, and purified by passage through sulphuric and pyrogallic acid. Its effect seems to be to prevent oxidation without decreasing the vapor density to any great extent. The arc burns readily in nitrogen, and the same time of exposure can be used as with air or oxygen. The carbons are consumed very slowly, and no calcium oxide is formed if a strong current of nitrogen is maintained. Still, from the ready burning of the arc, it is not probable that the vapor density in the arc is greatly different from the conditions in air, and it is different from the conditions in oxygen chiefly because with the latter gas more calcium is supplied to the arc in a given time.

The effects with nitrogen are entirely in accordance with this view. The tendency of nitrogen is to equalize the intensity of the pair and triplet considered above as types of the two sets of lines. The vapor density is probably small, but not smaller than we can obtain in air with a small amount of calcium and a strong current of air pumped through the chamber. In some photographs where a large amount of calcium was used with nitrogen atmosphere, the triplets were broadened and rendered diffuse in a way often observed in air and approaching the effect of oxygen. The fact that this broadening is possible under some circumstances with a strong current of nitrogen around the arc is very much against the theory that the broadening is due to oxidation, or due in any large degree to higher temperature, as we should expect the violent combustion in the oxygen atmosphere to raise the temperature of the arc as well as that of the electrodes.

The use of sodium vapor around the arc gives another test for the effect of increased vapor density without oxidation. A

stream of sodium vapor passing up on all sides of the arc from a cup of boiling sodium around the lower carbon would allow but little oxidation, while it might be expected to crowd the calcium vapor back into the arc and increase the vapor density in the arc. The photographs taken under these conditions show a decided difference in effect upon the two sets of lines, and the difference increases as the sodium vapor becomes denser. With the sodium boiling vigorously, the pairs remain sharp and show little change in intensity, while the triplets become broad and diffuse, and the reversal of *g* is much widened. The sodium vapor increases the conductivity of the arc so that the current is greater than with air and the electrodes are consumed more rapidly, a condition which should lead to an increase of vapor pressure. An explanation may thus be given for the observed fact that the effect of sodium vapor is much like that of oxygen, though we might expect the opposite.

Turning now to the effect of ammonia, we have here a gas which in all probability reduces the vapor density in the arc. As has been observed by others, the arc is very difficult to maintain in ammonia. When it is burning, the current and fall of potential show little change from the conditions in air, but the intensity is feeble and the arc is often extinguished. The exposure must be from twelve to twenty times as long as with the arc in air. With a strong stream of ammonia sweeping away the gases formed in the arc and the feeble action which we observe, the conditions seem very favorable for a minimum vapor density in the arc. None of the other atmospheres I have used give such a relative weakening of the lines belonging to Kayser and Runge's series as compared with the pairs at $\lambda 3159$ and $\lambda 3179$, $\lambda 3706$ and $\lambda 3737$, and the H and K lines. The pairs mentioned are stronger than in air in the case of several photographs in which the supply of calcium was kept the same as nearly as possible. The triplets, on the other hand, are very decidedly weakened, and their lines rendered sharper. The reversal of *g* is much narrower in ammonia; also that of H and K, when they are reversed. The effect of mercury vapor around the arc is to diminish the triplets more than the pairs. The *g* line shows a

narrower reversal, and the effects all point toward a lower vapor density than with the arc in air. The poor conductivity of mercury vapor may account for this, giving the effects observed to a much greater degree in the case of ammonia.

The use of copper electrodes with calcium filling gives another means of producing conditions in which the vapor density is probably small. The copper arc is of feeble intensity, a much longer exposure is needed than with the carbon arc to make the lines equally strong, and then relative differences appear in the two sets of calcium lines much like those observed when mercury vapor was used around the arc. Comparing two photographs of the calcium spectrum between λ3500 and λ4600, one with carbon and the other with copper electrodes, with the H and K lines, as well as the pair at λ3737 and λ3706 of the same intensity in both, we observe a very distinct difference in the behavior of the other calcium lines. The several triplets are much weaker in the copper arc, the greatest difference being in the diffuse triplets with lines of greatest wave-length at λ3644 and λ4512. The *g* line is narrow and not reversed in my plates taken with the copper arc.

STRONTIUM.

The spectrum of strontium has been studied by the writer from λ3000 to λ5000 with atmospheres of air, oxygen, carbon dioxide, nitrogen, ammonia, and sodium vapor. The relative differences found result in the segregation of a group of strong lines which make up three pairs having the same difference in wave-number, given by Kayser[1] as follows:

λ	Vibration-Difference
4305.60	801.6
4161.95	
4215.66	801.5
4077.88	
3475.01	801.1
3380.89	

These are homologous to the three pairs in the calcium spectrum which were observed to be affected by surrounding atmos-

[1] *Handbuch der Spectroscopie*, 2, 541,

phere differently from the lines belonging to Kayser and Runge's series. In the strontium spectrum we again find a difference in the effect upon these two classes of lines; but in this case it is the sharp lines forming the pairs which are the more sensitive to changes of physical condition.

A comparison of a large number of photographs shows conclusively that the lines of these pairs are strengthened by atmospheres of oxygen, carbon dioxide and sodium vapor to a greater degree than the remaining lines of the spectrum, and are correspondingly weakened by atmospheres of nitrogen and ammonia. The sharp line at λ 3464.58 is acted upon in the same way as the pairs, but seems to have no companion with a similar behavior. The broad reversed line at λ 4607.52 is not strengthened by oxygen to the same degree as the pairs. In carbon dioxide it is actually weaker than in air, while the pairs are much stronger, and the remaining lines, including those belonging to Kayser and Runge's series, show little difference. In ammonia λ 4607.52 sometimes suffers a double reversal, making it broad and hazy. The pairs are distinctly weakened by ammonia, giving a general effect opposite in character to that of the first three gases. The action of nitrogen is similar to that of ammonia, but not so pronounced, as the pure nitrogen seems to make but a slight change from the conditions of the arc in air.

A NEW STRONTIUM TRIPLET.

In studying the spectrum of strontium, the writer has observed that whenever the line λ 4077.88 is of considerable intensity, three faint lines appear close to it. The wave-lengths of these three lines I have measured as follows:

4087.80
4071.22
4091.52

Not being able to identify these with any known metallic lines, I have applied several tests to determine whether they belong to strontium, to some impurity, or whether they may be "ghosts" of λ 4077.88. The last possibility

The three lines are not at all symmetrical

λ4077.88 [illegible]
prismatic spectrum [illegible]
a dispers [illegible]
trum, and [illegible]
grating. [illegible]
impurities as [illegible]
of both the [illegible]
with very [illegible]
not likely [illegible]
as they are [illegible]
or ammonia.

While [illegible]
lengths of [illegible]
λλ4[illegible]
certainly [illegible]
stand [illegible]

Perhaps [illegible]
strontium [illegible]
differences [illegible]
differences [illegible]
Runge [illegible]
have their [illegible]
The four [illegible]

[illegible]

[illegible]

The fifth member of this group of triplets seems to be the three lines:

4087.86	
4071.22	100.0
4061.52	58.6

The members of these five triplets are the more diffuse lines in the strontium spectrum. There is a decided similarity in appearance between the last two triplets, though the one to which I have called attention is much the weaker.

MAGNESIUM.

The writer has made but few observations of the spectrum of magnesium, and these have for the most part given negative results, as far as relative differences are concerned. This may be due to the fact that nearly all of the prominent lines belong to the triplets of the first and second subseries of Kayser and Runge, and so may be expected to show the same behavior with change of physical conditions. The spark line λ4481 does not appear in my photographs.

My plates show a very slight difference in the behavior of the triplets belonging to the first subseries in atmospheres of oxygen and nitrogen from that of the second subseries in the same atmospheres. The triplets of the first subseries are relatively stronger in nitrogen, those of the second subseries in oxygen. This is the case with the triplets with lines of greatest wave-length at λλ 3838, 3097, 2852 of the first subseries as compared with those at λλ 3337, 2942, of the second. The difference is most noticeable in the case of the very broad line λ 2852. The results so far obtained do not point to any more general conclusions than the similar behavior of lines belonging to the same series.

BARIUM.

A study of the spectrum of barium with the arc in atmospheres of air, oxygen, nitrogen, carbon dioxide, ammonia, and the vapors of copper, mercury, and sodium, shows a number of lines which are affected differently from the majority of the barium lines, and always affected in the same way by a given gas. While no

general series relation has yet been found for the barium lines, my results serve to strengthen the evidence that certain lines are related to each other.

The opposite effects of oxygen and ammonia upon a number of lines give a basis for separating a certain group from the main body of the spectrum. These lines are as follows:

5853.91	4525.19
4934.24	4166.24
4900.13	4130.88
4700.64	3906.20
4554.21	3891.97

A comparison of a large number of plates shows these lines in every case to be strengthened by oxygen and weakened by ammonia more than the remaining lines of the spectrum. Nitrogen produces scarcely any change from the appearance of the lines in air. A slight relative weakening sometimes appears, showing that the tendency of nitrogen is in the same direction as ammonia. Carbon dioxide gives a definite strengthening of these lines, comparable to that of oxygen. Both mercury vapor and sodium vapor strengthen the lines, the effect of the latter being almost as pronounced as that of oxygen. Copper terminals filled with barium salt also strengthen the lines as compared with barium in the carbon arc. With all these atmospheres, the most sensitive lines are those at λλ4166, 4525, and 4900. With the exception of λ3906, the group of lines given above are among the strongest in the spectrum, and are sharp with a moderate quantity of metal in the arc, though they broaden and reverse with a large quantity. They are, however, always different in appearance from the broad diffuse lines at λλ4323, 4325, 4489, and 4494, which are not affected by change of atmosphere. This greater sensitiveness of the sharper, more prominent lines of barium is similar to the effect of corresponding atmospheres upon the strontium spectrum, and unlike that upon the calcium lines. The changes in the line λ4525, as compared with its more diffuse companion λ4523, are very striking.

Looking now at the relations between these lines which are affected by surrounding atmosphere, we find that almost the

only definite regularity so far discovered in the barium spectrum is the existence of four pairs of lines with constant difference in wave number, as follows:

λ		Vibration-Difference
6497.07	- - - - - - - -	1691.05
5853.91		
4934.24	- - - - - - - -	1691.16
4554.21		
4900.13	- - - - - - - -	1690.90
4525.19		
4166.24	- - - - - - - -	1691.47
3891.97		

It will be observed that all of these lines occur in my list of sensitive lines, except λ6497, which is beyond the region covered by my photographs. My results for the barium spectrum, then, serve to separate these lines, together with a few others, from the main body of the spectrum, as forming a series of related pairs, which appear to be relatively strengthened by increased vapor density in the arc.

ZINC.

My data for the zinc spectrum cover only the region from λ3000 to λ4100 with a comparison of the effects of oxygen and air. Brass terminals were used for the arc, so that the stronger copper lines appear. My results show a decided strengthening in oxygen of the group of lines

3282.42	3345.13
3702.62	3345.62
3303.03	3346.04

belonging to the first subseries of Kayser and Runge.

The lines

3572.90	3740.12
3671.71	4019.75
3683.63	4058.02

between which no relation has been found, are weaker in oxygen than in air. The difference is not as great as in the case of the series group. This strengthening of the series lines by oxygen is similar to that observed with the series lines of calcium, while

the effect on the unrelated lines is like that which we shall see in the case of the line pairs of copper.

COPPER.

The spectrum of copper shows a large number of relative differences in intensities of lines, when different atmospheres are used around the arc, the most important changes being given by an atmosphere of oxygen. The copper arc in oxygen burns with difficulty and frequently goes out, in decided contrast to the behavior of the carbon arc in oxygen. This action may be due to the formation of a non-conducting coating of copper oxide on the terminal, though it is doubtful if this forms in the arc itself. The oxidation of copper is not accompanied by the increased heat and wasting of the electrodes which accompanies the formation of carbon dioxide in the carbon arc.

In general, the more diffuse copper lines are most reduced by oxygen, the best examples of this being the pairs of wavelengths

3247.65	3654.60
3274.06	3688.60

These pairs are almost blotted out by a strong current of oxygen. The strong pair λ4023 and λ4063 are weakened by oxygen, though not so much as the preceding; while many of the sharper lines are quite unaffected, notably those of wavelengths:

3308.10	4378.40
3599.20	4587.19
3602.11	4651.31

The lines from λ3100 to λ5300 most reduced by oxygen are as follows:

3194.17	3925.40
3208.32	4015.80
3247.65	4022.83
3274.06	4056.80
3337.95	4062.94
3384.88	4480.59
3530.50	4531.04
3654.60	5105.75
3688.60	5153.33
3825.13	5218.45
3861 88	

It will be observed that these twenty-one lines include those of the pairs given by Kayser[1] as belonging to the first and second subseries of copper; also the pair λ4015.80 and λ4056.80, and the strong pair λ3247.65 and λ3274.06 which Kayser thinks may be a member of the principal series of copper. The lines of these several pairs have nearly the same difference in wave-number (248). Besides these pairs we have a weakening by oxygen of the seven lines:

3194.97	3530.50
3208.32	3925.40
3337.95	5105.75
3384.88	

In addition to these known relations, I have found that a number of these lines affected in the same way by oxygen form pairs with a constant vibration difference of about 3400 as follows:

λ		Vibration-Difference
3247.65	3654.60	3428.7
3274.06	3688.60	3432.5
3384.88	3825.13	3400.2
3925.40	4531.04	3405.1

This gives an interesting relation between the pair λλ3247.65, 3274.06 and one of the pairs of Kayser's first subseries. The corresponding lines of these two pairs are seen to have nearly the same difference in wave-number. The two pairs of the second subseries have a line of each pair related to an outside line by this constant difference; but in one case it is the upper line of the pair which has an outside companion, and in the other case the lower line. The connection indicated by these constant differences does not appear to be a simple one. It is probably a result of some relation among the the vibrations giving rise to this set of lines which are affected in the same way by change of physical conditions.

[1] *Handbuch der Spectroscopie*, 2, pp. 530-531.

The use of nitrogen gives effects but little different from those with air around the arc, except in the case of the pair $\lambda 3247.65$ and $\lambda 3274.06$. These are slightly weaker with nitrogen, and the reversal is narrower, while scarcely any change can be perceived in the other lines which are weakened by oxygen.

The effect of mercury vapor on the copper lines is similar to that of oxygen, the same lines being weakened, and to almost the same degree, as by a strong current of oxygen. The action is the same with sodium vapor around the arc, though not so pronounced, only the most sensitive lines being changed.

Mixtures of other vapors with that of copper were tried by using brass electrodes, giving strong zinc lines mixed with those of copper, and by using one copper and one carbon terminal, giving the carbon and cyanogen bands. These experiments were made with the arc in the open air. No relative differences could be detected in either case by comparing the plates with those taken with copper terminals.

Looking now to see if changes in vapor density would produce the effects we observe, the first fact to be noted is that the effect of oxygen upon the burning of the arc with copper terminals is very different from that when carbon electrodes are used. With carbon terminals, the arc burns with great vigor and brightness in oxygen, the carbons being rapidly consumed, probably with the formation of large quantities of carbon dioxide. With copper terminals, on the other hand, it is difficult to maintain the arc in oxygen. Some copper oxide is formed, but no change in the flame is observed except a weakening. A longer exposure is required for the same general intensity of the spectrum, as compared with the arc in air. This agrees with the observation of Porter,[1] who used the rotating arc with electrodes of magnesium, tin, zinc, and iron in oxygen, and found the metallic lines weakened. This difference between the carbon and metallic arcs must be considered in questions of vapor density.

The conditions with oxygen about the carbon arc appeared favorable for greater vapor density, and its action in intensifying certain lines strengthened the hypothesis that the change

[1] *Loc. cit.*

was produced by greater vapor density. With the copper arc, however, the weak action in an atmosphere of oxygen appears favorable for a low vapor density in the arc, and the reduction of the broad diffuse lines by oxygen can be attributed to this cause. Both mercury and sodium vapors weaken the arc. The action of sodium vapor is not the same as with the carbon arc. The current remains the same as with the arc in air, so that the sodium does not appear to increase the conductivity, and the arc burns with difficulty, frequently going out. Nitrogen might be expected to give little change of vapor density from that of air, and it is noted that only the most sensitive lines are changed by nitrogen, whose effect is slightly to reduce the vapor density, if my hypothesis is correct.

DISCUSSION OF RESULTS.

The first general result to be noted as given by a change of atmosphere around the arc is the power of the method as a means of separating the lines of an element into groups. The several groups may then be investigated with respect to series relations between the lines. It thus offers possibilities of finding new relations between lines, especially in the case of elements having so large a number of lines that it would be difficult to obtain a starting-point without some means of separating certain groups.

The magnetic field is a well-known means of separating lines into groups, and an interesting comparison of the two methods is given by comparing my results with those given by Kayser in his discussion of the Zeeman effect.[1] From this it will be seen that a number of lines in the spectra of calcium, strontium, barium, and copper, which belong to certain types in the classification of Zeeman phenomena, are among the lines which I find affected by a surrounding atmosphere differently from the other lines in those spectra.

The fact that a certain atmosphere may not produce the same effect on lines of the same kind (sharp or diffuse) in the spectra of different metals, might present a difficulty. Especially is

[1] *Handbuch der Spectroscopie*, 2, 671.

this the case with the behavior of calcium and strontium in the carbon arc. The structure of these two spectra show many similarities, yet the diffuse calcium lines belonging to Kayser and Runge's series are the more sensitive to change of atmosphere; while in the case of strontium, the strong lines not in Kayser and Runge's series respond most readily. It is very probable, however, that the groups of strong lines actually form the principal series in these two spectra, though no numerical relation has yet been found between the lines except the constant differences. If this is the case, the state of affairs seems to be that a change of atmosphere produces the greatest difference on the principal series of strontium and on the first subseries of calcium—an effect which might easily result from the differences between the two metals, in spite of their many similarities. The more rapid volatilization of the calcium salt in the arc might bring about a change of condition sufficient to account for the different behavior of the two series.

In regard to the probable action of the surrounding atmosphere in bringing about the changes in the spectral lines, the weight of evidence seems to indicate that chemical action has little or no part in producing the effects, except as it may affect the vapor density in the arc. The chief arguments against the effect of an atmosphere of oxygen being due to chemical action are (1) that the use of pure nitrogen produces little change as compared with the arc in air, and occasionally its action is in the same direction as that of oxygen; (2) that a cloud of sodium vapor around the arc leaves most of the lines unchanged, and slightly enhances those of one group, as has been noted in several cases.

The hypothesis that seems best to accord with my results and those of others is that the effects are to be ascribed to changes in vapor density which effect the vibrations producing the lines belonging to a certain group—vibrations which are probably similar in general character, judging from the behavior of the resulting spectral lines. This theory has been kept in view during the discussion of the results, and seems to be the one which best co-ordinates the various phenomena, both as to the

changes observed with different atmospheres, and those in which the supply of metal was varied with the arc in air.

It is possible that temperature changes may play a large part in giving the effects we observe, as the most distinct changes in the spectrum are associated with conditions which probably involve change of temperature in the arc. However, the facts that the carbon arc in oxygen does not give changes proportional to its probable change of temperature, and that these changes are brought about in some degree by other conditions which should not involve much change of temperature, point to the conclusion that temperature is not the direct cause of the effects observed, though it may assist in bringing about the state which is the actual cause. Temperature changes would be closely connected with changes in vapor density, to which it seems reasonable to attribute many of the results.

It is the writer's opinion that not enough attention has heretofore been given to the effects of changes in vapor density upon the intensities of lines. Is it not possible that density is the determining element for the relative intensities, as pressure seems to be for displacements and the use of the magnetic field for polarization properties? Any notions as to the action of change of vapor density in modifying the electrical properties of the vibrating particles must be entirely hypothetical, but it is easy to conceive that such actions might be selective in their nature, resulting in a change for only a portion of the lines.

Comparing my results with some observations of the spark spectrum which have been published, we find some striking evidences that the use of atmospheres which apparently give lower vapor density tend to make the arc spectrum similar to that of the spark discharge. Sir William and Lady Huggins [1] found that when the density of calcium vapor in a vacuum tube was decreased by gradually removing the calcium from the electrodes, the H and K lines, together with the pairs at λλ3737, 3706, and 3179, 3159 were weakened much less rapidly than the other lines and finally were the only lines visible. These are the lines which I find but slightly altered by changes in vapor

[1] Astrophysical Journal, 6, 77, 1897.

density in the arc. Similarly, the photograph of the spectrum of the spark discharge in calcium vapor given by Eder and Valenta[1] shows the lines λλ 3737, 3706, 3179, 3159 to have many times the intensity of λλ 3644, 3630 and the other series lines in that region. This state is approached by some of my photographs taken with the arc in ammonia, which appeared to give the lowest vapor density of the gases I have used. Again, the table given by Eder and Valenta,[2] in which their measurements of the spark lines of copper are compared with those of Kayser and Runge for the arc, shows that if the standards of intensity adopted by the two pairs of observers are at all alike, then most of the lines which I find weakened by an atmosphere of oxygen around the copper arc are much weaker in the spark spectrum than in that of the arc.

If, as is very probable, the vapor density in vacuum tubes and in the spark discharge is much less than that in the electric arc, we may arrive at a simple explanation of the action of hydrogen in intensifying certain spark lines, as observed by Crew and Porter. As the hydrogen atmosphere around the metallic arc seems to have much the same action as ammonia in causing the arc to burn with difficulty, probably reducing the vapor density, we are led to the idea that those lines which appear strongly in the spark, and faintly or not at all in the arc, may be lines whose vibrations require a low vapor density. The action of hydrogen in strengthening such lines in the arc would then be explained by the reduction of the vapor density, thereby bringing the conditions of the arc to approach those of the spark.

Further evidence in support of this view is given by the work just published by Hartmann,[3] who finds λ 4481, as given by the arc between magnesium terminals, much increased in intensity when a current as small as 0.4 ampere is used, the arc being constantly interrupted, with only a slight heating of the terminals and little vaporization—conditions which should give a low vapor density. The results show a similarity to the

[1] *Denkschriften der K. Akademie der Wissenschaften zu Wien*, **68**, 534, 1900.

[2] *Ibid.*, **63**, 189, 1896.

[3] ASTROPHYSICAL JOURNAL, **17**, 270, 1903.

spark spectrum and to the effects given by the arc when surrounded by atmospheres which should give low vapor density.

My results, then, considered in connection with those of others, show a definite selective action in the effects of surrounding atmospheres on spectral lines—an effect evidently connected with the series relations of the element. All of the results point to a change of vapor density in the arc as the direct cause of the effects observed. The tendency of a low vapor density seems to be to make the arc spectrum more like that of the spark.

In conclusion, I wish to express my thanks to Professors Slate and Lewis for constant interest in the work and many helpful suggestions.

UNIVERSITY OF CALIFORNIA,
Berkeley, May 1903.

ON THE SPECTRUM OF THE SPONTANEOUS LUMINOUS RADIATION OF RADIUM AT ORDINARY TEMPERATURES.[1]

By SIR WILLIAM HUGGINS and LADY HUGGINS.

THE discovery of an element possessing such remarkable and novel properties as radium, which in its separate and distinct form as a new chemical element we owe to the researches of Professor and Mme Curie, has already thrown many beams of suggestive light into the very obscure regions of the constitution of matter. In radium we have a body which appears to be spontaneously and without ceasing giving off energy in several forms. According to Professor Rutherford,[2] following upon the work of Becquerel, M. and Mme Curie, and others, the emanations going off from radium are at least of three kinds: first, an emanation of heavy corpuscles, larger in mass than the hydrogen atom, moving with a high velocity, and carrying a positive charge; secondly, of negatively charged electrons which form a powerful and penetrating cathode emanation;[3] and, further, of a radio-activity which diffuses from the radium as if gaseous in its nature. In addition, M. and Mme Curie have found that radium spontaneously maintains a temperature about 1°.50 C. above the surrounding temperature, and therefore emits heat radiations of wave-lengths falling within the infra-red part of the spectrum.

Now, in addition to these forms of radiant energy, the glowing of radium in the dark shows that it emits a luminous radiation

[1] From advance proofs of an article to appear in the *Proceedings of the Royal Society.*

[2] *Phil. Mag.*, (6) **5**, 445, 561, 1903.

[3] As an illustration of the penetrative power of the radio-active effects of pure radium bromide, the following experience may be recorded here. About 1 centigram of radium bromide (Buchler & Co., Brunswick) had been placed in an upper drawer of my writing table, while in a lower cupboard of the same table was a store of photographic plates. After a week or two, all the plates, in boxes lying upon each other three or four deep, were found to be as completely fogged as if they had been exposed to light.

spontaneously at ordinary temperatures. It appeared to us probable that in this glow we had not to do with either phosphorescence or fluorescence as usually understood, but with an independent and continuous radiation set up by those more active molecules which are supposed, in consequence of a condition of internal instability, to be the source of all the phenomena of radio-activity, and which can scarcely fail themselves to be violently agitated, in connection with disruptive molecular changes — especially the flinging off of the heavy corpuscles — during which, part of the energy stored up within the molecule is liberated in the kinetic form.

Taking this view of the luminous radiations visible to the eye, it seemed highly probable that the molecular motions by which they were set up, whether we suppose all the radium molecules alike to be concerned, or those only which are in active change, would be so far analogous to the vibrations produced artificially, when radium vapor is rendered luminous in a flame, or by the blow of an electric discharge, as, in like manner, to set up radiations of certain definite wave-lengths or, in other words, to furnish a spectrum of bright lines.

A preliminary prismatic examination of the glow from pure radium bromide was attempted by eye. In consequence of the feebleness of the light under dispersion a slit spectroscope could not be used. A thin fragment of some length of radium was selected, which in the dark shone as a narrow line of light; when this was viewed through a direct-vision prism, it was seen to be dispersed into a spectrum which extended from the blue down to about D, where it became too faint to be traced farther in the direction of the red. Within this faint spectrum certain spots were distinctly brighter, due, in all probability, to the presence of bright lines at those positions in the spectrum.

The success of this preliminary observation encouraged us to hope that it might be possible by availing ourselves of the accumulative power of continuous photographic exposure, to obtain a record of the blue, violet, and ultra-violet regions of the spectrum, if the glow radiations extended so far.

We made use of a small quartz spectroscope which had been

constructed some years ago for very faint celestial objects. It consists of a compound quartz prism of 60°, consisting of two prisms of 30° of right-handed and left-handed quartz respectively. The quartz lenses are of short focus and of large angular aperture, being about $f=\frac{1}{3}$. The focal length of the lenses is $5\frac{3}{4}$ inches; they are plano-convex, the marginal parts of the convex surfaces being "figured" to diminish spherical aberration.

The solid radium bromide was placed at about a millimeter distance in front of the slit, which had to be wider than if a bright object was being photographed; the width was about $\frac{1}{40}$ inch. In the case of the spark spectrum of radium and the comparison spectrum of nitrogen, a slit of less than half this width was used.

With the exposure of twenty-four hours, faint traces of two lines were seen on the plate. After several trials the negative reproduced on the accompanying plate was obtained with an exposure of seventy-two hours. The reproduction is enlarged two and a half times. The spectrum consists of eight bright lines, and at least eight faint lines, together with a faint trace of continuous spectrum in the blue region, which does not come out in the reproduction.

In consequence of the wide slit and the small scale of the spectrum, it is not possible to measure with certainty to the fourth figure, but the probable error is, we think, not greater than two units in the fourth place.

It was seen at once that the two very strong characteristic rays of the spark spectrum of radium, in this part of the spectrum, namely, λ 3814.5 and 3649.6[1] were not present on the plate. It was clear that the spectrum was not of the radium molecule when excited by the electric discharge. It was indeed not improbable that if the radiation came alone from the most active molecules, which were suffering loss by material emanations, then, if we may accept the analogy from sound, like a filed tuning-fork they would no longer give radiations of the same wave-lengths as before.

[1] For spark spectrum of radium, see DEMARÇAY, *Comptes Rendus*, 129, 786, and 131, 258; Exner and Haschek, *Wien. Akad. Sitzber.*, 110, July 1901; RUNGE, ASTROPHYSICAL JOURNAL, 12, 1.

On comparing the new spectrum with the band spectrum of nitrogen, seven of the strongest lines were found to agree, not only in position, but also in intensity, and in character with those of the three nitrogen bands in this part of the spectrum.

The positions of the three bands are, according to Ames,[1] λ3576.85, 3371.2 and 3158.9.

Indications of other lines, besides those which can be seen in the reproduction, can be faintly glimpsed on the negative. There seems little doubt that with a longer photographic exposure a more complete spectrum will be obtained. We have now secured some radium bromide prepared by the Société Centrale de Produits Chimiques, and it is our intention to take photographs of this salt as well as photographs of the German salt with longer exposures. It may then be that indications of helium, and possibly of radium itself, may be forthcoming.

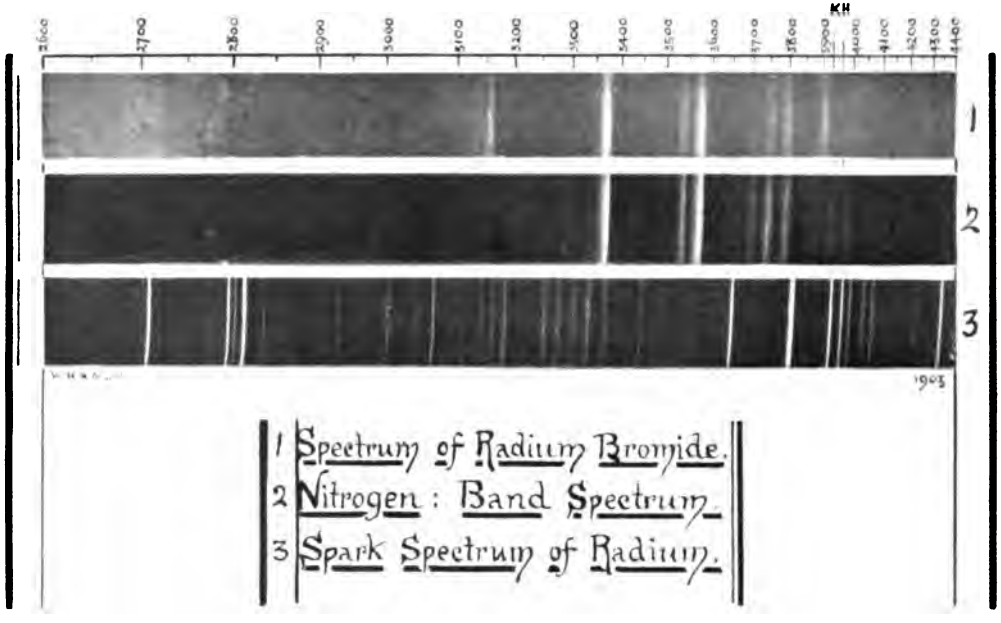
Nearly the whole of the ultra-violet radiations appear to come from nitrogen, and we think it best to refrain from any discussion at this moment. Have we to do with occluded or with atmospheric nitrogen? The remarkable fact should be pointed out that in radium we have a body which at the ordinary temperature sets up radiations which are similar to those which have hitherto only been obtained in connection with the electric discharge.

Description of the plate.—At the top is placed a scale of approximate wave-lengths. Immediately below is a reproduction, enlarged two and a half times, of the spectrum obtained from the radium bromide with an exposure of seventy-two hours. As has been already explained, this has been shifted to bring the lines into position with those of nitrogen photographed from a vacuum tube. The identity of the two spectra seems complete. The third band is faint in the nitrogen spectrum on account of the absorption of the glass of the tube.

Below is a spark spectrum of radium bromide from the Société Centrale de Produits Chimiques. The H and K lines of calcium are present, as well as faintly some of the stronger lines of barium. The characteristic lines of radium at λ3814.59 and

[1] *Phil. Mag.*, (5) 30, 57, 1890. See also DESLANDRES, *Comptes Rendus*, 101, 1256; and PERCIVAL LEWIS, ASTROPHYSICAL JOURNAL, 12, 8.

Spectrum of Radium Bromide



1 Spectrum of Radium Bromide.
2 Nitrogen : Band Spectrum.
3 Spark Spectrum of Radium.

3649.7 come out strongly, as well as the strong line recorded by Demarçay at λ4340.6. A strong line about λ2710 was placed by Berndt[1] at λ2708.6. The strong line a little beyond, about λ2814 is due to radium. We do not recognize several lines recorded by Exner and Haschek[2] in this part of the spectrum.

[1] *Phys. Zeitschr.*, **2**, No. 12 (1900–1).

[2] *Sitzb. Ak. Wiss. Wien.*, **110**, July 4, 1901.

Reviews.

Problems in Astrophysics. By Miss A. M. Clerke. London: A. & C. Black, 1903.

It is not possible to separate entirely the child Astrophysics from the parent Astronomy. The two are mutually dependent in much of their instrumental equipment and in many of their methods, and their results are often intimately related. Radial-velocity determinations, for example, are based largely upon astrophysical methods, but the direct results are mostly astrometrical. However, the newer term serves an exceedingly useful purpose, and has become permanently fixed in our literature. In general, we may say that the older astronomy is concerned with the determination of the structure of the sidereal universe; with the arrangement of the sidereal units in space, and with their motions and relations in accordance with Newton's law of gravitation. Astrophysics, on the contrary, is interested in determining *what the stars really are*, and what the history of their development has been.

The beginning of astrophysical inquiry is old, but until the last half-century its results were surprisingly few. Theories of planetary evolution by Kant and Laplace, the observations and wonderfully sagacious deductions of the elder Herschel, and studies of planetary conditions and of Sun-spots, constituted the main body of the science up to 1859. But the spirit of inquiry as to the nature and history of the heavenly bodies was latent in many quarters; and Kirchhoff's superlatively important discovery that "the light which reveals to us the existence of the heavenly bodies also bears the secret of their constitution and physical condition," opened a gateway to Comte's forbidden field which many were ready to enter. The pioneer work of Secchi, Huggins, Vogel, Young, and their colleagues in the next quarter-century was very fruitful. Their results, originally scattered through the journals, were collected and systematized by Roscoe, Schellen, Kayser, and later by Scheiner and Clerke;[1] but the speed of development in the past fifteen years has been such that astrophysical literature is again widely scattered. This statement will not, I trust, be con-

[1] Miss Clerke, *The System of the Stars*, 1890.

strued into an overlooking of the colossal work on general spectroscopy now issuing from the able pen of Professor Kayser. A historical and critical survey of astrophysics is greatly needed, and the appearance of Miss Clerke's volume could scarcely have been better timed.

The author's intention, as announced in the preface, "is not so much to instruct as to suggest. It represents a sort of reconnaissance, and embodies the information collected by scouts and skirmishers regarding practicable lines of advance and accessible points of attack. . . . . Before attempting to add to our store of learning, we must realize what is already possessed." This statement of purpose has been consistently adhered to. By way of suggestions for future lines of research this book is the richest one known to me.

The physical study of the Sun forms the basis of astrophysical research more fully perhaps than astronomers themselves have realized. It is the only star near enough to be studied in some geometrical detail, all other stars giving integrated images which practically are mathematical points. It is likewise the only star bright enough to supply satisfactory metrical standards demanded in the study of other stars. Miss Clerke devotes the first third of the volume to a treatment of the Sun. After outlining "The Progress of Solar Physics," there are chapters on "The Chemistry of the Sun," "The Reversing Layer," "The Photosphere," "The Sun spots," "The Faculæ and Prominences," "The Chromosphere," "The Corona," "The Sun's Rotation," "The Solar Cycle," and "The Sun as a Whole."

The difficulties of solar study, in spite of comparative nearness and intense brightness, are very great. It is not generally appreciated that we are unable to study the body of the Sun, except by very indirect methods. Its interior is invisible; the photospheric veil hides it completely from view. The photosphere, chromosphere, prominences, corona, etc.—the only portions accessible for direct observation are an insignificant part of its mass. They are literally the Sun's outcasts. Our knowledge of the Sun as a whole is based almost entirely upon a study of these outcasts. We might hope to reach safe conclusions as to the characteristics of a hermit nation by making a careful study of its banished subjects, provided the observed types correspond with types produced by our own civilization; but if new types, new customs, new forms, presented themselves on a stupendous scale, and were observable only at long range, our conclusions as to the country from which they were expelled should come slowly and uncertainly. It is difficult to say what the chromosphere is. To determine what the conditions

within the body of the Sun must be in order to create and maintain such an outcast shell is far more difficult.

Again, the ordinary photographic or visual image of the Sun is an integrated one. A coronal photograph secured at a total eclipse is the result of a projection upon and into one plane, at right angles to the line of sight, of all that remains of the Sun after subtracting the (approximate) cylinder of matter hidden by the Moon. The tops of some coronal streamers, the intermediate portions of others, the bases of those near the limb, the corresponding parts of prominences and chromosphere, are all projected into one point, illuminated in part by reflected and in part by inherent light. Whether every man who has gone forth to solve the riddle of the corona has fully realized the odds against success is doubtful.

Imperfections in atmospheric conditions are perhaps more liable to introduce uncertainties in solar observations than in any other class. Whether Janssen's *réseau photosphérique* phenomena are of solar or terrestrial origin is still uncertain, as Miss Clerke states on p. 17, but Hale's recent results with the spectroheliograph, in which the image is built up from one limb to the opposite limb in some two or three minutes, confirm the pretty general belief that the effect is purely terrestrial.

The impossibility at present of reproducing solar phenomena in our laboratories is recognized as our most severe limitation. There is scarcely one result to be obtained by interpolation: extrapolation is demanded everywhere, and at great distances beyond the range of observed points in the curves. Such decided extensions of our resources as the electric furnace provides seem at best but beginnings toward what is required. And, as Miss Clerke repeatedly points out, our knowledge of the effect of the presence of one vapor upon the spectrum of another vapor is almost wholly wanting. Evidence is not lacking, however, that such effects must be both common and potent.

There can be no doubt that the Sun is a very tempestuous body. It is probable that convection currents are on a scale so great and have at times a speed so high that we have difficulty in adequately realizing their immensity. I fear, therefore, that Miss Clerke's division of the solar outcasts into "several distinct envelopes" overlying the photosphere (p. 16) - reversing layer, chromosphere, corona, etc. — is somewhat too definite, even with her qualifying remark that none of them is "apparently in a condition of atmospheric equilibrium." Our direct evidence of disturbance in the photosphere is limited largely to the

visible spots, but the faculæ and prominences are almost certainly the results of disturbances generally invisible. In this connection it would be well, I think, to modify the statement on p. 74, "but faculæ exist abundantly where there are no spots," to read where there are no visible spots. Faculæ accompany very small spots as well as large ones. It is natural that there should be countless invisible spots, and that they as well as the large ones should be surrounded by faculæ. Spots one second of arc (650 kilometers) in diameter could easily escape notice. There is little doubt that the coronal streamers springing from every part of the sphere are produced in some way by powerful but invisible disturbances in all parts of the surface. There is certainly a close relation between spots and faculæ, and the latter are not confined to the spot zones; between chromosphere and prominences; and between prominences and at least the inner part of the corona. The tremendous cone of disturbance in the corona of 1901 (Sumatra) and in its apparent situation immediately above the large, and only Sun-spot visible near that date, discovered by Perrine[1] — perhaps inadvertently not mentioned in *Problems in Astrophysics* — is also very significant. Not to dwell on the details longer, it seems to me that the strata above the photosphere must be highly confused, though there is no doubt that the order of elevation generally assigned to the reversing layer, chromosphere, etc., is correct. If the zeal displayed by a successful comet hunter were devoted to searching for changes in the solar spectrum, rich rewards could reasonably be expected.

These statements are not opposed to the fact that progress must be made by differentiating and isolating various kinds of phenomena; and those instruments and methods which have this power most markedly are the most promising. The spectrographic method of differentiating, including the line-displacement principle, has had no equal; and the extension of it by the two-slit spectroheliograph has been a real advance. Much can reasonably be expected from Hale's improved form of this instrument.

The value of a solar eclipse as a partial differentiator, through the elimination of photospheric light, should not be overlooked. Surprising, indeed, is the impetus given to solar study by eclipse observations. Our knowledge of the *existence* of the reversing layer, the chromosphere, the prominences, and the corona, is an eclipse product; and so are many of our present everyday methods of studying them. The

[1] *Lick Observatory Bulletin* No. 18, and CLERKE'S *History of Astronomy*, 4th ed., p. 190.

richness of eclipse results, considering the remarkably short intervals available for observation, is unique in science. Nor are eclipse problems all solved; every such event brings its returns. Whether the reversing layer and chromosphere can ever be studied equally well without an eclipse is a question. They probably cannot, by means now available, but Hale's observation of carbon bright lines near the photosphere shows clearly that the effort should be made. Limitations in quality may be more than balanced by continuity and timeliness. The reviewer agrees with Miss Clerke (p. 135) that "the prospect is dim of realizing" success in observing the corona without an eclipse, at least along the lines of attack thus far attempted. Any chance for even moderate success would seem to be limited to the gaseous inner portion, and a daily record of this would no doubt be extremely valuable, but the real problem of the corona would remain unsolved.

Miss Clerke refers to the possibility that electrical or magnetic forces are playing prominent parts in solar and stellar phenomena; and while positive evidence of their action has not yet been recognized, the idea is a valuable one, and observers would do well to apply tests for Zeeman and other effects whenever opportunity offers. In this connection, I have been more than surprised to find no mention of Arrhenius's theory of light- and heat-pressure effects in the production of the more extended solar phenomena. Nichols's proof that theory is in agreement with observation on this question requires us to consider whether such forces at the Sun's surface may not be sufficient to create observable movements of finely divided matter existing at photospheric temperatures. Is it not possible that the resultant of gravity and light- and heat-pressures is small for very minute particles, and that herein lies the explanation of the hypothetical principle of "levity" referred to by the author, on p. 51 and elsewhere?

The salient points in solar progress have received masterful treatment from Miss Clerke. Especially well is attention called to the gaps in our knowledge. Many of them could be filled, at least in part, by the energetic use of apparatus now in existing observatories, while others demand new and expensive equipments, with ample provision for their systematic application. The Sun as the great exemplar of our science is receiving but scant attention.

Part II, 375 pages on "Sidereal Physics," is a very able presentation of facts and theories for all known types of stars, and for nebulæ. It is not so systematic as the author's treatment of "Solar Physics," but the nature of the subject, and the scattered character of observed

facts, forbid. There are chapters on all the principal spectrum types, on all the types of variable stars, on all known classes of nebulæ, on the spectra of double stars, on spectroscopic binaries, new stars, dark stars, and the Milky Way, etc.; forty-one chapters, each rich with the most sallent facts of observation, with theories very wisely and conservatively discussed, and with constant suggestion as to points most in need of strengthening by observation. The impossibility of doing justice to so extensive a work in a brief review will be evident to all.

The field of stellar and nebular investigation has its own difficulties. The quantity of light received from a star or nebula is really very minute; a stellar spectrum is an integrated result, for photosphere, reversing layer, chromosphere, prominences, and corona—all fall in one small point, as viewed from the Earth; the distances, diameters, and in nearly all cases the masses, are unknown, leaving us without basis for correlating masses, quantity, and intensity of light; the amount of measurement accurate according to today's standard is small; little effort has been devoted to the ultra-violet region, so important from temperature considerations; and laboratory accompaniments are but just beginning on a requisite scale. Nevertheless, the mass of solid fact brought within the realm of knowledge in the past forty years is exceedingly encouraging.

The foundation of every philosophic treatise on the stars must rest, so far as information now avaible is concerned, upon a classification of stars according to spectral types, preferably in the order of their supposed evolution from nebulæ to dark stars. Miss Clerke treats nebulæ last of all. This is not the order of nature, but the plan has its advantages, principally because we know too little about the nebulæ and their processes of transition to stellar states. The astrophysical treatise of the future will probably begin with the nebulæ—or perhaps with the origin of the nebulæ! Miss Clerke assigns all stellar spectra to eight classes: four showing absorption spectra only, and four marked by both absorption and emission spectra. The first five classes are: I, helium stars, brilliantly white, hydrogen and helium absorption predominating, the *Orion* and *Pleiades* stars as illustrations; II, hydrogen stars, with intense hydrogen absorption, feeble and thin calcium H and K, feeble iron lines, with *Vega* and *Sirius* as examples; III, solar stars, with innumerable metallic lines, the lower hydrogen lines, and broad and heavy H and K, with the Sun and *Arcturus* as types; IV, stars with fluted spectra, containing a great number of fine dark lines, with superposed heavy absorption flutings whose heads are toward

the violet, α *Orionis* and α *Herculis* as illustrations; V, carbon stars, with innumerable dark lines, very feeble in violet, strong carbon absorption flutings and many apparent bright lines, 19 *Piscium* being typical.

This scheme of classification differs very little from Secchi's four-type system. Classes I and II are subdivisions of Secchi's Type I. Class III is Secchi's II, and IV and V are Secchi's III and IV. It is in effect, also, a condensation of Miss Maury's system of more than twenty groups. It is of course understood by all readers that any such scheme of classification will divide the stellar spectra into compartments only very imperfectly: there will be many spectra on the border lines between classes, especially in Classes I, II, and III. Classes IV and V have more individuality than the others.

Secchi's classification has been a most fortunate and useful one. For general purposes it is satisfactory even today, when the observational data are almost incomparably more numerous and accurate than when Secchi formed it; and I doubt whether the time has come to replace it with another. The Harvard system is very useful, and perfectly justifiable, as an expression of the results of systematic examination and study of a great number of spectra. But its general adoption in the daily work of astrophysicists would seem to be unwise for the present, in view of our very imperfect knowledge of the temperature and other basic conditions underlying the real system of stellar development. New determinations of the spheroidal constants of the Earth, or of the aberration constant, based upon the latest data, are extremely valuable; but to adopt each improved spheroid as a basis for government surveys, or each improved aberration constant in astronomical yearbooks, would lead to hopeless confusion. For similar reasons I regret the division of Secchi's I into Miss Clerke's I and II, and the mechanical shifting of the former's II, III, and IV to the latter's III, IV, and V. The highly desirable distinctions which Miss Clerke desired to call attention to in stars of Secchi's I could have been accomplished more wisely, I think, by some subclassification analogous to Vogel's — notwithstanding the very important calcium-absorption distinctions covered by her plan. Again, her Class VI stars, having fluted spectra and showing hydrogen and other bright lines, includes only the variables of the o *Ceti* type. They are certainly very closely related to her Class IV stars. The spectra of the variables apparently differ from those of Class IV only in having bright lines, induced probably by the forces which produce variations

in brightness; and in this case a subclass of IV would certainly be satisfactory and more logical. Helium stars with bright lines and Wolf-Rayet stars are assigned to Classes VII and VIII. Miss Clerke states that "no hypothesis of growth and affinity is implied by the order of succession given to the bright line objects," though "it certainly rests upon broad and unmistakable distinctions." Class VII is certainly closely related to the Class I helium stars, and a subdivision for these under I would seem to me both advantageous and natural. Of their close relationship there is no doubt, and many facts could be cited in that connection. Space prevents reference to more than the bright lines in spectra of *Alcyone*, *Pleione*, μ *Centauri*, etc., and the absence of the bright lines from many otherwise almost identical spectra of Class I. The assignment of a special class to the Wolf-Rayet stars — Miss Clerke's VIII, Pickering's V — seems necessary at present, as no one is now able to fix their relation to other spectra. These stars, like comets, seem to stand out from any system of stellar evolution, but we should be fully justified, I think, in saying that they are comparatively new stars. It is also possible that they, as well as other stars of peculiar spectral types, are alien to the main sidereal system, and have no place in the sequence of development of ordinary stars; but this seems very improbable.

In the absence of full information as to what conditions produce bright lines in stellar spectra — whether in γ *Cassiopeiae* or in o *Ceti* — it is difficult to say why certain stars have bright lines and others have not. It is a constant surprise to me that bright-line spectra are not vastly more numerous than we have found them to be. Miss Clerke's reference to Classes I and II as *unveiled*, in contradistinction to the Sun's photospheric veil, may prove to be a very useful one. It probably has a basis in fact, though of this we have no positive proof. It would be surprising if the formation of bright lines is not favored by the absence of a photospheric veil. The study of bright line spectra is one of the most fascinating and important subjects in spectroscopy. They appear to represent critical periods in stellar development, or to accompany rapid changes, as in variable stars. Systematic study of such spectra is greatly needed. The field is undoubtedly a rich one. It is but a few years since the discovery was made of both bright and dark lines of hydrogen in the same spectrum, the bright lines growing weaker from red to violet, and the dark lines growing stronger. Enough additional evidence has been collected to indicate that a general principle is here involved; and Kayser's explanation, or some other equally

simple one, will probably establish its fundamentality. As a case in point, since Miss Clerke wrote (p. 220), "the invisibility (up to the present) of hydrogen in the carbon stars is not easily accounted for," a long-exposure spectrogram secured with the Yerkes two-foot reflector has shown strong dark $H\gamma$ and $H\delta$ lines in the red star 19 *Piscium*, whereas $H\beta$ is bright or missing in all observed stars of that type. The laboratory investigations of Hale and others on spark spectra obtained under high pressure show somewhat analogous phenomena, but of their correct interpretation I do not feel sure.

It is very unfortunate that our knowledge of temperature relations in stars of various types is so defective. The paucity of laboratory results is perhaps the major cause. The great importance and attractiveness of work along this line is evident to those who have followed investigations on the magnesium line at $\lambda 4481$ by Scheiner, Huggins, and others, or on the relation between arc and spark spectra by Hartmann. Very few such researches are sufficient to modify greatly our views on stellar temperature relations.

In this connection, attention should be called to the great need for extensive observations of stellar ultra-violet spectra, along the lines pointed out by Huggins. If the truly continuous portion of the ultra-violet in solar types is relatively stronger than in helium stars, that surprising fact should be fully established.

Miss Clerke's computations for the masses and radiative powers of various stars, leading to results very widely different from those for our Sun, are extremely interesting, and they in a sense mark the opening of a new field of great significance. Unfortunately, the available parallaxes for many of these stars are so uncertain as to introduce large uncertainty into her numerical results. The need for better and more numerous parallax data is at least as pressing in astrophysics as in the study of the distribution of the stars in space, and it is fortunate that extensive plans are forming at several observatories to supply the deficiency. However, granting the sufficiency of the author's parallax data, the tremendous radiative powers arrived at for certain stars need not wholly surprise us. There are certainly no more startling facts in all astronomy than those of *o Ceti* varying in brilliancy four-thousand-fold, with very little change in spectral type; of *Nova Cygni* varying from third to fifteenth magnitude, the spectrum at the present lower limit being continuous, without any evidence of bright lines; and of *Nova Persei* decreasing from the first in brilliancy in the northern hemisphere to below the tenth magnitude. In these cases the ratio of mass

to quantity of light radiations has had a remarkable range. It is impossible to say what the brightness of the Sun would be if its photospheric veil were suddenly removed.

Among the most interesting chapters in the book are those on variable stars. Excepting the new stars, a half-dozen *Algol* and δ *Cephei* types, β *Lyrae* and *o Ceti*, our knowledge of variable star spectra is remarkably slight. No well-equipped observer has devoted himself entirely or in the main to that subject. They offer a great variety of phenomena, and afford almost exclusive opportunities to observe spectral changes in rapid progress. Miss Clerke emphasizes the need for more work. There is in my opinion no richer field awaiting systematic cultivation. The number of variables known is about 1200.

The author's urgent recommendation that the bright lines in variable star spectra be tested for Zeeman effect is timely. The reviewer has held the subject in mind for several years in connection with the triple $H\gamma$ and $H\delta$ of *o Ceti*, but when the star was in favorable position the lines have been either single or too faint for such observation. The Lick tests on the broad bands of *Nova Persei* in 1901, and of *Nova Geminorum* in 1903—no decrease in width resulting from a rotating Nicol prism—seem to be the only applications of such tests to stars thus far made.

Many interesting features of variable- and double-star spectra, of nebulæ, star clusters, etc., merit the reviewer's attention, but lack of space prevents.

In a book written in so broad a spirit, it would be invidious for the reviewer to enumerate the fifteen or twenty minor errors noticed, but a few should be referred to. On p. 500, Keeler's result is misinterpreted—that in the hydrogen vacuum tube "the third line ($H\gamma$) always vanishes experimentally before the first ($H\alpha$), while in a nebula it ($H\gamma$) shines unfailingly, although $H\beta$[1] be imperceptible." The $H\beta$ is, I think, visually much brighter in all nebulæ than is $H\gamma$, though $H\gamma$ is brighter than $H\alpha$. Again, at the top of p. 501, the uninformed reader will undoubtedly credit the discovery of variations in the relative intensities of the *Orion* nebular lines to one of the several observers who confirmed, and not to the discoverer who bore the brunt of the unusually hostile criticisms following the announcement of his discovery.

On p. 394, l. 13, the erroneous dates "7th and 13th November"

[1] Since this review has been put in type, it seems probable that $H\beta$ in the quotation is a misprint for $H\alpha$. — W. W. C.

instead of the correct dates 9th and 13th, are extremely unfortunate, though the dates were stated correctly in the author's *History of Astronomy*, 4th ed., p. 401.

It is to be regretted that the illustrations of nebulæ, clusters, etc., are not the most modern obtainable.

Miss Clerke's book is in my opinion most ideally planned: its history is fairly, but inconspicuously, complete; only relevant facts are stated; theory has not run away with observation; the defective corners in our field of knowledge are pointed out; the style is lucid, attractive, and logical; and to me its most remarkable feature is the wonderfully correct estimate of the relative values of observations, by an author with little or no experience in making observations.

W. W. CAMPBELL.

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A REVISION OF ROWLAND'S SYSTEM OF WAVE-LENGTHS.

By J. HARTMANN.

1. ROWLAND'S system of wave-lengths has become the basis of all spectroscopic measurements made in recent years, and with the progressive increase in the precision of these measures the necessity has now arisen of testing the reliability of that important basis, and, in case it should not appear as adequate, of correcting it by new series of observations in order thus to create a foundation sufficient for all demands.

Aside from the earlier and provisional publications of Rowland, his system of wave-lengths as at present accepted appears in three different forms:

a) The standard wave-lengths in the solar spectrum, and

b) The standard wave-lengths in the arc spectra of different metals which was first published in *Astronomy and Astro-Physics* (**12**, 321, 1893) under the title "A New Table of Standard Wave-Lengths." For brevity I shall denote the wave-lengths of these two series by ⊙ St. and M. St. Rowland gave in 1896 in the *Memoirs of the American Academy of Arts and Sciences* (**12**, No. 2, 101) a more extended account of the origin of these standards.

c) The measurement of the whole solar spectrum which appeared in the ASTROPHYSICAL JOURNAL, Volumes 1 to 5, in the years 1895–97, under the title "Preliminary Table of Solar Spectrum Wave-Lengths." The wave-lengths in this table I shall designate by P. T.

The question which first arises, as to how far these three series of wave-lengths are to be regarded as coincident, so that we can speak of a single Rowland system, may be answered in two different ways—first, on the basis of the history of the development of these series, and secondly, by a numerical comparison of them.

2. The history of the development of Rowland's wave-lengths is briefly as follows. By taking the mean of the absolute measures of Ångström, Müller and Kempf, Kurlbaum, Peirce, and Bell, Rowland derived for the Fraunhofer line D_1 the wave-length

$$\lambda = 5896.156.$$

To this line he connected thirteen further "primary standards" (P. St.) in the visible spectrum by visual measures of coincidences with the use of several concave gratings of 21½ feet focus. These P. St. are single lines in only certain instances; being for the most part mean values from the wave-lengths of groups of two, three, and four lines, which are separated by distances as great as 76 tenth-meters. As the result of numerous measurements he obtained twenty-six equations of condition of the form

$$nA - mB = D,$$

where n and m represent the order of the coinciding spectra concerned, B the wave-lengths of the observed lines, and D the separation of the lines measured with the eyepiece micrometer. The measured distances of the lines observed as coincident are in part very large: only in eight cases are they smaller than ten revolutions of a screw for which one revolution = one tenth-meter in the first order of the grating concerned; in eleven cases it is more than twenty revolutions, and in two cases it even exceeds one hundred revolutions. Although the micrometer screw employed by Rowland, which he himself made in the most careful manner

and investigated, is undoubtedly of the highest excellence, there is nevertheless a certain danger in the measurement of such long distances, particularly as these distances frequently could be measured only on one side, so that it was not possible to interpolate the lines of the one order *between* those of the other order. How Rowland obtained the screw-value with sufficient accuracy for such long distances is not to be readily ascertained from his publications, which, indeed, contain so few data as to the measurements themselves that a test of them is impossible.

3. Rowland did not solve the twenty-six equations of condition for the wave-lengths of the fourteen P. St. by the method of least squares, although this would have been quite simple, as each equation contains only two unknowns with integral coefficients ranging from two to seven. He employed *per contra* the process of successive approximations, which had the advantage of greater clearness, and without doubt also led to a system of values which satisfied very well the equations of condition.

From this process of adjustment Rowland deduced for the line D_1 the wave-length 5896.160, and, in order to bring this value into better agreement with the above-mentioned mean from the absolute measurement, he reduced[1] all the wave-lengths found in the solution by $-\frac{1}{200,000}$. In this way he now obtained for D_1, $\lambda = 5896.157$.

The P. St. determined in this way lie between the limits λ 4215.6 and λ 7040.1. Subsequently the Fraunhofer line A was also once connected in the spectrum of the first order with the nearest normal in the second order, for which purpose it was necessary to measure with the screw over a distance of 84 tenth-meters; a further connection was also obtained by the coincidence of the second and third order.

4. The 798 ⊙ St. and the 404 M. St. were now connected with these 15 P. St. as follows: The visual portion of the solar

[1] The corrections actually applied by Rowland (*loc. cit.*, p. 139) do not agree with this statement in the case of several of the standards. It cannot be ascertained whether this is due to typographical errors, of which the publication unfortunately contains a very large number, or are errors of computation, or whether certain of the standards were intentionally further altered for some reason which was not given.

spectrum between λ 4048 and λ 7714 was repeatedly measured with a micrometer screw 5 inches long, with the use of various gratings. The portions separately measured with the screw were then united by means of the overlapping portions into thirteen long strips, which rendered possible not only an interpolation of the standards, but also an adjustment of the P. St.

Eighteen photographs were made with the solar spectrum of one order situated between two strips of the solar spectrum of another order for the continuation of the work into the ultra-violet region. A large number of plates was also made which contained a solar spectrum and beside it metallic spectra of different orders. The measurement of all these plates, which was carried on by Mr. Jewell, now produced numerous combinations between the different parts of the spectrum, and thus there always occurred, along with the interpolation of the standards, also an adjustment of the P. St. Thus for each wave-length of the standards there were developed a number of individual determinations from which finally the mean was taken; these are the numbers published as ⊙ St. and M. St.

5. It follows from this origin that the whole system of ⊙ St. is entirely coincident with that of the P. St., but that the wave-lengths of the single lines in the two systems may differ by small quantities from each other. Thus the line D_1 appears under the ⊙ St. with the wave-length

$$\lambda = 5896.154.$$

The ⊙ St. are to be regarded in every case as the better adjusted system in comparison with the P. St.

6. The conditions are less simple with the M. St. Rowland remarks[1] as follows:

> In every plate having a solar and metallic spectrum upon it, there is often — indeed, always — a slight displacement. This is due either to some slight displacement of the apparatus in changing from one spectrum to the other, or to the fact that the solar and the electric light pass through the slit and fall on the grating differently. In all cases an attempt was made to eliminate it by exposing on the solar spectrum both before and after the arc, but there still remained a displacement of 0.01 to 0.02 division of Ångström, which was determined and corrected for by measuring the difference between

[1] *Memoirs of the American Academy*, 12, 116.

the metallic and coinciding solar lines, relating a great number of them, if possible.

It follows from this remark that the wave-lengths of the M. St. which were directly measured and hence accurately referred to the system of the ⊙ St., in case the apparatus was in order, were subsequently so altered by empirical corrections, now larger and again smaller, that as accurate a coincidence as possible was attained between the lines of the arc and the solar spectrum.

Jewell was the first to call attention to the erroneousness of this process, in his very valuable paper on "The Coincidence of Solar and Metallic Lines."[1] It cannot now be ascertained by what amounts the individual M. St. were falsified by the application of these corrections. More definite information could not be given to me by Mr. Jewell in response to a letter which I addressed to him several years ago. We must therefore first remember that all the M. St. are erroneous by small amounts, which cannot be recomputed, lying in the vicinity of 0.01 to 0.02 tenth-meters, and in general too large relatively to the system of the ⊙ St. In certain parts of the spectrum the ⊙ St. may be affected to a small amount by the application of these empirical corrections, since Rowland repeatedly also employed the M. St. in order to derive the ⊙ St. in a spectrum of another order by the method of coincidences.

7. While the progression of the errors which arose from the application of these corrections is very gradual, so that the error for neighboring lines is always of nearly the same amount, I must now call attention to another source of error which may have produced larger deviations of *individual* lines. As already mentioned, the ⊙ St. and the M. St. are mean values from a number of different sorts of determinations; and the number of plates employed is very different for each line. The following series of M. St. in the iron spectrum may serve as an example.

Even this short list shows that the M. St. measured on eight or nine plates either coincide very closely with the ⊙ St. or fall on the negative side of them, while all the lines measured only once or twice deviate, in some cases by very considerable

[1] ASTROPHYSICAL JOURNAL, 3, 89, 1896.

TABLE I.

M. St.	No. of Measures	⊙ St.	M. St. − ⊙ St.
4308.072	8	.071	+ 1
4325.932	8	.940	− 8
4352.908	1	.903	+ 5
4369.948	1	.943	+ 5
4376.108	1	.103	+ 5
4383.721	9	.721	0
4404.928	9	.927	+ 1
4415.298	8	.299	− 1
4447.912	2	.899	+13
4494.756	1	.735	+21

amounts, toward the positive side. In particular I would refer to the uncertainty of the two lines at λ 4448 and λ 4495.

The direct measures made by Rowland and Jewell on each separate plate certainly possess a very high degree of accuracy, so that the relative position of the lines was determined to a precision of a few thousandths of a tenth-meter; but since each plate has a systematic error, chiefly in consequence of the application of the empirical correction, there occur, through the above-mentioned lack of uniformity in the number of plates employed, displacements in the relative position of the individual lines which may rise as high as several hundredths of a tenth-meter.

8. The relative wave-lengths of the ⊙ St. are also injuriously affected by the large variation in the number of measurements employed for the individual lines. However, the errors arising in the individual wave-lengths of the solar spectrum are for the most part rendered harmless by the extensive measurement of the whole solar spectrum which was carried out in a uniform manner, the results of which are given in the P. T. The wave-lengths of the P. T. depend upon the measurements of the photographic plates of the whole solar spectrum carried out by Jewell. In this work the newly measured lines were not merely referred to the ⊙ St. by interpolation, but a solution was made for every plate by which the ⊙ St. were also freed from their accidental errors, so that they constitute a homogeneous system with the lines newly connected to them. There accordingly exists between

the wave-lengths of the P. T. and the ⊙ St. a relation similar to that between the ⊙ St. and the P. St. (see 5). The *whole system* of the P. T. is also here to be regarded as entirely coincident with the system of the ⊙ St., but the wave-lengths of the individual lines in both systems may differ from each other by small amounts. For instance, the line D_1 appears in the P. T. with the wave-length 5896.155.

9. It appears from this statement of the history of the origin of Rowland's wave-lengths that the P. T. constitutes the definitive and most reliable form of the Rowland system. If, for instance, the question should arise which is the most correct of the four values for the wave-length of the line D_1 in the solar spectrum, viz.:

λ=5896.156 (starting value)
.157 (P. St.)
.154 (⊙ St.)
.155 (P. T.)

and which most precisely represents the Rowland system, the answer would be the value 5896.155.

10. The conclusions which were reached as to coincidence of the three systems P. St., ⊙ St., and P. T. on the basis of the history of their origin are confirmed by a direct comparison of the numerical values. The relation of the ⊙ St. to the P. St. may be seen from Table II.

The difference ⊙ St.−P. St. amounts at the maximum for a line to 0.007 tenth-meter; the sum of all these differences properly should be exactly zero; for the thirty-four lines together, however, it amounts to +0.016, whence the mean value of this difference is +0.00047 tenth-meter. Hence the two systems P. St. and ⊙ St. are sufficiently in coincidence.

11. On account of its great extent I will not print here the comparison of the system ⊙ St. and P. T., and I will limit myself to giving in Table III for intervals of 100 tenth-meters under the heading *S* the sum of the differences P. T.−⊙ St. In an entirely rigorous comparison *S* should be, at least for long stretches of spectrum, zero. The number of ⊙ St. employed is denoted by *n*. Several of Rowland's ⊙ St. had to be omitted in this com-

TABLE II.

P. St.	⊙ St.	⊙ St.-P. St.	P. St.	⊙ St.	⊙ St.-P. St.
4215.665	.667	+2	5397.350	.346	−4
4222.381	.381	0	5405.984	.987	+3
4376.103	.103	0	5624.254	.253	−1
			5624.764	.768	+4
4494.729	.735	+6	5826.582	.580	−2
4497.048	.041	−7	5890.184	.182	−2
4501.444	.444	0	5896.157	.154	−3
4508.460	.456	−4	5914.386	.384	−2
4690.323	.324	+1	6246.530	.530	0
4691.575	.581	+6	6318.241	.242	+1
			6322.912	.912	0
4903.484	.488	+4			
4924.110	.109	−1	6563.049	.054	+5
4924.956	.955	−1	6569.461	.461	0
5060.250	.252	+2			
5064.834	.833	−1	6750.409	.412	+3
5068.946	.946	0			
			7023.747	.747	0
5269.717	.722	+5	7027.724	.726	+2
5270.497	.495	−2	7040.056	.058	+2

parison because the measurement was made upon the center of gravity of a double line, the components of which were separately measured in the P. T. M denotes the mean error of coincidence, that is, the quotient $\frac{S}{n}$, and the last column contains the largest one of the differences P. T.−⊙ St. which occur in the particular portion of the spectrum.

Table IV contains a similar comparison for stretches of 1,000 tenth-meters.

It may be seen from the numbers of Tables III and IV that the P. T. are well related to the system of the ⊙ St., if we except the short region from λ 7000 to λ 7400, the measurement of which gave special difficulty. The difference P. T.−⊙ St. amounts in the mean to +0.00040 tenth-meter for the whole spectrum from λ 3000 to λ 7000. The errors of individual lines of the ⊙ St. exceed 0.03 tenth-meter only in isolated cases; as a rule they amount to from 0.01 to 0.02 tenth-meter.

TABLE III.

λ	*S*	*n*	*M*	Maximum
3000-3100	−0.014	21	−0.0007	+0.018
3100-3200	− 13	12	− 11	− 13
3200-3300	+ 59	10	+ 59	+ 21
3300-3400	+ 15	8	+ 19	− 12
3400-3500	− 15	16	− 9	+ 20
3500-3600	+ 50	17	+ 29	+ 18
3600-3700	− 20	29	− 7	+ 30
3700-3800	− 28	32	− 9	± 16
3800-3900	+ 77	23	+ 33	+ 20
3900-4000	+ 44	25	+ 18	+ 16
4000-4100	− 18	17	− 11	− 11
4100-4200	+ 1	9	+ 1	+ 6
4200-4300	+ 28	14	+ 20	+ 12
4300-4400	+ 33	11	+ 30	+ 10
4400-4500	− 18	13	− 14	− 18
4500-4600	− 10	9	− 11	− 3
4600-4700	− 4	12	− 3	− 7
4700-4800	− 15	5	− 30	− 12
4800-4900	+ 29	6	+ 48	+ 31
4900-5000	− 28	13	− 22	− 10
5000-5100	− 13	11	− 12	− 8
5100-5200	− 32	31	− 10	− 15
5200-5300	+ 41	28	+ 15	+ 28
5300-5400	+ 18	17	+ 11	+ 30
5400-5500	− 33	15	− 22	− 27
5500-5600	− 43	16	− 27	− 31
5600-5700	− 3	16	− 2	− 14
5700-5800	+ 8	18	+ 4	± 15
5800-5900	− 7	13	− 5	− 12
5900-6000	+ 1	13	+ 1	+ 5
6000-6100	− 19	13	− 15	− 10
6100-6200	− 26	17	− 15	− 16
6200-6300	+ 70	17	+ 41	+ 26
6300-6400	− 29	13	− 22	− 24
6400-6500	+ 90	15	+ 60	+ 29
6500-6600	+ 72	12	+ 60	+ 49
6600-6700	− 39	6	− 65	− 38
6700-6800	− 37	10	− 37	− 37
6800-6900	+ 46	35	+ 13	− 32
6900-7000	+ 36	25	+ 14	+ 24
7000-7100	+ 157	13	+ 121	+ 51
7100-7200	− 21	7	− 30	− 21
7200-7300	+ 61	12	+ 51	+ 41
7300-7400	− 14	3	− 47	− 26

TABLE IV.

λ	S	n	M	Maximum
3000–4000	+0.155	193	+ 0.0008	+0.030
4000–5000	− 2	109	0	+ 31
5000–6000	− 63	178	− 4	− 31
6000–7000	+ 164	163	+ 10	+ 49
7000–7400	+ 183	35	+ 52	+ 51
3000–7000	+0.254	643	+0.00040	+0.049

12. In Table V, I give a comparison of the systems ⊙ St. and M. St. A few lines had to be excluded here also, for which it appeared, from the remarks made by Rowland, that not precisely the same lines were measured in the solar spectrum and the arc spectrum.

TABLE V.

λ	S	n	M	Maximum
3000−3500	+0.109	29	+0.0038	+0.027
3500−4000	+ 284	80	+ 35	+ 36
4000−4500	+ 90	47	+ 19	+ 21
4500−5000	− 64	15	− 43	− 25
5000−5500	− 62	21	− 30	+ 72
5500−6000	− 64	8	− 80	− 80
3000−6000	+0.293	200	+0.0015	−0.080

An almost perfect coincidence has also been effected between the systems M. St. and ⊙ St., as is seen from the mean value holding good for the whole spectrum, $M = +0.0015$ in Table V. It has already been mentioned above under 6 that this does *not* actually occur, and it is particularly striking that the difference M. St. − ⊙ St. is prevailingly positive in the stretch from λ 3000 to λ 4500; while the lines of the solar spectrum, in consequence of the greater pressure under which the absorbing strata of vapor exist in the solar atmosphere, should in general be somewhat displaced toward the red as compared with the arc lines at atmospheric pressure. Hence the difference M. St. − ⊙ St. should be prevailingly negative; as may be seen from Table V, this is true, however, only between λ 4500 and λ 6000.

The conclusions drawn above are accordingly confirmed by the direct comparison of Rowland's tables of wave-length: the P. T. constitutes the most reliable form of Rowland's wave-lengths; the ⊙ St. and the P. St. coincide on the whole with the P. T., but nevertheless individual lines have large accidental errors; the M. St. similarly contain large accidental errors, and are, moreover, systematically displaced by measurable amounts as compared with the system of the P. T.

13. At this point I will also refer to another system of wave-lengths which bears the same relation to the M. St. that the P. T. bears to the ⊙ St. This is the "Normale aus dem Bogenspectrum des Eisens," which Kayser published in the year 1900.[1] Kayser's standards, which I shall briefly designate with K., are — aside from a region near λ 3400 — precisely connected to the system of the M. St., and therefore contain the constant systematic errors of the M. St., but the careful measurement of numerous plates has so far reduced the accidental errors of the separate lines in K. that the mean error of a line amounts at the most to 0.003 tenth-meter.

14. We shall certainly not be far from the truth if we assume that the accidental error of the single lines in the P. T. is about $\pm$ 0.003.[2] It is true that a direct test of the P. T. has never been made, but everyone who has occasion to employ Rowland's wave-lengths will have become convinced of their great reliability, in so far as the relative position of lines within small regions of spectrum are concerned. The conditions are, however, changed in respect to the accuracy of the relative wave-lengths as soon as widely separated portions of the spectrum are compared with each other. Attention has already been called by various individuals to the fact that there are irregularities which slowly increase in Rowland's values, as a result of which for considerable portions of the spectrum all the wave-lengths are from 0.01 to 0.02 tenth-meter too large, and in other regions are too small. Müller[3] was the first to point out the possibility

[1] *Ann. der Phys.*, (4) 3, 195; ASTROPHYSICAL JOURNAL, 13, 329, 1901.

[2] A proof of the correctness of this estimate follows, under 16.

[3] "Beobachtungen auf dem Gipfel des Säntis;" *Publ. des Astroph. Observatoriums zu Potsdam*, 8, 49, 1893.

of such errors on the occasion of his making a comparison of the Potsdam system of wave-lengths with the ⊙ St. He found that the difference "Potsdam — ⊙ St." had a prevailingly positive sign from λ 4900 to λ 5400, while it was negative from λ 5400 to λ 6100, and then positive again up to λ 6350. Kayser[1] expressed himself more definitely when he found that the values of several M. St. at λ 3400 are too large by from 0.02 to 0.03 tenth-meter. An entirely certain explanation as to the magnitude and the course of these errors in a part of the visual spectrum was, however, first given in the research by Fabry and Perot,[2] who determined with their interference apparatus the absolute wave-lengths of 33 lines of the solar spectrum and of 14 lines of the iron spectrum. In order to compare their results with Rowland's wave-lengths they computed for the 33 lines the quotient

$$F = \frac{\text{P. T.}}{\text{F. and P.}\odot},$$

which must be constant if Rowland's figures are free from systematic errors. The values of F show, however, a clearly expressed progression which precisely coincides in respect to sense with the results of Müller. Eberhard[3] then made a more precise comparison of the Potsdam and Rowland systems, utilizing 104 lines of the P. T., and was able to furnish another complete confirmation of Fabry and Perot.

15. There can accordingly no longer be any doubt that the wave-lengths of the P. T. are up to 0.02 unit too small from λ 4900 to λ 5370; that they are up to 0.02 unit too large between λ 5370 and λ 6050, and then again too small up to λ 6500. We may assume that similar errors will be revealed in the remaining portions of the spectrum which have not yet been checked, and the question therefore arises in what manner a correction can best be applied to the Rowland system. At the same time it would be necessary to determine the systematic

[1] *Loc. cit.*, p. 198.

[2] "Mesures de longeurs d'onde en valeur absolue," ***Ann. de Chim. et de Phys.***, (7) **25**, 98; ASTROPHYSICAL JOURNAL, **15**, 270, 1902.

[3] "Systematic errors in the Wave-Lengths of the Lines of Rowland's Solar Spectrum," ASTROPHYSICAL JOURNAL, **17**, 141, 1903.

difference between the systems of the P. T. and the M. St., which have been extensively discussed above.

Fabry and Perot write, after determining the values of the reduction factor F: "All recent spectroscopic measurements which have been based upon Rowland's figures contain the same errors. In order to correct them, and at the same time to reduce them to the value of the wave-length of the cadmium line found by Michelson and Benoit, one has only to divide the values by F." I would, however, by no means recommend this procedure, for, in the first place, the systematic error above mentioned between the ⊙ St. and the M. St. is not contained in the F, and since most observers have used the iron spectrum for comparison and not the solar spectrum, the true value of the wave-length would not be obtained by division by F. Secondly, it certainly could not be recommended to alter so greatly all previously determined wave-lengths as would be necessary to pass from the Rowland system to that of Michelson. The reduction factor F has at this place the value 1.000034, whence it follows that all the wave-lengths in the visual spectrum would have to be diminished by about 0.2 tenth-meter if they were to be transferred to the Michelson system. This would produce great confusion.

16. I would, on the contrary, recommend that each correction of Rowland's wave-lengths should be undertaken in such a manner that his numerical values are thereby altered only by the *smallest possible* amount. This may be accomplished in the following manner. As has already been said, Fabry and Perot give the quotient $\frac{\text{P. T.}}{\text{F. and P.}} = F$ for each of the 33 solar lines they measured. The values of F lie between the limits 1.0000286 and 1.0000381. If we take the arithmetical mean of these thirty-three values,

$$F_0 = 1.0000340 ,$$

and multiply all the wave-lengths found by Fabry and Perot by this factor, we shall evidently obtain a system of wave-lengths which fits that of Rowland as closely as possible, but is free from its systematic errors. If we designate these errorless wave-

lengths on the Rowland system by λ, then $\lambda = (\text{F. and P.}) \cdot F_0$. I have computed the values of λ in Table VI, in which the first column contains the wave-lengths directly measured by Fabry and Perot, the second the value of λ, and the third the corresponding number in the P. T. The difference $\lambda - \text{P. T.} = C'$ given in the fourth column therefore represents the correction which must be applied to Rowland's values in order to free them of their systematic errors. The sum of all the positive values of C' is $+0.178$ tenth-meters, of all the negative values -0.183; a proof that the λ system coincides precisely with the P. T.

TABLE VI.

F. and P.	λ	P. T.	C'	C	$C-C$
4643.483	4643.641	.645	− 4	− 7	+ 3
4704.960	4705.120	.131	−11	− 6	− 5
4736.800	4736.961	.963	− 2	− 5	+ 3
4783.449	4783.612	.613	− 1	− 4	+ 3
4859.758	4859.923	.928	− 5	− 1	− 4
4923.943	4924.110	.107	+ 3	+ 1	+ 2
5001.881	5002.051	.044	+ 7	+ 5	+ 2
5090.787	5090.960	.954	+ 6	+11	− 5
5123.739	5123.913	.899	+14	+14	0
5171.622	5171.798	.778	+20	+17	+ 3
5247.063	5247.241	.229	+12	+21	− 9
5247.587	5247.765	.737	+28	+21	+ 7
5339.956	5340.138	.121	+17	+13	+ 4
5345.820	5346.002	.991	+11	+11	0
5367.485	5367.667	.669	− 2	0	− 2
5409.800	5409.984	.000	−16	−11	− 5
5434.544	5434.729	.740	−11	−13	+ 2
5497.536	5497.723	.735	−12	−17	+ 5
5506.794	5506.981	.000	−19	−18	− 1
5586.778	5586.968	.991	−23	−20	− 3
5715.095	5715.289	.308	−19	−19	0
5763.004	5763.200	.218	−18	−18	0
5862.368	5862.567	.582	−15	−15	0
5934.666	5934.868	.881	−13	−11	− 2
5987.081	5987.284	.290	− 6	− 7	+ 1
6016.650	6016.855	.861	− 6	− 4	− 2
6065.506	6065.712	.709	+ 3	+ 2	+ 1
6151.639	6151.848	.834	+14	+13	+ 1
6230.746	6230.958	.943	+15	+15	0
6322.700	6322.915	.907	+ 8	+11	− 3
6335.340	6335.561	.554	+ 7	+10	− 3
6408.027	6408.245	.233	+12	+ 5	+ 7
6471.666	6471.886	.885	+ 1	+ 1	0

The accidental errors of measurement of the individual lines are still contained in the values of C'; in order to free them of

these I have graphically represented their progression by the curve reproduced in Fig. 1. From the curve we now obtain the true values C of the corrections, which are collected in Table VII. The values $C' - C$ given in the last column of Table VI correspond to the accidental errors of observation in the difference F. and P.— P. T. If we regard as equally accurate the absolute measurements of Fabry and Perot and the relative wave-lengths of the P. T., we obtain for the mean accidental error of a wave-length in the two series of measurements, $\pm$ 0.0025 tenth-

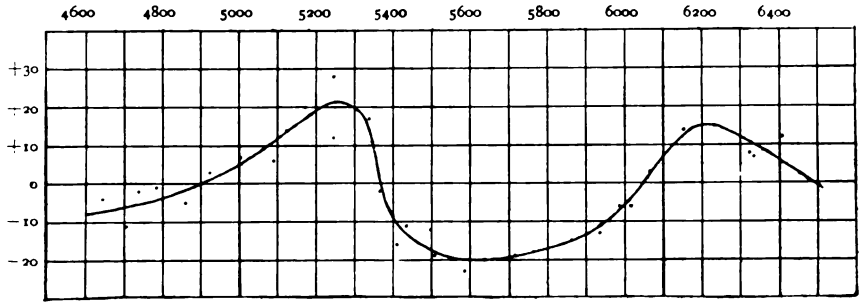


FIG. 1.

meter. The sum of all the values of $C' - C$ in Table VI is zero, whence it follows that the curve was drawn in Fig. 1 with entire accuracy.

17. It is seen that the corrections given in Table VII refer only to the stretch of the solar spectrum from λ 4600 to λ 6490 observed by Fabry and Perot, and hence there arises the necessity, on the one hand, of determining the values of C for the remaining parts of the P. T., and, on the other, of constructing a similar table of corrections for the M. St.

First, in respect to the determination of C for other portions of the solar spectrum, I made the attempt of utilizing for this purpose the plates used by Rowland himself by a new reduction. In all cases where a portion of the solar spectrum, falling within a range of Table VII, was photographed in coincidence with any other portion of the solar spectrum, the values of C for this particular new portion of the spectrum can be obtained by applying the correction from Table VII. It unfortunately

TABLE VII.

CORRECTIONS C OF THE P. T. FROM λ 4600 TO λ 6490.

(Unit is 0.001 tenth-meter.)

	0	1	2	3	4	5	6	7	8	9
46.....	− 8	− 8	− 8	− 7	− 7	− 7	− 7	− 7	− 7	− 6
47.....	− 6	− 6	− 6	− 5	− 5	− 5	− 5	− 4	− 4	− 4
48.....	− 4	− 3	− 3	− 2	− 2	− 2	− 1	− 1	− 1	0
49.....	0	+ 1	+ 1	+ 1	+ 2	+ 2	+ 3	+ 3	+ 4	+ 4
50.....	+ 5	+ 6	+ 6	+ 7	+ 8	+ 8	+ 9	+10	+10	+11
51.....	+12	+13	+13	+14	+15	+16	+16	+17	+18	+18
52.....	+19	+20	+20	+21	+21	+21	+21	+21	+21	+20
53.....	+20	+19	+18	+16	+13	+ 9	+ 4	− 1	− 5	− 7
54.....	− 9	−11	−12	−13	−14	−15	−15	−16	−16	−17
55.....	−18	−18	−19	−19	−19	−20	−20	−20	−20	−20
56.....	−20	−20	−20	−20	−20	−20	−20	−20	−20	−20
57.....	−19	−19	−19	−19	−19	−19	−18	−18	−18	−18
58.....	−17	−17	−17	−16	−16	−16	−15	−15	−14	−14
59.....	−13	−13	−12	−12	−11	−10	− 9	− 9	− 8	− 7
60.....	− 6	− 5	− 4	− 2	− 1	0	+ 1	+ 3	+ 4	+ 5
61.....	+ 7	+ 8	+ 9	+10	+11	+12	+13	+14	+14	+15
62.....	+15	+15	+15	+15	+15	+14	+14	+14	+13	+13
63.....	+12	+11	+11	+10	+10	+ 9	+ 9	+ 8	+ 7	+ 7
64.....	+ 6	+ 5	+ 4	+ 4	+ 3	+ 3	+ 2	+ 1	+ 1	0

appeared that the material available for this purpose was altogether insufficient, since, for the reasons above mentioned, all measurements had to be excluded in which the wave-lengths of the solar spectrum were derived from coincidences with a metallic spectrum of another order. I was able to derive the values of the correction C collected in Table VIII, which are of very slight accuracy, from the few coincidences between two parts of the solar spectrum which I could utilize.

TABLE VIII.

3500	−0.004 tenth-meter
3600	− 2
3700	+ 2
3800	+ 7
3900	+ 4
4000	− 2
4100	− 7

The corrections, therefore, appear to be under 0.01 tenth-meter for this reason. We may regard as comparatively safely

determined only the stretch from λ 3890 to λ 3990, as here as many as seven plates in two different orders were used for each line. In the region of λ 4300 C appears to attain more appreciable values, but for this place, as well as for the other parts of the spectrum, there are so few measurements available that it is impossible to derive even a tolerably certain value of C from Rowland's observations.

18. The necessity is thus shown of securing new observations in order to obtain the corrections to Rowland's wave-lengths, and since the apparatus necessary for the execution of such fundamental measurements is not available to me at present, I should be glad to suggest to others the prosecution of these investigations. On account of the great importance for all spectroscopic researches of the production of as accurate and errorless fundamental system as possible, it is desirable that these researches should not be carried out at one place only, but should be independently undertaken by as many observers as possible.

There are three different problems to solve which mutually complete and check each other: *first*, the corrections C for all portions of the P. T., hence for the entire solar spectrum, should be determined; *second*, the amount of the displacement of the iron lines in the solar spectrum should be accurately determined by photographs giving the solar spectrum with an adjacent arc spectrum of iron; and *third*, a table of standard lines in the arc spectrum of iron covering the whole spectrum, and rigorously reduced to the corrected Rowland system, should be constructed.

19. If absolute measurements are made for the purpose of obtaining the corrections C, either by the interference method of Fabry and Perot or in any other way, there will always be obtained in addition to C a new determination of the reduction factor F_0. However, for the determination of C it is only necessary to make relative measurements in the solar spectrum, as by observations of coincidences with a large concave grating.

20. For the determination of the displacement of the lines of

the solar spectrum in respect to the arc lines, Jewell[1] has already contributed a very valuable series of measures, which embraces the iron spectrum from λ 3424 to λ 4495, and also includes the lines of numerous other elements. Such measurement should be repeated and extended to all regions of the spectrum. The determination of these differences, which I shall designate by *D*, the sense being solar spectrum minus arc spectrum, is at the present moment a pressing necessity in astrophysics. An exhaustive study of these displacements would not only render possible important conclusions as to the distribution of the pressure and the occurrence of the metallic vapors at different altitudes in the solar atmosphere, but would be of fundamental importance in the determination of the radial velocities of the stars. It will doubtless for a long time be the case that for all stars of the second type, which on account of the sharpness of their lines are particularly adapted for accurate determinations of velocity, the most reliable wave-lengths will be taken from the solar spectrum, since we may assume that conditions similar to those in the Sun prevail in the atmospheres of those stars. If in the computation of the velocity of such a star one should employ for the comparison spectrum the wave-lengths determined in the laboratory and referred to Rowland's M. St.—as, for instance, Kayser's iron standards—the whole amount of *D*, which Rowland did away with by empirical corrections, enters into the velocity and vitiates it. If we assume that *D* has a value of + 0.015 tenth-meter in the region of spectrum near λ 4400 chiefly employed for stars of the second type, then every velocity would be found about one kilometer greater than it should be. We should, however, commit precisely the same error if we were to employ the *same* wave-lengths for the stellar and comparison spectrum, as is always the case, for instance, when the method of coincidence measures is used.

21. The above-mentioned series of measures by Jewell is the only one which gives wave-lengths of the iron spectrum strictly referred to the system of the P. T., and I have therefore employed it in order to derive at least provisional values of the corrections

[1] *Loc. cit.*, p. 109.

which must be applied to Kayser's standard in order to make them also comparable with the P. T. Table IX contains the comparison of the lines common to the two observers.

TABLE IX.

Jewell	*K*	*k*	Jewell	*K*	*k*
3424.444	.430	+14	3815.988	.987	+ 1
3440.761	.762	− 1	3820.568	.573	− 5
3441.145	.138	+ 7	3824.584	.591	− 7
3445.308	.301	+ 7	3826.025	.028	− 3
3490.722	.721	+ 1	3827.969	.967	+ 2
3497.990	.989	+ 1	3856.515	.515	0
3536.704	.694	+10	3860.052	.054	− 2
3581.338	.348	−10	3886.415	.426	− 9
3617.934	.944	−10	3928.059	.073	−14
3618.912	.918	− 6	4022.022	.029	− 7
3622.155	.158	− 3	4045.964	.978	−14
3647.983	.997	−14	4062.597	.605	− 8
3687.597	.609	−12	4063.751	.755	− 4
3705.704	.714	−10	4071.898	.901	− 3
3720.075	.083	− 8	4199.263	.256	+ 7
3724.519	.527	− 8	4202.194	.195	− 1
3731.084	.102	−18	4222.396	.387	+ 9
3735.005	.016	−11	4271.917	.933	−16
3737.270	.278	− 8	4325.921	.941	−20
3745.691	.710	−19	4352.900	.910	−10
3748.406	.409	− 3	4369.937	.954	−17
3749.628	.634	− 6	4376.097	.104	− 7
3758.376	.381	− 5	4383.705	.724	−17
3763.932	.940	− 8	4404.911	.929	−18
3767.336	.339	− 3	4415.286	.301	−15
3788.023	.031	− 8	4447.886	.907	−21
3795.144	.149	− 5	4494.749	.755	− 6

The differences in the third column, k = Jewell − K., exhibit some rather large leaps between the individual lines, which are chiefly due to accidental errors of measurement by Jewell. I have smoothed out these figures graphically and give in Table X the values of the correction k thus found.

22. The construction of a table of standard lines of the arc spectrum of iron rigorously referred to the corrected Rowland system, which should serve as the basis for all laboratory measurements, can be accomplished in two different ways. In the first place, we may depart from the relation given above in section 16,

$$\lambda = (\text{F. and P.}) \cdot F \ ;$$

that is, we may reduce to the Rowland system the wave-lengths

TABLE X.

FOR THE REDUCTION OF KAYSER'S IRON STANDARDS TO THE SYSTEM OF THE P. T.

(Unit is 0.001 tenth-meter.)

λ	k	λ	k
3400	+ 9	3950	−11
3450	+ 6	4000	−10
3500	+ 2	4050	− 7
3550	− 3	4100	− 4
3600	− 7	4150	− 1
3650	−10	4200	0
3700	−11	4250	− 2
3750	− 8	4300	− 7
3800	− 5	4350	−13
3850	− 6	4400	−16
3900	− 9	4450	−14
3950	−11	4500	−10

in the iron spectrum measured by Fabry and Perot by multiplying them with the factor $F_0 = 1.0000340$ found from their measures in the solar spectrum, and then connect to them lines in all parts of the iron spectrum by the coincidence method. Instead of the iron lines of Fabry and Perot we may employ the other metallic lines determined by interference methods by these observers as well as by Michelson and by Hamy, the wave-lengths of which, as reduced to Rowland's system, I have collected in Table XI.

The following remarks should be made in regard to this list of standard lines. Rowland's system gives the wave-lengths in air at 760 mm pressure, and at + 20° C., while Michelson's cadmium standards are referred to air at 760 mm pressure and at + 15° C. Fabry and Perot, therefore, properly ought first to reduce their wave-lengths to + 20° before comparing with the P. T. But this correction is precisely proportional to the wave-length, for the stretch from λ 4000 to λ 7000, being here

$$\lambda_{20^\circ} = 1.0000047\, \lambda_{15^\circ}.$$

This reduction factor is therefore contained in the values of F computed by Fabry and Perot, and hence also in F_0; and accordingly the wave-lengths are at the same time reduced to + 20° when they are multiplied by F_0. The values of λ in

TABLE XI.

λ	Element	Observer	λ	Element	Observer
4358.491	*Hg*	F. and P.	5460.9281	*Hg*	F. and P.
4662.5117	*Cd*	H.	5465.675	*Ag*	"
4680.297	*Zn*	F. and P.	5506.970	*Fe*	"
4722.325	*Zn*	"	5586.965	*Fe*	"
4736.946	*Fe*	"	5615.848	*Fe*	"
4800.0739	*Cd*	M.	5763.219	*Fe*	"
4810.699	*Zn*	F. and P.	5769.7946	*Hg*	"
4859.928	*Fe*	"	5782.257	*Cu*	"
5002.057	*Fe*	"	5782.356	*Cu*	"
5083.518	*Fe*	"	5790.8562	*Hg*	"
5085.9969	*Cd*	M.	5890.165	*Na*	"
5086.0754	*Cd*	H.	5896.132	*Na*	"
5105.717	*Cu*	F. and P.	6065.695	*Fe*	"
5153.426	*Cu*	"	6230.945	*Fe*	"
5154.8363	*Cd*	H.	6325.3853	*Cd*	H.
5209.258	*Ag*	F. and P.	6362.561	*Zn*	F. and P.
5218.379	*Cu*	"	6438.6911	*Cd*	M.
5233.132	*Fe*	"	6495.213	*Fe*	F. and P.
5302.501	*Fe*	"	6708.074	*Li*	"
5434.710	*Fe*	"			

Table XI are consequently rigorously referred to the corrected Rowland system (P. T. + *C*). The true value of the reduction factor, depending only on the absolute measurement, or upon the comparison with the standard of length, is therefore

$$\frac{1.0000340}{1.0000047} = 1.0000293 .$$

The large difference between the wave-lengths of the cadmium lines at λ 5086, as determined by Michelson and by Hamy, may possibly be explained by the somewhat different character of the light sources used by the two observers; as to this point, see the articles by Fabry and Perot,[1] Hamy,[2] and Bell.[3] In view of the great importance which these figures have in the construction of a new fundamental system of wave-lengths, it is nevertheless necessary that the values of Table XI should be established with entire certainty by independent series of measurements by different observers.

23. Another way for constructing a list of iron standards accurately referred to Rowland's corrected system is as follows:

[1] *Comptes Rendus*, 130, 653, 1900; ASTROPHYSICAL JOURNAL, 16, 36, 1902.

[2] *Comptes Rendus*, 130, 700, 1900. [3] ASTROPHYSICAL JOURNAL, 15, 157, 1902.

A perfectly correct system of relative wave-lengths of iron (λ') should be established, at first wholly without regard to the absolute wave-lengths, either by interference measurements or by means of coincidences obtained with a large concave grating. The wave-lengths of the same lines on the system of the P. T. is obtained by the above-mentioned (20) measures of displacements in the solar spectrum. F is obtained by the division of the two wave-lengths thus found for each line, and thereafter the reduction factor F_0 is obtained by taking the means, and then the wave-lengths of the iron lines measured are transferred to the Rowland system by means of F_0. Hence the reduction is made by the formula

$$F_0 = \frac{1}{n} \sum \frac{\text{P. T.} - D}{\lambda'} ,$$

where n is the number of lines measured, and then

$$\lambda = F_0 \lambda' .$$

24. For more convenient examination I will summarize here the relationships so far employed and the conditions which connect them:

The true Rowland system at present current is represented by the wave-lengths (P. T.) of the "Preliminary Table of Solar Spectrum Wave-Lengths."

The P. T. contain slowly progressing inequalities, for removing which the corrections C are to be applied. The wave-length of a line of the solar spectrum on the corrected Rowland system is therefore

$$\lambda_\odot = \text{P. T.} + C . \qquad (a)$$

Rowland's standards in the arc spectrum (M. St.) have been systematically vitiated by the application of empirical corrections in a manner that cannot be checked, and therefore they cannot be used as a basis for accurate measures. The system of iron standards ($K.$) of Kayser, which is accurately referred to the M. St., must therefore receive the corrections k in order to be comparable with the P. T., and the wave-length of an iron line on the corrected Rowland system is accordingly

$$\lambda_M = K. + k + C . \qquad (b)$$

There exists between $\lambda_{\odot}$ and λ_M the relation

$$\lambda_{\odot} - \lambda_M = D \ , \qquad (c)$$

wherein it must be remembered that D may change its value from line to line by leaps, and therefore cannot be interpolated, as can C and k.

It follows from equations (a) and (c) that

$$\text{P. T.} - D = \lambda_M - C \ . \qquad (d)$$

The wave-lengths of metallic lines published by Jewell give the values P. T. $- D$, and must therefore, according to (d), be corrected by the amount $+ C$, in order to furnish the correct wave-lengths:

$$\lambda_M = (\text{Jewell}) + C \ . \qquad (e)$$

For the determination of k we get from (b) and (e)

$$k = (\text{Jewell}) - K. \qquad (f)$$

If λ' are entirely correct relative wave-lengths, in order to refer them to the solar spectrum, we have either

$$F_0 \quad \frac{1}{n} \sum \frac{\text{P. T.}}{\lambda'_{\odot}} \qquad (g)$$

$$\lambda = F_0 \lambda'_{\odot} \ ,$$

whence the values of C follow from (a); or, if the λ' referred to metallic lines,

$$F_0 = \frac{1}{n} \sum \frac{\text{P. T.} - D}{\lambda'_M} \qquad (h)$$

$$\lambda_M = F_0 \lambda'_M \ ,$$

whence C may be obtained from (d).

If the λ' in (g) and (h) were obtained by relative measurements, the values F_0 have no general meaning; but if they were determined by absolute measurements—as, for instance, in the system of Michelson or Fabry and Perot—then $\frac{1}{F_0}$ is the factor for converting the corrected Rowland system into the absolute meter system; that is,

$$\lambda_{\text{abs.}} \quad \frac{\lambda}{F_0} \ . \qquad (i)$$

According to Michelson's determination, if the wave-lengths have been previously reduced to the same temperature of the air, the value of

$$F_0 = 1.0000293 \ .$$

Use will be made of this conversion, however, only in very rare cases where the absolute values of the wave-lengths are employed. For most investigations, particularly in astrophysics, it is enough to employ the correct wave-lengths λ, which can easily be derived from all series of observations made up to the present and referred to the Rowland system, by the application of the small corrections C, D, and k.

POTSDAM, ASTROPHYSIKALISCHES OBSERVATORIUM,
June 1903.

THE PEROT-FABRY CORRECTIONS OF ROWLAND'S WAVE-LENGTHS.

By LOUIS BELL.

LAST year in this JOURNAL[1] I took occasion, in discussing the discrepancies between the wave-length measured by gratings and by the interferometer, to call in question the validity of the correction curve deduced by MM. Perot and Fabry for Rowland's relative wave-lengths. Their most courteous reply[2] seems to call for some comment, as it still leaves the main issue in doubt. The corrections deduced by them are actually of small magnitude, never greater than 0.03 tenth-meter and on the average less than 0.02 tenth-meter[3]—so small, in fact, that were they purely accidental, they would be negligible for most spectroscopic purposes. The serious feature of the case is that if the reality of these corrections as systematic errors is established, a grave doubt is raised as to the sufficiency of the grating for the determination of relative wave-lengths by the method of coincidences. The minute care with which Rowland's measures were made is well known, and if the method itself proves to have been faulty, spectroscopists are left entirely at sea for any practical method of establishing relative wave-lengths in ordinary spectrographic work. If an error of even 0.02 tenth-meter is due to systematic causes, errors many times greater may reasonably be expected under less favorable conditions.

For absolute measures the grating must be, as I have already shown, definitely put aside for good and sufficient cause, although I do not consider that even the beautiful work of Michelson and Benoit can yet be accepted as a final result within several times its apparent probable error. The purely metrological difficulties, well known to those who are familiar with the comparison of standards of length, are sufficient to imply some uncertainty. Michelson's value is, however, much more reliable

[1] ASTROPHYSICAL JOURNAL, **15**, 157, 1902.

[2] *Ibid.*, **16**, 36, 1902. [3] *Ibid.*, **15**, 272, 1902.

than any other, and is very properly accepted universally as the standard.

As regards relative measurements the case is different. The closeness of agreement instanced by Perot and Fabry regarding their own measures[1] can be duplicated without especial difficulty in the higher orders of a long-focus concave grating. The relative convenience of the two procedures depends on the nature of the research undertaken. Precision of setting, in methods both essentially micrometrical, is not the issue here involved. It is of fundamental importance to determine whether the grating method is or is not beset by systematic errors which, small in the case cited, may become serious. And I must here reiterate that, while systematic errors may exist in that or any other method, the evidence adduced by Perot and Fabry is insufficient to establish them.

In the first place one must recur to that extraordinary result of Hamy[2] for the wave-length of the green cadmium line. In this instance the relative values assigned by Hamy and by Michelson differ by 0.079 tenth-meter, a quantity four times greater than the differences which Perot and Fabry have sought to establish by reference to this same green line assumed as a standard. It is quite true that the experimenters used different sorts of vacuum tubes, but the observed pressure shifts of the *Cd* lines do not account for any such difference as that here found; and if they did, it would merely show such hypersensitiveness to slight changes in excitation as would indicate that the line in question is a very shifty standard. The other alternatives are a downright error in the application of interference methods, or a shifting of the apparent place of the line under differences of illumination. Until this matter is cleared up the green *Cd* line is certainly open to suspicion, and while Perot and Fabry have not heretofore had occasion to note its possible variability, they may actually have been measuring it in the case under discussion. The point is one which can be settled only by a thorough investigation.

Another source of incertitude in measures carried out by the

[1] *Ibid.*, 15, 263, 1902. [2] *C. R.*, 130, 489, 1900.

method of Perot and Fabry is one common to all micrometric comparisons of dark and bright lines—viz., the difficulty of treating such lines or diffraction rings just alike in making the settings. The consistency of the results does not carry proof that there is no error introduced in locating the centers of the respective objects, and that, if such an error occurs, it will remain constant when the color of the bright line or field changes. The experiments of Reese[1] and of Hasselberg[2] show very plainly that such errors occur in micrometric measures of photographic plates and that they are by no means negligible in amount. In these cases cited they partake of the nature of a personal equation which may or may not have the same sign for different observers. Whether the difference in the readings by two observers having personal equations of the same sign would be comparable in size with their common error would be a matter of chance. The smaller the absolute error, the less likely perceptible differences in it.

The general experience of spectroscopists is that it is extremely difficult to treat a bright line and a dark line just alike with the micrometer. The same difficulty extends to comparing lines of widely different appearance in general. And even supposing the personal equations of MM. Perot and Fabry to have been zero, it still remains to be shown that the dark diffraction ring as measured, having been obtained through a rather wide slit, correctly represents the wave-length of the line as it would be set upon in the grating spectrum itself. Analogy would suggest the negative, and it would be most extraordinary if the variation remained uniform in fields of different colors and intensities.

Difficulties of this sort appear in every branch of spectroscopy when dealing with quantities of the order of magnitude attacked by Perot and Fabry, and are most aggravated in comparing bright and dark lines. Even in Hasselberg's plates, which were on a large scale and uncomplicated by the question of color differences, the errors exceeded 0.01 tenth-meter.

In my previous paper I referred to a discrepancy between

[1] ASTROPHYSICAL JOURNAL, 15, 142, 1902. [2] *Ibid.*, 15, 208, 1902.

Perot and Fabry's correction curve and their value for the *Zn* line at λ 4810 +. The difference, as they remark, is in the direction which would be produced by pressure-shift; but Rowland's measures on the arc agree with his solar value to 0.002 tenth-meter, and the pressure shift for *Zn*, as found by Humphreys and Mohler,[1] accounts for only one-fourth of the outstanding difference of 0.02 tenth-meter. The error may be Rowland's, or Perot and Fabry's, or due to both, but it is much larger than the ostensible probable errors of either.

A far more serious source of error is the apparent neglect by Perot and Fabry of the corrections for the Earth's annual and diurnal changes of relative velocity. Perot and Fabry very carefully eliminated the velocity shift due to the Sun's rotation, but I find no record of corrections for the Earth's motion, which is almost equally important. This error affects all comparisons of solar and terrestrial sources, but of course does not appear in comparisons of solar or terrestrial sources by themselves.

Frost[2] has examined these errors and finds for the correction due to the eccentricity of the Earth's orbit a value varying from 0 to $\mp$0.01 tenth-meter as the Earth shifts from perihelion or aphelion to quadrature, the latter being reached in April and October. The value I here note is for the green of the spectrum. Similarly the variation with hour-angle amounts in the latitude of Paris to about 0.006 $\times$ sin t tenth-meter in the green. These quantities are quite comparable with those which Perot and Fabry have sought to evaluate, and must be reckoned with in every comparison of solar and metallic spectra, although generally, if not universally, neglected.

Any one of the sources of error which I have here noted is sufficient to account for the apparent systematic variations in Rowland's relative wave-lengths found by Perot and Fabry, and until they are eliminated, one and all, it is unsafe to assume that Rowland's relative values involve errors other than accidental. The differences found are much greater than the probable errors of reading either with the grating or the interferometer, and deserve further investigation; but they certainly should not be

[1] *Ibid.*, 3, 114, 1896. [2] *Ibid.*, 10, 283, 1900.

charged up to systematic errors introduced in Rowland's method of coincidences until the method used to determine them is cleared of the uncertainties which have here been noted. I do not hold Rowland's work as in the least sacrosanct, and I greatly admire the elegant method adopted by MM. Perot and Fabry; but the matter at stake is the validity of relative wave-lengths measured with the grating.

The whole discussion tends to confirm the opinion, expressed in my previous paper, that at about 0.01 tenth-meter the precision of wave-length measurements rests on a somewhat shaky foundation on account of numerous and varied small sources of error, which it is to be hoped will eventually be eliminated.

The attempt of Eberhard[1] to confirm the results of Perot and Fabry from the wave-length measurements of Müller and Kempf is interesting, but somewhat unconvincing. In Fig. 1 I have plotted the variations from Rowland's tables of the individual lines studied by Eberhard, together with some others of Müller and Kempf's lines easily identified on Rowland's map. The result is ragged in the extreme, and shows the futility of using such data to define supposed errors eight or ten times smaller than the variations in the data. The similarities between Perot and Fabry's and Müller and Kempf's curves, as shown by Eberhard, seem to be mainly due to the process of averaging. The differences are conspicuous. For instance, as between Rowland and Perot and Fabry there is practically a constant difference from λ 5900 to λ 5600. But in this particular region the difference between Rowland and Müller and Kempf is varying with great rapidity. The greatest abnormality between R. and P. and F. is at λ 5200, while between R. and M. and K. there are two regions of about equally great variation, one at λ 5300, the other at λ 6300. Add to this the wideness of the apparent variations shown by M. and K. as compared with those shown by P. and F., and I do not see how one can avoid the conclusion that the former are entirely worthless as confirmatory evidence. In Fig. 1 I have shown in addition the relation of Rowland's values to those obtained in absolute measure by Thalén.[2] These, as might

[1] *Ibid.*, **17**, 141, 1903. [2] *Nova Acta Upsala*, 1899.

be anticipated, are in vastly better accord with each other than the measures of Müller and Kempf, in which great skill was handicapped by poor gratings; but in relative values they do not agree with either M. and K. or P. and F. Thalén shows an abnormal region at about λ 5150, near that located by P. and F.,

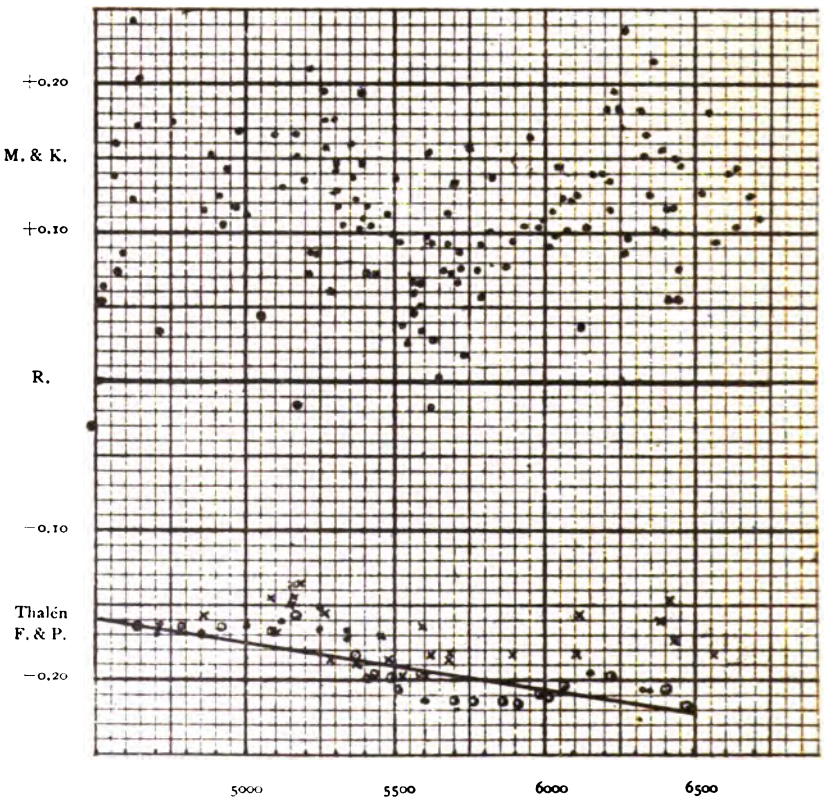


FIG. 1.

but toward the red the two sets diverge, Thalén indicating almost a constant difference from Rowland and showing no such extraordinary difference at λ 5500 as is found in M. and K. In short, Perot and Fabry, Müller and Kempf, and Thalén agree neither with Rowland nor with each other, the variations being conspicuously greater than the probable errors in each case. Rowland's values may very possibly be in error, but they obviously cannot

concurrently err by three different and considerable amounts at the same point. Considering the methods employed and the magnitudes of the supposed errors located by Perot and Fabry it is in no wise hypercritical to say that only the results of these latest experimenters can be taken seriously. I hope ere long, through the courtesy of Mr. Jewell, to have at hand a direct recomparison of certain of Rowland's lines which will confirm or reject the differences found by Perot and Fabry between the regions *circa* λ 5200 and 5700. Meanwhile these differences should be taken to heart as a lesson in the relation between probable and possible errors.

BOSTON, July 1903.

ON THE SPECTRUM AND RADIAL VELOCITY OF χ CYGNI.

By G. EBERHARD.

THE well-known variable χ *Cygni* has been observed by me at its last two appearances in 1901 and 1902 with the aid of the excellent spectrograph IV, designed by Vogel for the Potsdam photographic refractor (32.5cm). The results obtained during its appearance in 1901 have already been published in No. 3765 of the *Astronomische Nachrichten*. The observations of 1902, however, indicate departures in some respects from the star's behavior in the year preceding, and for this reason it may be

1901	G.M.T.	Plate	Observer[1]	Exposure	Slit-Width	Brightness	v from $H\gamma$	v from Fe 4308
	h m			m	mm		km	km
Aug. 2.....	9 28	IV 766	E.	150	0.025	.....	— 17.5	.....
9.....	9 10	772	L., E.	90	0.025	.....	— 17.5	.....
10.....	9 26	774	E., L.	240	0.025	4^m9–5^m0	— 19.9	.....
11.....	9 20	777	L., E.	30	0.025	.....	— 19.6	.....
12.....	9 4	778	E.	60	0.025	.....	— 18.1	.....
15.....	8 3	779	E., L.	55	0.015	.....	— 20.2	.....
20.....	8 2	784	L., E.	23	0.025	.....	— 21.5	.....
21.....	8 7	786	E., L.	30	0.025	4.7	— 20.5	.....
22.....	8 22	789	L., E.	30	0.020	.....	— 23.8	.....
23.....	8 10	793	E., L.	12	0.025	4.7	— 20.3	.....
24.....	10 5	794	L., E.	60	0.025	.....	— 20.6	.....
31.....	7 34	803	E., L.	60	0.025	.....	— 21.2	.....
Sept. 1.....	8 11	805	E.	38	0.025	4.9	— 20.9	.....
7.....	8 40	812	E.	60	0.025	5.0	— 21.2	— 18.6
17.....	6 59	814	E.	60	0 025	5.5 – 5.6	— 17.5	— 20.9
19.....	7 15	823	E.	60	0.075	5.6	— 22.5	— 24.2
26.....	7 30	822	E., S.	60	0.025	.....	— 20.7	— 25.3:
27.....	7 49	827	E., S.	60	0.025	5.8 – 5.9	— 19.1	.....
28.....	8 0	828	E.	60	0.025	.....	— 17.5	.....
Oct. 1.....	7 47	835	E., S.	60	0.025	6.4	— 17.2	— 16.8
3.....	8 23	838	L., S.	120	0.025	6.5 – 6.6	— 23.6	— 21.0
7.....	8 58	840	L., E.	60	0.025	6.7 – 6.8	— 18.3	.....
15.....	7 19	844	E., L.	120	0.025	7.0 – 7.2	— 19.2	— 16.9
20.....	6 15	845	E.	60	0.025	7.8 – 7.9	— 16.4:	.....
Nov. 9.....	5 40	864	L., E.	120	0.025	8.5 – 8.7	— 20.4	— 20.3
23.....	6 24	876	E., S.	180	0.025	9.0 – 9.2	— 15.7:	— 19.1:
						Mean	—19.7	—20.3

[1] Observer: E. = Eberhard, L. = Ludendorff, S. = Scholz.

well to insert here briefly, at the appropriate places, the principal results from the above-mentioned communication.

The star *χ Cygni* has a very weak absorption spectrum, on which an emission spectrum of very bright hydrogen lines and some essentially weaker metallic lines are superposed. While the bright *Hγ* line at the time of the light maximum of *χ Cygni* requires an exposure which corresponds to a star of the first type and second or third magnitude, it is necessary to expose for the continuous spectrum as long as for a star of the first type, but of about the sixth magnitude. On this account it is unfortunately impossible to obtain a good photograph of the continuous spectrum except at the time of the light maximum, and even then only on those evenings during which favorable atmospheric conditions prevail, while the bright *Hγ* line is easily photographable even when the brightness of the star is still less.

The measurement of the bright *Hγ* line and the equally bright *Fe* line at λ 4308 gave for the year 1901 the radial velocity indicated in the preceding table.

In good agreement with those figures is the radial velocity of the star obtained in 1902:

1902	G. M. T. h	G. M. T. m	Plate	Observer	Exposure m	Slit-Width mm	Brightness m	v from *Hγ* km
Sept. 22.....	7	17	IV 1158	E.	30	0.020	4.5	16.3
23.....	8	21	1161	E., S.	240	0.025	4.5	— 17.2
24.....	7	29	1162	E., S.	135	0.025	4.6	— 20.4
25.....	9	23	1163	E., S.	240	0.025	4.4	— 21.9
26.....	8	1	1165	E., S.	240	0.025	4.6	— 21.4
28.....	7	52	1167	E.	40	0.025	.....	17.5
29.....	6	25	1169	E., S.	15	0.025	4.4 - 4.5	- 22.3[2]
30.....	6	34	1174	E., S.	20	0.015	.....	- 20.7
Oct. 11.....	7	10	1176	E., S.	60	0.020	.....	— 24.0
11.....	8	7	1177	E., S.	45	0.010	4.6 - 4.7	— 21.7
15.....	5	38	1178	E.	40	0.015	4.8 - 5.0	— 20.3
21.....	5	13	1184	E., S.	120	0.025	.....	— 17.9
22.....	7	15	1195	E., S.	180	0.025	5.3	- 16.8
Nov. 2.....	6	42	1210	E.	40	0.025	5.2	— 24.3
7.....	6	10	1232	E., S.	30	0.020	5.3 - 5.4	— 23.6
18.....	5	29	1255	E.	40	0.025	6.4	- 25.9
30.....	5	28	1285	E.	50	0.025	7.5	18.0:
Dec. 12.....	5	9	1301	E., S.	90	0.025	7.8 - 8.0	— 27.0:
							Mean	— 21.0

[2] *Hγ* double?

In this table it is to be noted that the estimates of the magnitude of the variable in terms of the usual comparison stars were made only incidentally, and can therefore make no claim to precision, and the given magnitudes may perhaps be uncertain by several tenths. The $H\gamma$ line presented the same appearance during both years; it was hazy toward the red, as was also $H\delta$. This caused an uncertainty in the measurement, since it was not easy, particularly in the case of fully exposed plates, always to judge in the same manner as to the place of maximum intensity which is set upon throughout the measurement. The figures given for the single plates show this clearly. I sometimes got the impression, particularly on plate 1169, that $H\gamma$ was double; but this question could not be decided with certainty in view of the comparatively slight linear extent of the spectrograms. The much fainter *Fe* line at $\lambda 4308$ could be measured only on the plates of 1901, and it was only just at the close of the observations of the next year that it was visible at all, and then it was too faint for measurement.

Nothing in the way of a law appears in the separate figures of the above tables, so that we are well justified in assuming that the radial velocity of the portions of χ *Cygni* showing the emission lines is constant and about -20 km. The comparatively large departures of the individual values from the mean are probably less to be attributed to alterations in the form of the $H\gamma$ line than to the uncertainty of the observer as to the proper place at which to make the setting.

The continuous spectrum of χ *Cygni* varies much from that of the Sun between $\lambda 4200$ and $\lambda 4435$, perhaps most closely resembling that of α *Herculis*. The intensities of many lines—as, for instance, those of *Ca* and *Va*—are wholly different from those in a star of the second type. Many of the characteristic iron lines are partially concealed by the *Fe* emission lines. In all about seventy lines could be recognized between $\lambda 4200$ and $\lambda 4435$ on the fully exposed plates, and I have made the attempt to identify these with known lines of the elements.

The presence of *Ca*, *Ti*, *Cr*, *Va*, *Fe*, and *Mg* could be established, and, in fact, practically all the principal lines of these

metals were present. Several of the absorption lines of iron were in 1901, as already stated, partially concealed by the iron emission lines; for instance, those at λ4415.29, 4383.71, 4325.94, 4308.06. But, on the contrary, no such superposition could be determined, and, indeed, the *Fe* emission lines did not appear at all until the brightness of the star was again rapidly decreasing.

There were a large number of lines, particularly of the broader ones (presumably groups), which I could not assign to any element with any degree of probability. I do not give these in detail here, since a minute knowledge of the spectrum of *χ Cygni* will not be attained until a bright star of the same, or at least of a similar, character (such as *o Ceti* at a principal maximum) has been investigated with an instrument of the greatest possible dispersion. On account of our still too limited knowledge of the continuous spectrum, there is only a small number of tolerably unobjectionable lines at our disposal for a determination of velocity. Based upon twelve and eighteen such selected lines the radial velocity of the portion of *χ Cygni* producing the continuous spectrum resulted as follows:

1901	+2.5 km	Plate 772	1902	−1.3 km	Plate 1162
	+2.3	774		−3.3	1165
Mean	+2.4 km			−2.3 km	

I was at first of the opinion that this not particularly large difference of the values of the radial velocity was perhaps to be attributed to flexures in the apparatus during the comparatively long exposure of from two to four hours. This is, however, not very probable, since with these long exposures the *Fe* comparison spectrum was for precaution taken both at the beginning and end as well as at the middle of the exposure; moreover, the plates were made at neither very large nor very different hour angles, the middle of the exposures falling in every case near the meridian. Furthermore, if such flexures actually occur, the radial velocities of these plates, as deduced from the bright *Hγ* line, should exhibit especially large deviations from the mean values, which is not the case. Hitherto an appreciable effect of

flexure has not appeared on the plates taken with spectrograph IV, and temperature changes are excluded by the continuous careful supervision of the electric heating, so that we may assume for the present, with a high degree of probability, that the portion of χ *Cygni* producing the continuous spectrum has a variable radial velocity. Further observations will be required, however, for the certain decision of this matter, and I therefore plan to continue the spectrographic observations of χ *Cygni* during this year, particularly in respect to the above question, the solution of which is of the greatest importance for the explanation of the phenomena presented by a variable of this sort.

It is in any case quite remarkable that χ *Cygni* exhibits the same behavior as *o Ceti* in the fact that for both stars the emission spectrum is displaced toward the violet of the absorption spectrum, contrary to the manner in the case of the temporary stars. On June 1 (Plate D 115; exposure 90 m) and June 9, 1899 (D 116; exposure 150 m), two plates of χ *Cygni* were made by Dr. Ludendorff and myself with spectrograph III, which showed $H\gamma$, $H\delta$, $H\zeta$, $H\theta$, and $H\iota$ to be bright, $H\epsilon$ being lacking on account of the strong calcium absorption. $H\delta$ is by far the strongest line. A bright line having the wave-length 3905.8 also appears, and therefore coincides with the principal line at 3905.7 (Rowland) in the arc spectrum of silicon. I followed up this interesting fact and found that the line also occurred in a spectrum of *Mira Ceti* obtained by Dr. Ludendorff and myself in 1899, and that it was also doubtless measured by Vogel[1] in earlier plates of this star (λ 3906). The fact that the bright $H\beta$ line was not present, although observed by Miss Maury,[2] is to be attributed to the fact that light of this wave-length is united too far from the focus of the region ($H\gamma$ to $H\zeta$) ordinarily used, both for the photographic refractor and for spectrograph D.

On the plates of 1899, 1901, 1902 there is further present between λ 4000 and λ 4200 a series of strong brightenings, giving the impression of broad bright lines. There is a particu-

[1] "Ueber das Spectrum von *Mira Ceti*," *Sitzungsberichte der K. Akad. der Wiss. zu Berlin*, 17, 395, 1896.

[2] *Annals of Harvard College Observatory*, **28**, I, 98.

larly strong band of this sort at λ 4138, and one lying at the side toward red of *Hδ*. Inasmuch as spectrograph IV does not permit any accurate measurements of wave-lengths in this region, I have not been able to attempt the identification of these bright bands.

In concluding I would also mention that the bright lines *Hγ*, *Hδ*, and *Fe* λ 4308 altered their intensities in different ways in 1901. From August 2 until September 19 *Hδ* was considerably brighter than *Hγ*; from October 3 to 15 *Hγ* and *Hδ* differed little from each other; on October 26 they were equal; and on November 9 and 23 *Hγ* was brighter than *Hδ*. The *Fe* line λ 4308, on the contrary, increased in brightness the fainter the star became. While it was not present on August 24 and 31 with an exposure of 60 minutes, it appeared on September 7 in an equal exposure time and remained of this intensity until about the last plate, on which it was equal to *Hγ* in brightness. The other bright *Fe* lines — for instance, λ 4402 — were only just barely visible with the exposures chosen and were in any case not measurable.

POTSDAM, June 23, 1903.

POSTSCRIPT.

(July 14, 1903.)

In *Bulletin* No. 41 of the Lick Observatory Mr. Stebbins has just published his pretty investigations of the spectrum of *o Ceti*. It is evident that my similar paper on *χ Cygni* in *A. N.* No. 3765 was unknown to him, for otherwise the highly interesting fact would not have escaped him that both stars exhibit a precisely identical behavior, both as to the spectrum and as to the variations of the spectrum; whence it is highly probable that this sort of spectrum is typical for the long-period variables of that class.

ON DOUBLE REVERSAL.

By W. J. HUMPHREYS.

APPARENTLY not much attention has been given to double reversal; that is, the occurrence of a narrow bright line on the dark space of a broadly reversed one. Liveing and Dewar[1] speak of having often momentarily seen, though rarely photographed, this condition on the introduction of fresh material into the arc. Kayser[2] says that he has photographed it very rarely ("nur ungemein selten"), and that he doubts the genuineness of the two cases selected from his plates to illustrate this phenomenon; a doubt well founded, as will appear presently. Jewell,[3] on the other hand, states that a certain result exhibited by double and multiple reversal, displacement of the line due to quantity of material, "is shown in a large number of cases." Still in these different papers, and elsewhere, so far as I am aware, the phenomenon is soon dismissed, and no experiments are described that help to determine just what is essential to its appearance.

Double reversal has long been known to be both persistently and well marked in the Sun's faculæ, so that there can be no doubt about its actual existence, under proper conditions; and thereby its claim to specific investigations is established.

I do not know how many photographs I have taken of arc spectra, but certainly several thousand, and of practically all known elements that can be so used. The great majority of these were secured at atmospheric pressure, but a few hundred were obtained at higher pressures, up to nearly fifteen atmospheres. The volume of the current has been multiplied more than a hundredfold, and the quantity of material in the case of several elements variously altered from mere traces to solid rods of the metal in a reasonably pure state. Still with all these changes, until a method presently to be explained was adopted, I never observed visually a single case of double reversal, and

[1] *Proceedings of the Cambridge Philosophical Society*, 4, 264.

[2] *Handbuch der Spectroscopie*, 2, 363. [3] ASTROPHYSICAL JOURNAL, 3, 95, 1896.

only a few of my plates give any evidence of it. Besides in each of these few the phenomenon is, I am convinced, entirely spurious.

An interesting case of apparent double reversal is shown in Fig. 1 of the accompanying plate. This line in the reversal of D^1 is supposed by Kayser[1] to be due to sodium, and he regards it as possibly a true double reversal, and also says[2] that it shows the displacement of the center of the reversal when a large amount of material is used from the position of the corresponding line given by a small amount of the substance.

During the course of this investigation I got what I shall call the true double reversal of the sodium D's, as shown in Fig. 4. Here both the D's are doubly reversed, and the fine lines are centrally placed. After some further experiments it was found that by mixing iron and sodium together and burning them in a common arc Fig. 1 was always obtained, but never so when the iron was left out. The conclusion therefore is, and this is supported by careful measurements, that the fine line in question is the ultra violet iron line $\lambda 2948.00$, second order, superimposed on D_1, $\lambda 5896.16$, first order.

This must not be taken as a wilful criticism of Kayser's work. All of us who have the pleasure of making investigations in this fascinating field acknowledge ourselves immeasurably indebted to him, and therefore feel it a duty as well as a privilege to make, whenever we can, even the slightest addition to or correction of his masterly treatise.

Fig. 1 seems to indicate that, while the outer layers of relatively cool sodium vapor can strongly absorb light of the wavelength given by D_1, it cannot so cut out light of double that frequency. Indeed, no theory, so far as I know, would lead us to suppose that free particles, such as fill the electric arc, could absorb vibrations other than those which they can also emit. However, that idea in this particular case was experimentally tested, as follows:

Two arcs were connected up in series, and placed, about 5 cm apart, in line with the slit of the spectroscope. The arc next the

[1] *Handbuch der Spectroscopie*, 2, 363. [2] *Ibid.*, 365.

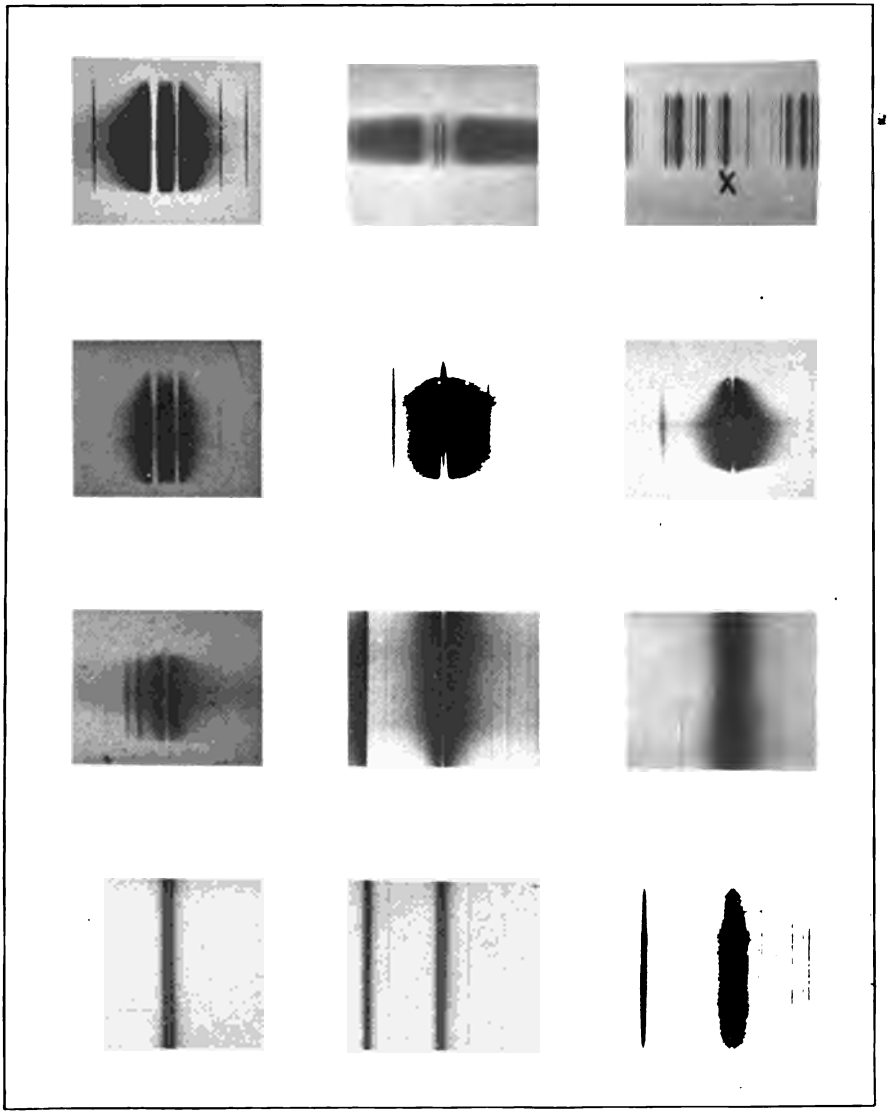
slit contained a large amount of sodium, while the one farther away had a small amount of iron. On developing the spectrogram secured with this arrangement, the fine iron line came out nicely on the reversal of D_1, and showed that its light had not been very greatly absorbed. When, instead of iron, a small amount of sodium was used in the more distant arc, no trace of the D's thus obtained could be seen on the broad reversals due to the sodium in the nearer arc. It would seem then, to borrow a musical term, that in this case the particlis that cut out one note cannot absorb its octave. Neither will it absorb the sub-octave, as is shown by mixing a trace of barium with thallium, by which the barium line λ 5535.69, first order, is superimposed on the reversal of the thallium line λ 2767.97, second order.

A somewhat different case of apparent double reversal is given by adding some iron to aluminium. Here the reversal of the aluminium line λ 3092.84 contains the iron line λ 3092.87. In this case the relatively cool vapor, in the outer parts of the arc, that cut out an aluminium radiation of a certain wave-length, either will not absorb an iron radiation of the same wave-length or else, as seems to me more likely, its absorptive power at this place is not equal to the combined corresponding emission of the two elements.

With materials free from iron, one could easily determine experimentally which of these suppositions is correct, but, as my carbons and aluminium both contain enough iron to render the results doubtful, I have not as yet undertaken it.

Occasionally two lines of the same element produce results markedly like double reversal. Thus the iron line λ 2966.99, third order, contains in its reversal the iron line λ 4450.44, second order. Also the two iron lines λ 3020.70 and λ 3021.15 both reverse in such way as to appear like a single broadly reversed line with a fine line a little to the violet of its center. But the best example I have seen of this is given, when a large amount of iron is used, by the lines λ 2973.17 and λ 2973.41. This combination, as it appears on one of my plates, is shown in Fig. 3. The fine line, nearly centrally placed in what seems to be the reversal of a single line, is, as shown by a careful determination

PLATE V.



Double Reversal.

of its wave-length, $\lambda 2973.28$, only the unabsorbed remnants of the red and violet sides respectively of the two reversed lines. Clearly then one side of each line is largely absorbed by the vibrating particles that produce the central parts of the other, a result in accord with the idea that in any particular case the absorption of radiant energy depends upon its wave-length and not upon its origin.

Fig. 2 illustrates probably the most common case of apparent double reversal. This line $\lambda 3383.00$ was obtained by putting a button of metallic silver in the arc. At first the arc burned violently, and most of the metal was quickly driven out. This period was followed by one of comparatively quiet action with a smaller amount of silver. The result was, therefore, a negative, produced by successive exposures, indistinguishable in appearance alone from true double reversal. In the same way I have obtained one plate that shows an apparent triple reversal of the silver lines.

As stated above, true double reversal is found in the faculæ, and it was during an attempt to explain to Professor Ormond Stone its probable origin in this case that it occurred to me to test the effect of two arcs connected in series and so placed that the light of one could reach the slit only by passing through the other. By putting a small amount of the substance to be studied in the arc nearest the slit and a large amount in the other, which was about 5 cm farther removed, the phenomenon of double reversal was always produced. On interchanging the arcs no such result appeared—only the dark reversals due to the large amount of the substance then in the nearer arc.

The following table gives a list of the lines that were thus obtained doubly reversed. All those in the visible region were repeatedly observed, and, with the exception of the lithium line, also photographed. This list, which a little care would extend, is, I trust, sufficient to show how double reversal may always be obtained; that is, by the use of two independent sources of the radiation in question, the one giving the broadly reversed line shining through the nearer source which contains a relatively small amount of the material.

The doubly reversed lines in the spectra of faculæ may therefore be interpreted as due to two practically distinct sources of light: a source deep in the Sun's atmosphere, where the material is abundant and the consequent reversals broad; and above this a more or less detached, self-luminous cloud containing a relatively small amount of the substance.

It might at first seem, if this is the explanation of double reversal, that ordinary arc spectra should often show the phenomenon, due presumably to the arc burning in two parts, one behind the other, and containing a different amount of the material. This, however, would really be two parallel arcs, and, as is well known, they will not burn that way. A split arc is therefore not a thing of any permanence, if indeed it has even a momentary existence; and thus true double reversals, as above explained, seldom, if ever, appear in arc spectra.

DOUBLY REVERSED LINES.

Element	Wave-length	Remarks	Element	Wave-length	Remarks
Ba	4554.21		*Pb*	4057.97	
"	4934.24		*Li*	6708.20	Small line complex
"	5535.69	Small line confused	*Mg*	2852.22	Fig. 7 of plate, small line also reversed
Cd	3261.17	Fig. 10 of plate			
Ca	3933.83		*Ag*	3280.80	Fig. 5 of plate, small line also reversed
"	3968.63				
"	4226.91	Fig. 8 of plate	"	3383.00	
Cu	3247.65		*Na*	5890.19	Fig. 4 of plate
"	3274.06		"	5896.16	Fig. 4 of plate
Pb	2614.26		*Sr*	4077.88	
"	2802.09		"	4215.66	
"	2823.28		"	4607.52	Fig. 9 of plate
"	2833.17		*Tl*	5350.65	Fig. 6 of plate
"	3639.71		*Sn*	3175.12	Fig. 11 of plate
"	3683.60	Fig. 12 of plate	"	3262.44	

The doubly reversed lines offer a favorable opportunity for studying the effect of quantity of material on wave-length. The lithium line $\lambda 6708.2$ shows a double reversal very excellently, and even the smaller line is also often reversed. In this case the larger part of the small line is to the red of its own reversal, while when not reversed the fine line is to the blue of the center of the larger reversal.

This line was found by Michelson[1] to be a confused double, the difference in wave-length between the components being about 0.14 tenth-meter. The components are unequal in intensity, but there is no record as to which has the longer wave-length. My own observations would be fully accounted for by assuming the line to be double, as Michelson found it, and with the feebler component to the red of the heavier one.

The barium line $\lambda 5535.69$ also seemed either confused, or closely accompanied by a line due to some other element, probably the iron line $\lambda 5535.52$.

It is well known that many lines are more or less complex, some of them exceedingly so, and it may be that the brightest component, when a small amount of material is burned, does not remain so when the substance is used in quantity. At any rate, the complexity and unsymmetrical spreading of many lines make it doubtful whether the wave-length of any given component is ever changed by merely varying the amount of material producing it.

However this may be, careful and repeated measurements in both directions over my plates fail in most cases to show with certainty any displacement of the fine line from the center of the broad reversal. Even the sodium D's, *g* of calcium, the strontium line $\lambda 4607.52$, and some others, that are greatly shifted by mechanical pressure are within errors of my measurements centrally placed.

UNIVERSITY OF VIRGINIA,
August 5, 1903.

[1] *Phil. Mag.*, September 1892.

PHOTOGRAPHIC OBSERVATIONS OF BORRELLY'S COMET AND EXPLANATION OF THE PHENOMENON OF THE TAIL ON JULY 24, 1903.

By E. E. BARNARD.

THE quick-acting lantern lens (which is really a doublet or portrait combination, is specially suited for photographing large diffused masses of faint light in the sky. For small objects or for minute details it is, of course, unsuited because of its small scale. This kind of lens, therefore, has its limitations as well as its advantages. As an auxiliary to a larger photographic telescope for the delineation of such phenomena as the extent of comets' tails, etc., it is indispensable. The rapidity of such a lens for cometary work is well shown in the photographs made here of Borrelly's comet of this year.

The only suitable lens available at the time for photographing the comet was a small lantern lens of 1.6 in. aperture and 6.3 in. focus (see ASTROPHYSICAL JOURNAL for January 1903). With this lens attached to the 12-inch equatorial a series of photographs was obtained of the comet. These were made with the intention of accurately locating the position of the comet's tail.

Mr. R. J. Wallace, of this Observatory, becoming interested in the subject, began an independent series of photographs with the same lens at the close of my exposures, and thus covered several nights on which other work prevented my photographing the comet; for it was intended to make the set as complete as possible. By this means there were several dates on which the comet was being photographed here from dark until dawn.

In general the two pictures for any one night showed no decided difference in appearance. On the night of July 24, however, a remarkable change was shown to be taking place in the comet.

During the period of visibility of this object with the naked eye, it appeared as a bright hazy star between the second and

PLATE VI.

July 23, $15^h\ 27^m - 18^h\ 2^m$ G.M.T.

July 29, $16^h\ 5^m - 18^h\ 5^m$ G.M.T.

E.

July 24, $17^h\ 59^m - 20^h\ 29^m$

July 24, $14^h\ 57^m - 17^h\ 34^m$

COMET *c* 1903 (BORRELLY).

third magnitudes. Feeble traces of the tail could be seen with the naked eye for some 4°. In a small telescope this could be traced for even a less distance. With every advantage in its favor—a close approach to both the Earth and Sun; above the horizon all night at a high altitude, with no moonlight to interfere; and with a head as bright as some of the "great" comets —it was decidedly disappointing so far as the naked eye was concerned. The nucleus seemed to be very inactive in the production of the ordinary phenomena of a comet's tail. The lantern-lens photographs, however, showed that the comet really had a considerable extent of tail—as much as 17°.

As stated, with the exception of the length and the accurate location of the tail, the small lens did not show much of interest until July 24, when a very remarkable and important phenomenon took place, which was almost unique in its character. Some two or three degrees back of the head the tail was apparently broken off and the outer portion shifted bodily toward the direction from which the comet was receding and in a line parallel with the remaining portion of the tail—as if some force had suddenly broken off a large section of the tail and pushed it to one side of its former position. Mr. Wallace's photograph, which was begun at the close of mine, shows that the apparent separation was taking place rapidly, as if the fragment were being left behind by the comet in its flight through space.

This singular phenomenon is not entirely unique. One of the photographs of Gale's comet of 1894 with the Willard lens showed a similar, though not so pronounced, appearance.

A photograph made on July 25 showed no traces of this peculiarity, the tail having apparently assumed its normal condition while another photograph on the 23d had shown nothing abnormal about the tail. The phenomenon was, therefore, confined to an interval of a few hours only.

The following measures were made of the photographs of July 24. "The tail" refers to the short tail attached to the head, while "the section" refers to the separate portion some 9° long, which was nearly parallel with the direction of the small tail:

PLATE I (E. E. B.).

Length of tail = 2°51′.
Distance from head to nearest end of section = 2°14′.
Distance between the axes at the point of separation = 0°12′.4.
Width of tail just back of head = 0°4′.

PLATE II (R. J. W.).

Length of tail = 4° 7′. It diffuses out beyond this.
Distance from head to nearest end of section = 2°48′.
Distance between the axes at the point of separation = 0°18′.0.
Width of tail just back of head, not so narrow — more diffused.

Following are the exposure times of the plates:

	Plate I	Plate II
Begin	$8^h\ 57^m$	$11^h\ 59^m$
End	11 34	14 20
Middle	10 15	13 14
Interval	$2^h\ 59^m$	

As will be seen, the duration of exposure was essentially the same in each case.

In the *Bulletin de la Société Astronomique de France* for August is reproduced a photograph of the comet, made by F. Quénisset, of the Observatory of Nanterre, on July 24–25 from $23^h\ 9^m$ to $0^h\ 9^m$. The scale of this photograph appears to be 5.05 times that of the lantern lens — from measures of stars on the two photographs. Assuming the above to be Paris times the following are the Greenwich times:

Begin July 24	$11^h\ 00^m$	G. M. T.
End	12 00	
Middle	11 30	

As my exposure was made at $16^h\ 15^m$ G. M. T. and Mr. Wallace's at $19^h\ 14^m$, it will be seen that Mr. Wallace's photograph was $3^h\ 0^m \pm$ later than mine, and that of M. Quénisset $4^h\ 35^m$ earlier. I have measured the distance between the head of the comet and the nearest end of the section on the Nanterre photograph, and get 1° 36′. By a combination of the times of these photographs, we get the following values of the motion of separation of the section and the head:

Nanterre and first Yerkes Observatory, - - 10.′1 hourly
First and second Yerkes Observatory, - - 10.7 hourly,

or, taking the mean of these—assuming the difference of the two results as accidental, which may not be the case—we have 10.′4 for the mean hourly motion of separation of the nearest end of the section and the head With this motion it can be shown that the separation of this section of the tail from the nucleus or the head of the comet occurred at 2^h 30^m G. M. T., July 24.

With the aid of the elements of the comet's orbit, and assuming that the direction of the tail was coincident with the prolongation of the radius vector—from which it will deviate but little at most—the hourly rate of separation was 104,000 miles, or 29 miles a second. A large portion of this was due to the comet's approach to the Sun, which amounted to 22 miles a second. The remaining 7 miles a second must, hence, be due to the motion of the tail away from the Sun, which therefore is the actual velocity of the particles of which the tail is composed.

This leads one directly to an explanation of the phenomenon observed on the photographs. The appearance in question would seem to be due to a sudden change in the direction of emission of the particles. This must have occurred between 2 and 3 o'clock, G. M. T., on July 24. If the emission had suddenly ceased altogether, then the tail would recede from the comet bodily until it dissipated in space or ceased to be luminous. Suppose, however, that, instead of stopping, the direction of emission should suddenly be shifted slightly to one side. So far as the tail at that moment was concerned, it would be the same as if the emission had ceased entirely and it would recede bodily into space; but at the same moment a new stream of particles would begin to form a new tail in the new direction, and we should have exactly the phenomenon presented by the comet on July 24. By the 25th the disconnected tail or stream of particles would have vanished by dissipation, or by ceasing to be luminous from some other cause, and the new tail would have assumed the normal appearance.

This theory may be illustrated by a jet of steam issuing from

a nozzle. If the nozzle is suddenly slightly changed in direction, the old stream will recede for a few moments before it dissipates, and a new one will follow it closely in a slightly different direction.

That the particles of the tail take a definite time to traverse its length shows that the tail may actually move as a body—a stream of small particles. Should this moving stream encounter a resisting medium of any kind, the tail would become distorted and broken, just as was shown to have occurred in the case of Brooks' comet in 1893. It is well known that swarms and streams of meteors exist within the solar system. Whether they are dense enough to produce the disturbances shown in the tails of some comets or not, it is not the purpose of this paper to say, but the explanation seems a probable one. Or there may be other currents or sources of resistance in space of which we know nothing.

There is one feature that must be taken into account in considering the motion of these particles. Previous to its free existence, the particle was moving toward the Sun with a velocity of twenty-two miles a second—as a part of the comet. When it became independent and subject to the repulsive action of the Sun, it would still approach until the repulsive force checked its speed, when for a moment it would become stationary, and then begin to recede, at first slowly, and then with rapidly increasing velocity. The hourly motion from the Yerkes Observatory plates is greater than that from the Nanterre and Yerkes Observatory plates. This would be in accordance with the above idea, except that one would expect a greater difference. The velocity derived from these photographs seems small. In many comets' tails the particles must move with a speed many times this amount.

An inspection of Mr. Wallace's photograph shows that the new tail, at its end, is growing more rapidly than the motion of the section. This would imply that some of the particles move with a much greater speed than others. By using the near end of the section, therefore, which is the only available method, one would get the velocity of only the more sluggish particles.

Following are the measures of the angles made by the tail with

the direction of the motion of the comet; that is, they are the angles the tail made with the star trails near the comet's head. It is not possible to determine this angle with great exactness, because the tail was seldom perfectly straight when the photographs were examined with a glass. The measures, therefore, represent the general direction of the tail with reference to a tangent to the apparent direction of motion of the comet.

PLATES BY B.

Date	Exposure, C. S. T.	Angle of Tail	Length	Sky
July 18.....	$10^h 24^m$ to $12^h 2^m$	42°	13°	Fair
19.....	9 22, about 20^m	39	13	Clouds
22.....	9 0 11 30	38	14	Clouds
23.....	9 27 12 2	38	13	Fair
24.....	8 57 11 34	40	15	Clouds
25.....	8 50 11 ±	36	12	Clouds
26.....	9 9	37	13	Clouds
29.....	10 5 12 5	34	8	Clouds
30.....	10 35 11	..	4	Clouds
August 16....	8 34 9 30	..	10	Thick sky

Mr. Wallace's plates have been measured in the same manner.

PLATES BY W.

Date	Exposure	Angle of Tail	Length	Sky
July 20.....	$9^h 40^m$ to $12^h 5^m$	40°	11°	
22.....	12 1 14 31	41	15	Good
23.....	12 40 14 43	38	14	
24.....	11 59 14 29	38	15	Very poor
29.....	12 30 13 40	41	7	Clouds

The long and unjustly neglected stereoscope when applied to certain astronomical subjects has become of great importance in recent years. The combination of two views of the Moon, similar with respect to phase, made on different dates, reveals its spherical form in a most beautiful manner. Dr. Max Wolf has used this instrument recently for detecting variable stars or moving asteroids on celestial photographs with great success. Two photographs of the same region of the sky with the same instrument being properly adjusted for the instrument, the asteroid or variable star will stand out conspicuously from the other stars. In 1902

he presented to the Royal Astronomical Society some stereoscopic views of Perrine's Comet of that year, in which the comet stood out in beautiful relief, apparently suspended in space before the observer. These were exhibited at the November Meeting of the Royal Astronomical Society in 1902.

In the August 1903 number of the *Bulletin* of the French Astronomical Society is published a stereoscopic picture of the present Borrelly's Comet made by F. Quénisset at Nanterre, France, which when placed in the stereoscope shows the comet in beautiful relief.

Mr. Wallace has combined one of his photographs and one of mine of this comet made on the night of July 22 for the stereoscope. The result is surprisingly beautiful. The comet appears to be suspended in space between the observer and the stars in a most realistic manner. The result is what really happens in space, but which cannot be seen in any manner except with the aid of the stereoscope. A reproduction of Mr. Wallace's beautiful combination is given here, Plate VII, with the other pictures of the comet, and when placed in a stereoscope will exhibit the comet in splendid relief.

Following are descriptions of all of the photographs of the comet obtained here:

1903, July 18 (B.).—Very slight curvature of the tail, convex side preceding. It is narrow back of the head and gradually widens a little. The axis is not symmetrical with the center of the head. There are indications of a thin tail or fragment of a tail symmetrical with the head and preceding the main tail.

July 19 (B.).—Tail faint and feeble and apparently straight.

July 20 (W.).—Tail narrow back of head, and gradually widens out considerably, symmetrical with axis.

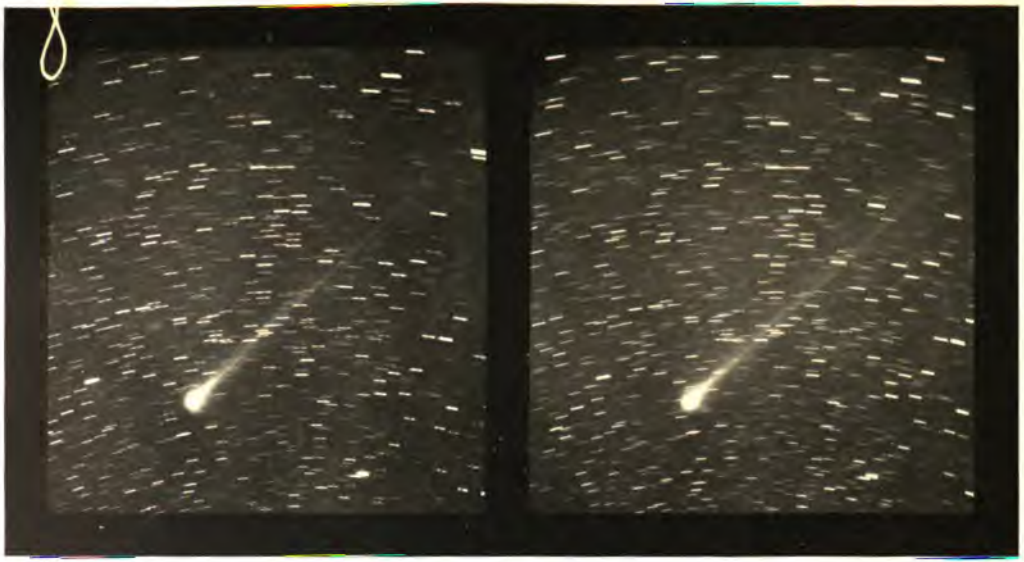
July 22 (B.).—Several strands in tail back of head, one of which is sharply defined and thread-like and is in the following side of the tail. About 6° from the head there becomes visible a faint strip 9° long, which runs parallel with the following side of the tail and distant from it 0°.2. This seems to have no connection with the head. Back of the head the tail is not so narrow as before.

July 22 (W.).—Three thread-like strands back of head. These fade out into the tail farther back. The faint strip shown on B.'s plate not visible.

July 23 (B.).—Indications of several strands from the head for some 3 to 4 . The tail bulges out slightly, preceding.

PLATE VII.

July 22, $15^h\ 0^m - 17^h\ 30^m$ G.M.T. July 22, $18^h\ 1^m - 20^h\ 31^m$ G.M.T.



COMET *c* 1903 (BORRELLY). FOR THE STEREOSCOPE.

July 23 (W.).—Shortly after leaving the head the tail is faintly double, as if a fainter portion ran along beside it following.

July 24 (B.).—The tail is very narrow back of the head, and is sharper on the following side. The section is equally sharp on both sides; the nearer end is narrow and more or less pointed.

July 24 (W.).—The main tail is narrow. Near the break it brushes out somewhat preceding. The section of tail is diffused on its preceding side, and apparently slightly double in places; the following edge of it is sharper.

July 25 (B.).—Faint and straight. No signs of the fragment of last night. The exposure was so interrupted by clouds that the utter dissipation of this fragment is not certain. If present, it must have been extremely faint.

July 26 (B.).—The tail is faint and straight, but is somewhat sinuous 3′ back of head, where it bulges out slighly, preceding.

July 29 (B.).—Slightly sinuous about 2°.2 back of head, and bulging out slightly, preceding. The tail gradually widens out. Its end is cut off by defect in negative.

July 29 (W.).—The tail is faint and apparently straight. If continued, it would cut through ϵ *Ursae Majoris*.

July 30 (B.).—Faint traces of tail for 3°.5. It seems to be straight, but is cut off by defect in negative.

August 16 (B.).—Tail extremely faint clear up to the head. It appears to be straight.

It is evident that the faint strip of tail shown on B.'s plate of July 22 is a stream of particles whose emission had ceased or changed its position some hours before.

In *Knowledge* for August is a reproduction of a photograph of the comet on July 24, made by Dr. Roberts. This shows the phenomenon exhibited on our plates for that date. Dr. Roberts has failed to give the time of this photograph, or of the others reproduced with it, and hence unfortunately the picture at present cannot be used for determining the motion of the fragment.

YERKES OBSERVATORY,
August 1903.

Minor Contributions and Notes.

AN APPLICATION OF THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY TO THE STUDY OF VERY FAINT SPECTRA.[1]

INTRODUCTION.

While engaged in photographing the brighter nebulæ with the Crossley Reflector, Professor Keeler, director of the Lick Observatory, was surprised at the relative photographic brightness of the central star in the Ring Nebula in *Lyra*, though the fact had been previously remarked by many observers. This star, which is visible only in the largest telescopes, was found to be brighter photographically than the nebula itself, which is an easy object in even a comparatively small telescope. It is true that a large faint area, such as a nebula, is more readily seen than a faint stellar point, but considering that the available telescopes magnify the Ring Nebula several hundredfold, and in effect leave the star unmagnified; that the image of the nebula photographed with the Crossley Reflector is virtually magnified but twenty-fold; and that the star photographs in about half the time required by the nebula, under good atmospheric conditions — making allowance for the physiological effect referred to, and for the proportions of the instrument, the photographic brightness of the star suggested that its spectrum contains an unduly large proportion of actinic rays. Professor Keeler also noticed that the central star in the Ring Nebula in *Cygnus* (H IV 13) behaved in the same manner. He tried to examine their spectra, both with a direct-vision spectroscope and with a prism held in the path of the rays, but in neither case was the spectrum bright enough to be seen. He therefore decided to construct a spectrograph for the study of these objects.

Such an instrument, to preserve and utilize the enormous advantages of the silver-on-glass reflecting telescope for work in the violet and ultra-violet regions, called for a design radically different from those of conventional spectrographs. It would evidently be difficult for a star near the limits of vision to be centered and kept upon a narrow slit; the method of guiding by means of a reflecting slit would not

[1] *Lick Observatory Bulletin* No. 35.

answer; and it would be important to avoid the usual losses due to a slit. The instrument as originally designed consisted, in outline, of a 50° quartz prism with circular aperture of 27 mm, placed directly in the converging beam of light from the great mirror, at a distance of 15 cm inside the focus; of a plate-holder suitably placed; and of a guiding eyepiece working on the same principle as that employed in ordinary nebular photography.

Director Keeler had thought that by placing the prism approximately at minimum deviation for the ray coinciding with the collimation axis, the confusion in the image due to the very large angle of the cone of incidental light, would be so slight as not to interfere with the qualitative purposes of the instrument; especially since the dispersion would be small—only 3 mm from λ 350 to λ 500. It should be said, however, that he had doubts as to the success of this plan, but felt that it would be well worth a trial.

The spectrograph was constructed by the Observatory instrument-maker. It was completed on the day Professor Keeler left Mount Hamilton for the last time, about a fortnight before his death.

A few weeks thereafter Dr. Campbell and I tried the spectrograph, and found that the beam of light could not be brought to a focus. On account of the approaching opposition of *Eros*, and the plans for completing, if possible, Professor Keeler's program of photographing the brighter nebulæ during the following spring, the instrument was set aside until April 1901.

Director Campbell then asked me to design such changes in the instrument as would permit the insertion of a double concave quartz lens immediately in front of the prism, to render parallel the rays incident upon the prism, and of a double convex similar lens immediately behind the prism to receive the refracted rays and form the image on the sensitive plate. The lenses were constructed by the John A. Brashear Co., from constants supplied by Professor Wadsworth. These additions to the optical train necessitated several changes in the support of the plate, in the guiding fixtures, etc. The alterations were completed by the instrument-maker on June 3, and to me was assigned the duty of testing its capabilities.

DESCRIPTION OF THE APPARATUS.

The Crossley Reflector has been described in detail by Professor Keeler.[1] Nothing need be said about it here, except that, for the sake

[1] ASTROPHYSICAL JOURNAL, 11, 325, 1900.

of completeness, its focal length is 534 cm and its clear aperture 92 cm, affording a ratio of aperture to length of 1 : 5.8.

The general features of the spectrograph are shown in Fig. 1. The tube *TT* fits into a similar tube on the side of the telescope, which holds it in position just as the plate-holder and guiding apparatus for ordinary photography are held in place. The latter tube can be moved in or out by rack and pinion to focus the plate. Since it would be

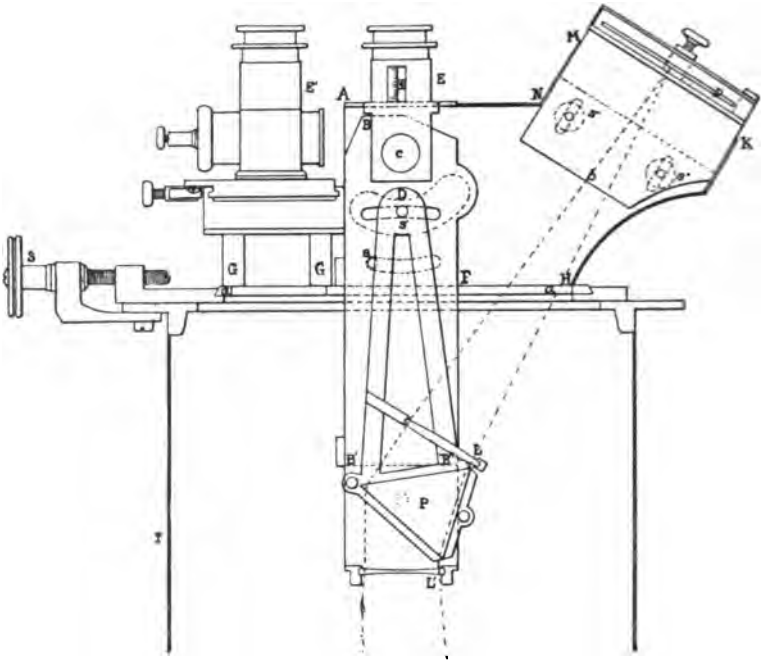


FIG. 1.

difficult to move the whole telescope to correct for any errors in following, the spectrograph is attached to the tube *TT* by two slides at right angles to each other, and the motions of these slides are controlled by two screws. In the drawing only the declination screw *S* is shown. The right-ascension screw would occupy a position in front of the middle of the apparatus, in the center of the drawing, and it is therefore omitted. The right-ascension screw has a coarse double thread to enable the observer quickly to correct the abormally large irregularities in the diurnal motion of the telescope, to which Professor Keeler's article refers at length. (The sources of much of this trouble

have been discovered and removed by Assistant Astronomer Perrine in the past year and a half, and the following in right-ascension is now much easier and better.) As the motion in declination is always small, the declination screw has a fine thread.

The base-plate *aa*, movable east and west between guides by means of the right-ascension screw, carries both the guiding eyepiece E' GG and the supporting framework AB' B'' $FHKMN$ of the spectrograph proper. This framework is built up of sheet brass, and is about 42 mm thick. It directly supports the plate-holder attachment bp, the finding and focusing eyepiece E, and the prism-box BL'; and the prism box in turn supports the minimum deviation apparatus DP.

The converging beam of light from the telescope first passes through the concave lens L', which converts it into a parallel beam; then through the quartz prism P; and finally through the convex quartz lens L, which brings it to a focus on the plate p. The prism and convex lens are attached directly to the minimum deviation apparatus. This consists of the framework extending from D down to and around the prism, turning about an axis P near the center of the prism. It is held in any desired position by the set screw S' which works in a slot in the prism-box. A much larger slot in the spectrograph case allows the screw to be manipulated when the apparatus is in place on the telescope. The convex lens should not, theoretically, be attached to the minimum deviation apparatus, but it was the only support available for it unless extensive changes were made in the design. However, it was set with the prism in an approximately correct position, so that its axis of collimation is nearly perfect.

The concave lens L' is attached directly to the prism-box. The plate-holder attachment bp can be moved toward or away from the convex lens in order to bring the plate into focus. It is held in any position by four set screws s'', two on either side. These screws work in slots in the spectrograph case. The prism-box moves about the tube C as an axis, so that the lenses and prism can be moved far enough to the right to allow the light from a star to pass without obstruction into the focusing eyepiece E. It is held in position by the set screw S, working in a slot in the spectrograph case.

The eyepiece E serves to identify the object under investigation, to set the telescope so that the spectrum will fall on the center of the plate, and finally to focus the spectrograph. The eyepiece can be moved in or out, and its position be read off on the accompanying

small scale. It is held at any desired reading by a set screw (not shown in the drawing). It is so adjusted that when the spectrum has by trial been focused on the plate, the *object itself* will be in focus in the eyepiece if the prism-box be moved aside. The cross-wires in the eyepiece are illuminated by a small electric lamp placed outside the spectrograph case. It shines through the tube *C*, about which the prism-box turns.

The guiding eyepiece E' is connected with the staging through two slides working at right angles to each other, similar to the two main slides carrying the entire spectrograph. When the star under observation has been brought to the intersection of the cross-wires in the focusing eyepiece E, the eyepiece E' can be moved over an area $15' \times 45'$ in search of a suitable guiding star. When such star is found, the eyepiece is clamped to its staging, with the star on the intersection of the cross-wires. Since this staging is rigidly attached to the plate *aa* carrying the spectrograph, the spectrum will remain in correct position so long as the guiding star is kept on the cross-wires. The lamp for the guiding eyepiece is shown just below E'. The electric light used for illuminating the wires in both eyepieces is first passed through red glass, to guard against fogging the sensitive plate.

ADJUSTMENTS AND METHOD OF OBSERVATION.

The first adjustment consists in bringing the plate to the focus of the convex lens, and this is done before prism and concave lens are mounted. A positive eyepiece is fitted in the place for the plate-holder, and is so adjusted that its focus will be in the plane of the plate. The instrument is then turned to a distant object and focused by shifting the plate-box into position, whereupon the box is clamped. In this adjustment it is necessary to use blue glass between the eye and eyepiece, as the convex lens by itself is not achromatic. When the adjustment is made in this manner, the blue light is the only light which will pass through the prism in a parallel beam, but the angles of the cones of the other colors will be too small to do harm.

After the prism and concave lens have been inserted, the focusing eyepiece and minimum deviation apparatus are adjusted. For this purpose the telescope is turned to a bright star; and using the eyepiece employed in focusing the convex lens, the prism is set at minimum deviation by shifting the deviation apparatus in the usual manner, and clamping it in the correct position.

The eyepiece E is focused best by a modification of Hartmann's

method. In this case, a rectangular diaphragm is placed over the center of the concave lens with its longer dimension at right angles to the edges of the prism. When the instrument is out of focus we shall see two parallel lines, each the continuous spectrum of the star. The instrument is now focused approximately by means of the eyepiece, after which the prism is turned to the side and the eyepiece E is focused. The reading of the scale on this eyepiece gives approximately the correct position of the eyepiece. To find the accurate position of the focus, we use Hartmann's method of taking two photographs of the spectrum, one considerably inside focus, and the other considerably outside, measuring the distance between each pair of images, and from these data interpolating the true position. In this case we set the focusing eyepiece to read, first, one or two millimeters less than the approximately correct reading, and then one or two millimeters more than its correct reading. After each setting the whole spectrograph is moved until the star is in focus in this eyepiece, then the prism is turned into position and a short exposure is made. During the exposure it is necessary to guide very carefully, as any error in guiding tends to widen the two spectra. After determining the correct focus from the two plates thus obtained, the eyepiece is set in the computed position and clamped. The spectrum then and thereafter is focused merely by moving the spectrograph in or out until the object is in the focus of the eyepiece.

When the object to be photographed has been brought to the center of the field of view, the instrument is focused and a guiding star found. This should be of about the eighth magnitude, though a fainter one can be used if necessary. It is unavoidably situated so far from the axis of collimation that its image is always very much distorted. The shape of the image can be altered by moving the guiding eyepiece in or out. For some reason, the focus of the telescope changes during a long exposure, and by noting any alteration in the form of the star image a change in the focus can be detected. By using a form of image which is sensitive to slight changes, we have a very accurate method of adjusting the focus in the progress of the exposure. After adjusting the guiding eyepiece, the optical train is turned into position and the exposure begun.

In observing some of the stellar nebulæ whose positions are not accurately assigned, difficulty is often encountered in identifying the objects. If they are brighter than the eleventh magnitude, they can usually be identified by their green color; but when they are fainter

than this, recourse must be had to their spectra. Nearly all of their visual rays are concentrated in the two green nebular lines, which are so close together in this instrument that they cannot be resolved. The result is that their spectra appear simply as dots, while a star with its continuous spectra appears as a line.

The length of exposure depends entirely on the brightness of the lines. Usually the extreme ultra-violet lines require much longer exposure than the others, though there are a few cases in which they record themselves as quickly as some of the lines in the lower part of the hydrogen series of the same spectra. The faintest continuous spectrum photographed was that of a fifteenth-magnitude star, with an exposure of four hours. The spectrum was not intense, but it was abundantly strong for purposes of interpretation.

As there is no slit, sky light and moon light exert their full effects on the plate, and it is therefore not possible to photograph a faint object on a moonlight night. Several photographs of fairly bright objects have, however, been obtained within three or four days of full moon, without serious injury to the spectrum.

The plates were measured on a Zeiss comparator. With this it is possible to measure to $0.1\,\mu$; but as only rough determinations of wave-lengths could be obtained with such a small dispersion, they were measured to $1\,\mu$ only. Even this is accurate enough to determine the wave-lengths by computation to one Ångström unit, but the probable errors were so great that the results were taken merely to the nearest $\mu\mu$. To determine the wave-lengths of unknown lines, a dispersion curve was first constructed. This curve was based upon the measure of known lines on several plates, after combining them by the method of least squares, using the Cornu-Hartmann formula. It was found that the least square determination was unnecessary, as the corrections which it gave were practically zero. This curve was plotted on such a scale that the measures could be laid off to 0.01 mm, and the wave-lengths read at a glance to $1\,\mu\mu$.

In working with a spectrograph of this kind, where no comparison spectrum is possible, all the measures must refer to some known line or lines. The interpretation of unique and unknown types of spectra would have to depend upon a comparison of the positions of the object and surrounding stars with the relative positions of the unknown spectral features and surrounding spectra on the same plate. Fortunately, in the spectra photographed thus far with this instrument, many features could always be identified; and I have in every case used the bright $H\gamma$ line as the point of reference and departure.

OBSERVATIONS.

As soon as the spectrograph was completed and adjusted, photographs were taken of the spectra of various objects to find out for what work an instrument of this kind is best adapted; and to determine its limitations. It is evident that, because of the small dispersion, it cannot as a rule be used to advantage on dark-line spectra, but some stars have been photographed with dark bands broad enough to show. This limits it very largely to bright-line spectra, though it has great value for determining the nature of very faint spectra of all kinds. Moreover, since there is no slit, the object must not be so large that the various monochromatic images will overlap too much, and this condition confines the field to objects no larger than planetary nebulæ.

Large numbers of objects having various spectral types promised to yield valuable results; but, on account of the limited time at my command, my program was confined, with the director's approval, to comparatively few objects. These included:

a) Such planetary nebulæ as were in observing position, both for purposes of comparison with results reached by observers using other instruments, and in the hope of discovering new features in the ultra-violet region. It was not expected that results already obtained in the $H\epsilon$–$H\beta$ region of bright objects, by observers using large telescopes and powerful spectroscopes, would be surpassed or even equaled, on account of the feeble dispersion of this instrument.

b) Such new stars as were in observing position.

c) The central star in the Ring Nebula in *Lyra*, and other similar stars and stellar nuclei.

d) A few Wolf-Rayet stars.

After the most of my observations were made, Professor Pickering suggested that the long-period variables be observed, to determine just when the bright hydrogen lines disappear; but lack of time prevented me from doing so.

An interesting series of observations somewhat similar to mine was made by von Gothard in 1892 (*Astronomy and Astro-Physics*, 1893, p. 51). His program included seven of the brightest planetary nebulæ, the Ring Nebula in *Lyra*, and *Nova Aurigae* after its reappearance. His telescope was a small reflector, the scale of his photographs was about the same as mine, and his results for the brighter nebulæ in the region $H\eta$–$\lambda 5007$ agree well with mine. Some of his nebulæ, however, were too large for the Crossley instrument.

Altogether, twenty-one nebular spectra were photographed by me between June and September 1901. After my departure, Mr. Joel

Stebbins, Fellow in the Lick Observatory, kindly secured for me with the same instrument some very good spectrograms of *Nova Aurigae*, *Nova Persei*, and a Wolf-Rayet star.

The general character of the different spectra is shown in Fig. 2. The scale of the original negatives was too small to permit photographic enlargements to a convenient size to be made, and they have been enlarged by drawing them to a scale of one inch to the millimeter. In the drawing, the edges of the disks appear much sharper than on the original negative.

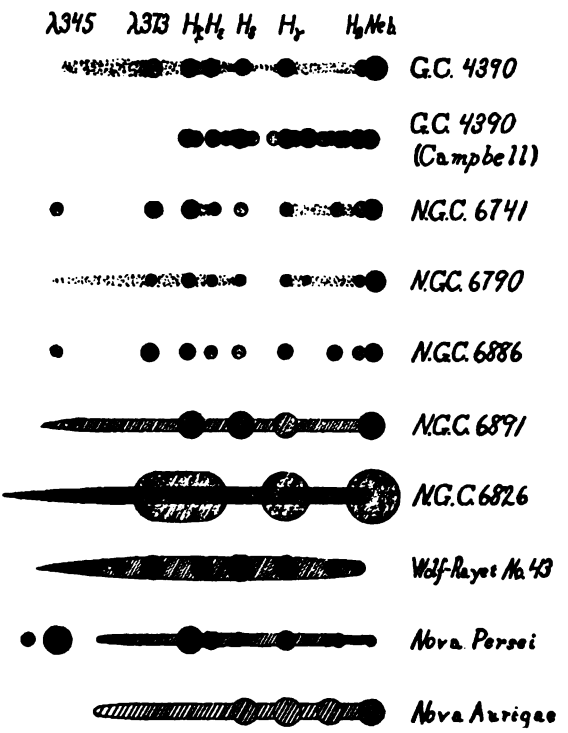


FIG. 2.

It will be noticed that in general the intensity of the continuous spectrum of the nebulæ has a minimum between $H\delta$ and $H\gamma$. This fact is confirmed by nearly all the negatives, and is too general a characteristic to be passed without notice. It seemed hardly possible that the spectrum of the nucleus, to which it was supposed that this belonged, should have an absorption band at this place, and another explanation was sought. This was found by plotting Dr. Campbell's observations of *G.C.* 4390 on the same scale, and comparing the result with the spectra obtained with the slitless spectrograph. His observations were made with a large spectroscope attached to the 36-inch refractor. A glance at the spectrum marked *G.C.* 4390 (Campbell) will show that he found the bright lines much fewer in the region where the continuous spectrum is weak in the spectrograms taken with the small instrument. His observations of *N.G.C.* 7027 give the same results. From this it seems safe to infer

that the apparently greater relative strength of my spectrograms in the $H\gamma$–$H\beta$ region is largely due to the presence of many faint overlapping bright lines, whereas, in the region $H\delta$–$H\gamma$ fewer bright lines exist. A further support of this theory is afforded by nebulæ *N.G.C.* 6826, and *N.G.C.* 6891, each of which has a stellar nucleus. In each case the spectrum of the nucleus has an almost uniform intensity throughout its entire extent.

A simple inspection of Fig. 2 will show one of the marked advantages of the reflector and quartz prism and lenses over the large refractor and glass optical train, in spectrum observations. Even in the favorable nebula *G.C.* 4390, Campbell's results stop at $H\zeta$, whereas the slitless spectrograph record extends to about λ 337.

NEBULA *N.G.C.* 6210.

This is described as very bright in Dreyer's *New General Catalogue*. Three photographs were taken with exposures of 1 minute, 5 minutes, and 30 minutes, respectively. The last is the best, the others being underexposed. The images are rather large, as the nebula is not stellar, but most of them are pretty distinct. The continuous spectrum seems to be of the kind made up largely of overlapping bright lines, with its maximum intensity in the neighborhood of $H\delta$. The following bright lines were found: principal nebular ($=\lambda$ 501 $+\lambda$ 496), $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, λ 386 and λ 373.

N.G.C. 6439.

This is a stellar nebula rated at thirteenth magnitude in the *N.G.C.* Satisfactory plates could not be obtained because there is a pretty bright star about 2′ due north of it, and the nebular spectrum appears to overlap the stellar spectrum. Three plates were made, the first with 1 hour and the other two with 2 hours' exposure. The principal nebular lines, $H\beta$, $H\gamma$, and $H\zeta$, are shown, though the last two are faint.

N.G.C. 6537.

This is a stellar nebula, and the images are small. Two plates were taken, with exposures of 1 and 1¾ hours, respectively, but both are underexposed. The latter shows the following lines: nebular, $H\beta$, λ 466, $H\gamma$, $H\delta$, $H\zeta$. There is no continuous spectrum recorded.

N.G.C. 6543.

This is a planetary nebula, marked "v B, p S" in the *N.G.C.* It has a diameter of about half a minute of arc, which is as large as can be photographed to advantage with this instrument. Two photo-

graphs of it were taken, with exposures of 15 minutes and 40 minutes, respectively. The former is the better of the two. The lines are very little brighter than the continuous spectrum; and being large they could not be measured accurately. Those identified were: nebular, + $H\beta$, $H\gamma$, and $\lambda 373$. Between $H\gamma$ and $H\beta$ the continuous spectrum is narrow, whereas beyond $H\gamma$ it is almost as wide as the nebula, as though above $H\gamma$ it were composed of overlapping bright lines. On a nebula as large as this one, it would take very few lines to cause such an appearance, the hydrogen series alone being practically sufficient. This continuous spectrum also extends above $\lambda 373$ to $\lambda 336$ as bright as it is below $\lambda 373$. It would seem that there must be other lines present in that region.

N.G.C. 6567.

This is a stellar nebula of the eleventh magnitude. But one photograph was obtained, with an exposure of 1 hour 30 minutes. The images are quite small, though not small enough to show the $H\beta$ line separated from the nebular lines. It contains the following lines: nebula, + $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, and $\lambda 373$.

The continuous spectrum recorded is of the type described above as probably due largely to overlapping bright lines. Between $H\gamma$ and $H\delta$ it is faint, and between $H\zeta$ and $\lambda 373$ it is almost invisible, but everywhere else it is bright.

G.C. 4390.

The spectrum of this nebula is shown in Fig. 2, as well as the spectrum obtained by Dr. Campbell, reduced to the scale of these spectrograms. The nebula is nearly stellar, described as "v B." Two photographs were obtained, with exposures of 10 and 15 minutes, respectively. The former is the better, although both are good.

N.G.C. 6741.

This spectrum is also shown in Fig. 2. It is a stellar nebula, but an exposure of three hours was able to bring out a continuous spectrum only from $H\beta$ to $H\gamma$ and from $H\epsilon$ to $H\zeta$. Another exposure of one hour failed to show any continuous spectrum. A new nebular line is strongly suspected at $\lambda 345$, but it is not absolutely certain.

N.G.C. 6790.

This is a stellar nebula of the 9.5 magnitude; and as it has been observed by Campbell with a larger spectroscope, little need be said about it here. Three photographs of 20 minutes', 1 hour's, and 3 hours'

exposure, respectively, were obtained. The first two show no continuous spectrum, but the third records one with a gap between $H\gamma$ and and $H\delta$. This spectrum is illustrated in Fig. 2. The continuous spectrum above $\lambda 373$ seems to be fluted, with condensation at $\lambda\lambda 363$, 356, and 352, but they are very faint and their existence is doubtful.

N.G.C. 6803.

This is a stellar nebula showing only a very faint continuous spectrum. Two photographs were obtained, one with an exposure of 1 hour, and the other with 2 hours 30 minutes. The latter shows just the faintest trace of a continuous spectrum from $H\beta$ to $H\gamma$, and from $H\delta$ to a little way beyond $\lambda 373$. The bright lines which were identified are: nebular, $H\beta$, $\lambda 468$, $\lambda 446$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, $\lambda 373$, and perhaps one at $\lambda 365$, though the existence of the last is very doubtful.

N.G.C. 6807:

This is a stellar nebula without the slightest trace of a continuous spectrum. Three photographs were obtained, with exposures of 45 minutes, 57 minutes, and 3 hours, respectively. They contain: nebular, $H\beta$, $H\gamma$ $H\delta$, $H\epsilon$, and $H\zeta$ lines.

N.G.C. 6818.

This is a nebula described in the *N.G.C.* as "B, v S," but is nevertheless almost too large for this kind of work. Two photographs were obtained, with exposures of 10 minutes and 32 minutes, respectively. The former is underexposed. The spectrum consists of a pretty strong continuous spectrum extending out to $\lambda 372$, with the following bright lines: nebular, $H\beta$, $H\gamma$, $H\delta$, $\lambda 375$ and $\lambda 372$.

N.G.C. 6826.

This is described as "B, p L" in the *N.G.C.* The nebula is rather large, and contains a bright stellar nucleus which gives a continuous spectrum. A drawing of the spectrum is given in Fig. 2. Two photographs were obtained, with exposures of 15 minutes and 1 hour, respectively. The former is the better, though in both the spectrum of the nucleus is overexposed. The lines are probably: nebular, $H\beta$, $H\gamma$, $H\zeta$, and $\lambda 373$. The continuous spectrum extends to $\lambda 333$.

N.G.C. 6833.

This is one of the finest stellar nebulæ. It is the only case in which $H\beta$ was suspected visually, the image seen in this case being somewhat elongated. On the plate it is distinctly separated from the nebular

lines. But one plate was taken, with an exposure of 1 hour 55 minutes. It contains the following lines: nebular, $H\beta$, $\lambda 446$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, and $\lambda 373$. The continuous spectrum extends to $\lambda 348$, with a slight minimum of intensity between $H\gamma$ and $H\epsilon$. The variation in the brightness of the continuous spectrum is less noticeable in this case than in most of the others.

N.G.C. 6881.

This is a stellar nebula, with spectrum quite different from most of the others. Two photographs were taken, with exposures of 2 hours and 3 hours, respectively, and they show only: nebular, $+ H\beta$, $\lambda 468$, $H\gamma$, $\lambda\lambda 422$ and 388. There is a continuous spectrum from about $\lambda 450$ to $\lambda 360$, which seems to be fluted between the $\lambda 422$ and $\lambda 388$ lines. Still, the spectrum is too faint to allow us to decide. An exposure of at least four hours would be necessary to make sure of this point, and opportunity to secure it did not offer.

N.G.C. 6884.

This is a typical stellar nebula. The one photograph secured with exposure of 1 hour shows the following lines: nebular, $H\beta$, $\lambda 467$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, and $\lambda 376$. There is a continuous spectrum from $H\beta$ to $H\gamma$, and from $H\epsilon$ to $\lambda 348$.

N.G.C. 6886.

This is a fine stellar nebula. But one photograph was obtained, with an exposure of 1 hour. It is an excellent one, and shows the images round and small. A drawing of it is given in Fig. 2. This was the first nebula in which the new line $\lambda 345$ was noticed. There is no trace of a continuous spectrum.

N.G.C. 6891.

This is a planetary nebula with a stellar nucleus. One photograph was taken with an exposure of 30 minutes. A drawing of it is given in Fig. 2.

N.G.C. 6905.

This is described as "!!, B, p S" in the *N.G.C.* It is simply a patch of nebulous matter, shading off gradually toward the edges. One photograph was taken with an exposure of 1 hour, which gives only one image of the nebula, due, probably, to the principal nebular lines.

N.G.C. 7009.

This is described as "!!!, v B" in the *N.G.C.* For so small a nebula it shows a great deal of structure. Three photographs were taken, with exposures of 5 minutes, 10 minutes, and 20 minutes, respectively. They record the nebular, $H\beta$, $H\gamma$, $H\delta$, and $H\zeta$ lines, as well as a continuous spectrum belonging to the center of the nebula. This continuous spectrum extends out to $\lambda 345$. At the $H\delta$ line it widens out to the width of the nebula, as though from there out to the ultra-violet the whole nebula gives out faint continuous light, or that there are several overlapping faint lines.

N.G.C. 7027.

This is the brightest of the stellar nebulæ photographed, being given as 8.5 magnitude in the *N.G.C.* Two photographs were taken, with exposures of 15 minutes and 30 minutes, respectively. The latter is the better of the two, although the nebular lines are so overexposed as completely to cover $H\beta$. In this nebula the $\lambda 345$ line is as bright as $H\delta$. There seems to be a very faint line at $\lambda 337$. It is very faint, but its existence is scarcely doubtful. The continuous spectrum has a minimum between $H\gamma$ and $H\epsilon$, but it does not extend quite so far as the $\lambda 345$ line.

N.G.C. 7354.

This is described as "B S" in the *N.G.C.* One photograph was taken with an exposure of 2 hours, which shows simply a single fairly bright image of the nebula, due to the nebular lines, and an exceedingly faint continuous spectrum.

NOVA PERSEI.

Several plates of *Nova Persei* were taken from September 1901, to March 1902, but the best plate was secured for me by Mr. Stebbins on October 14, with an exposure of 30 seconds. A drawing of this is given in Fig. 2. A note on this spectrum has been published by Mr. Stebbins in *Lick Observatory Bulletin* No. 8. In general the spectrum is like that of the stellar nebulæ, but resembling that of *N.G.C.* 7027 most strongly. The main difference between them is that in *Nova Persei* the brightest lines are in the ultra-violet, the green nebular lines being faint. The line suspected in *N.G.C.* 7027 at $\lambda 337$ is present here, to add to the similarity; and the line at $\lambda 345$ discovered by me in *N.G.C.* 6741 and *N.G.C.* 6886 is an exceedingly strong line in *Nova Persei*. The continuous spectrum also ends at about the same place in *N.G.C.* 7027 and the *Nova*.

NOVA AURIGAE.

Three photographs of this star were taken for my use, at the director's request, by Mr. Stebbins on September 12 and 13, 1901, with exposures of 30 minutes, 1 hour 30 minutes, and 3 hours 20 minutes, respectively. The last is the best, and is shown in Fig. 2. The visual magnitude of the star was about 13. Von Gothard[1] observed this with his objective prism on September 15, 1892, when its magnitude was 10.5. His observations are the best to compare with mine, because his dispersion was almost the same as mine. He found seven lines in this part of the spectrum, and a rather faint continuous spectrum, with a maximum brightness in the neighborhood of $H\epsilon$. Our plate made nine years later shows four lines on a nearly uniform background of continuous spectrum almost as bright as the lines themselves. Plainly the bright lines are growing relatively weaker.

NOVA CYGNI 1876.

One photograph was obtained on August 12, 1901, with an exposure of four hours. This showed that the spectrum has become continuous. At that time the *Nova* was estimated at about the fifteenth magnitude, yet the spectrum is very distinct. The plate records spectra of several stars as faint as fifteenth visual magnitude. The last published observation of this star was by Copeland and Lohse (*Copernicus*, II, 105) in 1881. At that time they surmised that the spectrum had become continuous, because they could see nothing of it, whereas, had it been a bright-line spectrum, they thought it should have been visible.

WOLF-RAYET, NO. 43.

$\alpha = 19^h\ 30.9^m$, $\delta = +30^\circ\ 19'$.

Mr. Stebbins took two photographs of this star, with exposures of 15 minutes and 60 minutes, respectively. The spectrum is shown in Fig. 2. The most noticeable difference between this and the nebulæ is the absence of the nebular lines. The continuous spectrum is nearly uniform.

RING NEBULA IN *LYRA*.

As the central star in this nebula was the principal object in mind when Professor Keeler designed the spectrograph, it was one of the first objects observed after the instrument was completed and adjusted. Three plates were taken of it, two with exposures of 30 minutes, and one with 2 hours; but the results obtained for the star were mostly

[1] *Publications A. S. P.*, 1893, 152.

negative. The rings are so large that they overlap, and only the two brightest images can be distinguished—those due to the nebular lines $+H\beta$, and to $\lambda 373$. The hydrogen lines are probably present, as the whole region between these images is darkened, probably by the overlapping images. The peculiar thing about it is that the $\lambda 373$ line is so much brighter than all of the others. Von Gothard also noticed this fact. His instrument was better adapted to this object than the Crossley Reflector is, because, having a shorter focal length, the rings were smaller; but having the same dispersion, their centers were just as far apart and consequently did not overlap so much. The only trace of the central star on these photographs is an extremely faint line, present on all three, extending from the violet edge of the green nebular ring to the red edge of the $\lambda 373$ ring. This line is too faint to be seen, except with the unaided eye. The two-hour exposure shows a dot near the middle of the $\lambda 373$ ring, but if this is due to the central star the image is a little more refrangible than $\lambda 373$. This dot is also present in the 30-minute exposure, though it is so faint that its presence would not be suspected had it not been first noticed in the long exposure.

From the fact that the spectrum of the central star is not conspicuous on these plates, it seems probable that it is almost wholly continuous, with perhaps a bright line near $\lambda 373$. The photographs taken by Professor Keeler showed that photographically the star is brighter than the nebula; and the theory that its light is spread out into a band, while the light of the nebula is concentrated in a few bright lines, seems to be the only one which explains the observed phenomena.

CONCLUSIONS.

One of the principal purposes of this investigation was to determine the efficiency of the slitless quartz spectrograph attached to the Crossley Reflector, for the photography of very faint spectra. Its power in this respect is surprisingly great. It is well illustrated in the case of *Nova Cygni*. The visual magnitude of this star was estimated by Professor Barnard in 1901 to be 15.6. My four-hour exposure, with fair following and focus, recorded its continuous spectrum in good strength, well up into the ultra-violet. The character of the spectrum could have been determined had the image been considerably less intense. This spectrum may be relatively stronger in the photographic regions than ordinary stellar spectra, though the spectra of many adjacent fifteenth-magnitude stars are recorded satisfactorily

on the same plate. It would be perfectly practicable to carry the exposures up to ten hours or more; the guiding could be more accurate, now that the principal sources of the irregularity in diurnal motion have been removed; and perhaps the more stable new telescope mounting will eliminate the present unfortunate changes in the focus during exposures. Considering these facts, I am reasonably confident that this instrument can record the continuous spectrum of the faintest star visible in the 36-inch telescope under the corresponding atmospheric conditions. Further, in the case of stellar nebulæ, etc., whose spectra contain well defined bright lines, I do not doubt that the instrument will record the principal bright lines of objects too faint to be seen in our most powerful telescope.

It is only for work on very faint spectra that this spectrograph is efficient. Its dispersive power is very low, and it should seldom be used on spectra, or on those portions of spectra, bright enough to be recorded by relatively longer exposures with instruments giving greater dispersion.

For qualitative studies of general spectral features this spectrograph is efficient on spectra of small objects, whether continuous or bright line; but for quantitative results its usefulness is limited largely to small and faint bright-line spectra.

For most of the objects observed a 60° prism, and for some of the objects two 60° prisms, would have been an improvement over the present 50° prism; requiring, of course, the reconstruction of the instrument.

For some of the stellar bright-line objects a convex lens of twice the present lens's focal length would have had an advantage in separating close lines on the photographic plate, permitting at the same time more accurate determinations of wave-lengths. This would probably alter the character of the field of the combination somewhat.

It is possible that a similar instrument on twice the linear scale, *i. e.*, with lenses and prism having 50 mm apertures—with perhaps a 60° prism—would be efficient and practicable for the objects of medium brightness on my list.

New nebular lines at λ 337 and λ 345 were discovered. These lines were later found to exist in the spectrum of *Nova Persei*, from the plate taken October 14, 1901. The nebular character of the *Nova* spectrum is thus established for the extreme ultra-violet region, as had previously been done for the other regions.

The spectrum of *Nova Cygni*, which in 1876 was of the usual "new star" type, and later changed into the nebular type, has now become continuous, with no signs of bright lines.

The bright lines in the nebular spectrum of *Nova Aurigae* are now relatively faint, and the spectrum appears to be approaching the continuous type.

These complete and astonishingly rapid changes of spectral types observed in the cases of *Nova Cygni* and *Nova Aurigae*—and likewise those observed in *Nova Normae*, *Nova Sagittarii*, *Nova Persei*, etc.—leave little doubt that the masses of these objects are small.

All the new stars of recent years should be re-observed with this spectrograph every year, to keep account of the progressive changes in their spectra. The importance of carrying out this plan can scarcely be overestimated.

The relative intensities of nebular and other bright lines, as well as of continuous spectra, are shown more accurately on the plates secured with this spectrograph than on those made with instruments absorbing the blue and violet very strongly.

The spectrum of the central star in the Ring Nebula in *Lyra* seems to be not of an unusual type, unless perhaps it contains a bright line near $\lambda 373$. It is quite possible that the star is somewhat brighter photographically than visually; at least, photographic evidence is not opposed to this theory.

The great intensity of the $\lambda 373$ ring in the Ring Nebula perhaps explains one point in nebular photography. Professor Keeler found that for most nebulæ an exposure of three hours gave the best results, whereas the best photograph of the Ring Nebula was obtained in ten minutes. While the Ring Nebula is brighter than the average bright nebula, this brief exposure of ten minutes is out of proportion to its visual brightness; but its radiations being mostly in the ultra-violet, it should have a somewhat greater photographic than visual brightness.

I wish to thank Mr. Joel Stebbins, Fellow in the Lick Observatory, for assistance in taking some of the photographs, and for making several exposures during my absence from the Observatory; and Dr. Campbell for his many suggestions during the course of the work.

HAROLD KING PALMER.

JUNE 1, 1902.

ERRATUM.

Plate III, in the September number of this JOURNAL was wrongly printed. It should be removed, and the one sent here with inserted in its place.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

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THE

ASTROPHYSICAL JOURNAL

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AND ASTRONOMICAL PHYSICS

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SPECTROGRAPHIC OBSERVATIONS OF STANDARD VELOCITY STARS (1902–1903).

By EDWIN B. FROST and WALTER S. ADAMS.

DURING the past year one part of the program of work with the Bruce spectrograph of the Yerkes Observatory has been the observation of the list of stars adopted for co-operative observation by those engaged in line-of-sight work with the largest spectrographs now in use. The scheme[1] of co-operation that seemed to represent the views of those interested called for three spectrograms per year of each of the ten stars (all having spectra of the solar type), taken if possible at about the date of the star's opposition and thirty days before and after opposition. It was recommended that additional observations should be made of *α Boötis* and *α Persei* throughout the year. The choice of comparison spectrum and mode of reduction was to be left wholly to the different observers, variety being considered desirable. A style of publication was suggested, uniform as regards the minimum amount of data to be communicated.

We have endeavored to carry out the plan of co-operation as

[1] "Co-operation in Observing Radial Velocities of Selected Stars," ASTROPHYSICAL JOURNAL, 16, 169–177, October 1902.

closely as possible since July 1902, but the weather conditions on the two nights weekly allotted to the spectrograph have not permitted us to obtain the plates as nearly on the schedule dates as we desired. In fact, we were able to obtain but a single plate of *β Leporis* during the past winter. The list of stars, with the approximate dates at which they were to be observed, if possible, is as follows:

α Arietis,	Oct. 1, Nov. 1, Dec. 1.
α Persei,	Oct. 23, Nov. 23, Dec. 23.
β Leporis,	Nov. 10, Dec. 10, Jan. 10.
β Geminorum,	Dec. 12, Jan. 12, Feb. 12.
α Crateris,	Feb. 14, March 14, April 14.
α Boötis,	March 13, April 13, May 13.
β Ophiuchi,	May 15, June 15, July 15.
γ Aquilae,	June 22, July 22, Aug. 22.
ε Pegasi,	July 24, Aug. 24, Sept. 24.
γ Piscium,	Aug. 13, Sept. 13, Oct. 13.

With the exception above noted, we have obtained three measurable plates of each of the above stars, and several extra plates of *α Boötis*, but unfortunately no extra plates of *α Persei*. We have also secured a few plates of the supplementary stars proposed by M. Bélopolsky as substitutes for the southern stars of the above list, which would be beyond his reach. Control plates of the Moon and planets have also been taken at intervals to test the performance of the spectrograph. The results for these plates are given below for the period covered by the plates of standard stars.

A detailed description of the Bruce spectrograph, with illustrations, has been given in this JOURNAL,[1] and need not be repeated here. The principal changes which have been made since that article was written have been: (1) the use of the lens instead of the mirror for projecting on the slit the image of the spark which furnishes the comparison spectrum; (2) the use of a small disk of ground glass at a distance of 20 mm in front of the slit, for diffusing the light of the spark, and insuring the complete and uniform illumination of the collimator by the rays from the spark; (3) a Hastings isokumatic collimator lens has

[1] 15, 1–27, January 1902.

been substituted for the triple of the same size (51 mm) and focal length (958 mm) at first used, resulting in some slight improvement in the previously excellent performance of the collimator.

The camera lenses which have been employed in the work included in this paper are designated as A, B, and B′. A is a Zeiss anastigmat of the "Protar" type, of 71 mm aperture and 449 mm focus, purchased of Bausch and Lomb; B is a triple lens, designed by Professor Hastings and constructed by Brashear, of 76 mm aperture and 607 mm focus; and B′, a triple from the same source as B, is of 57 mm aperture and of the same focus as B.

As the scale of the plates taken with camera B is one-third larger than for A plates, and as their definition is somewhat better at the center, we have naturally preferred to use camera B, which practically requires the same exposure time as A. B, however, has at times shown a disposition to become astigmatic, probably due to pressure on its cemented surfaces within its cell, and A has always been used when the trial plates suggested any lack of sharpness in the lines given by B. Hence A has been employed for about one-third of the plates concerned in this paper. Despite its smaller scale, camera A performs quite as well, if not better than B, as inferred from the accordance of the velocities deduced from different lines on the same plate. (See the values of ϵ in the details of measurements.) Camera B′ is also a cemented system, but has 25 per cent. less aperture than B, and is also mounted in a spring cell, so that no untoward effects are to be feared from pressure.

The performance of the great refractor has been most satisfactory in work with the spectrograph. The addition of a long-focus finder, without tube, having a six-inch object-glass attached beside the cell of the forty-inch lens with an eyepiece near the spectrograph, enables the observer to bring a star very quickly upon the slit, regardless of the flexure of the great tube.

The temperature-case of the spectrograph has also operated in a very satisfactory manner, as will appear from an examination of the records of the temperature in the outer and inner cases given in the journal of observations.

Additional details regarding the instrument, together with a new photograph of it and its accessories, may be found in our paper on the "Radial Velocities of Twenty Stars having Spectra of the *Orion* Type" in the forthcoming Vol. II of the *Publications of the Yerkes Observatory*.

The dispersion and scale of the plates for cameras A and B are as follows:

Wave-Length	Angular Dispersion for One Tenth-Meter	Tenth-Meters per mm	
		Camera A	Camera B
4300	45.7	10.0	7.4
4500	33.8	13.5	10.1
4700	26.0	17.6	13.1

On well-exposed spectra of the solar type a sufficient number of good stellar lines is usually found within a range of about 130 tenth-meters (or 13 mm for camera B), with the center at about λ 4480, which is the ray passing the prisms at the angle of minimum deviation.

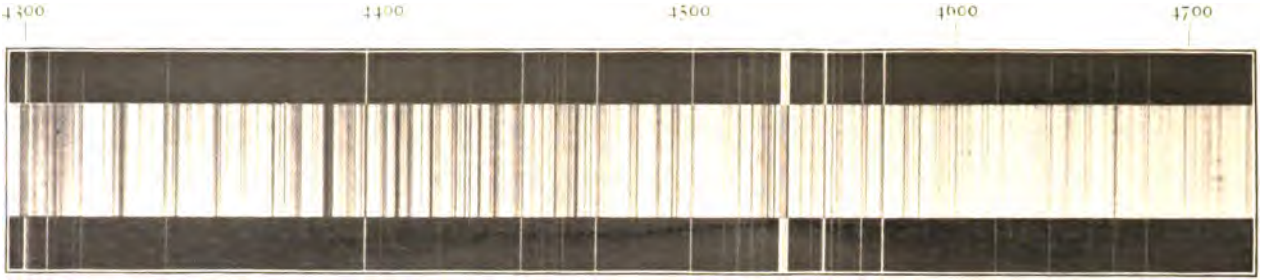
The titanium spark has been chiefly employed for the comparison spectrum on these plates, but electrodes of iron and chromium, and a helium tube can be readily rotated into the proper position for use when desired. Plate VIII, taken from the already cited article in the *Publications* of this Observatory, illustrates the distribution of the comparison lines of the *Ti* spectrum, and shows the quality of stellar spectrum obtainable. The use of a small self-induction coil obviates the presence of any air lines in the comparison spectrum, which, without the coil, are less pronounced with *Ti* than with most metallic electrodes.

MEASUREMENT OF THE PLATES.[1]

The plates have been measured with two screw-comparators constructed by Wm. Gaertner & Co., of Chicago. The screws have a pitch of 0.5 mm, and the head is large, so that the single

[1] An extended discussion of this topic, and of the various sources of errors in our determinations of radial velocities, is given in the volume of the *Publications* already alluded to, which has at this writing been in type for about eight months. For details the reader is referred to that paper.

PLATE VIII.



α Boötis, with Titanium Comparison Spectrum.
Enlarged vertically fortyfold, horizontally fourfold.

divisions (0.001 mm) can be easily read through a small magnifier and estimated to tenths. A distance of less than 20 mm is ordinarily measured on the spectra here in question—in fact, for A plates usually only about 10 mm, and for B plates about 13 mm. Four settings are made on each of the comparison and stellar lines which the observer desires to employ, and then the plate is reversed under the microscope (so that the violet side shall be toward the right instead of the left), and the process is repeated.

The measurements in the two positions are combined to means, and then reduced with the aid of the simple Hartmann formula, the constants of which are determined from three standard comparison lines selected near the middle and ends of the portion of spectrum measured. The computed wave-lengths of the various other comparison lines distributed along the plate, always selected so as to be as near as possible to the best star lines, serve to give successive checks upon the accuracy of the "fit" of the formula. The corrections to formula thus obtained are applied to the corresponding or intervening star lines, with such (linear) interpolation as may be necessary. We have feared the dangers of abitrariness in any process of smoothing out the accidental errors of settings by a curve, so that the accordance of the velocities deduced from different lines on the same plate is less than it might be otherwise; but the mean of the values probably may be regarded as quite as trustworthy, and is certainly free from errors due to prepossession in curve-drawing. From the values of the apparent wave-lengths of the stellar lines thus derived, the displacements, and hence the corresponding velocities, are immediately inferred. It is hardly necessary to remind the reader that the full amount of any error in the relative wave-length[1] of any line enters into our result, a relative error of 0.01 tenth-meter producing an error of 0.7 km per second in the velocity deduced for that line.[2] It will, however, be easy to correct our results later, as subsequent laboratory

[1] Taken from ROWLAND'S "Preliminary Table of Solar Spectrum Wave-Lengths."

[2] Unless the star line is identical with the comparison line, so that a direct displacement is measured.

observations shall gradually remove the relative errors in Rowland's tables. In our computations we carry the wave-lengths to the third decimal place, as given by Rowland, but of course we do not attach significance to measures on any given plate beyond the hundredth of a tenth-meter. Similarly, though we carry out our reductions to the hundredth of a kilometer, it is clear that the uncertainties of the wave-lengths, quite aside from the accidental errors of measurement, seriously affect the tenths of a kilometer.

Tests made by ourselves and by others indicate that the measurement of the plate in the two positions under the microscope largely eliminates the physiological error due to the difference in the nature of the process of setting a dark thread upon a white star line and the same dark thread upon a dark comparison line (on the negative). Our systematic errors of this sort would be quite large, about 4 km for Frost and 2 km for Adams, but with opposite signs.

Preliminary investigations of the periodic errors of the screws of the two comparators were made during the progress of this work, and errors were found, reaching a maximum of 4μ for one machine and of 3μ for the other. It was found that they changed when the nut was more tightly clamped to the screw, which was occasionally necessary, as the screws tapered slightly from both ends toward the center. Corrections have therefore not been applied for the errors of the screws in this year's work, and new screws are being supplied to replace the present ones. However, for reasons fully set forth in our article on the *Orion* stars, it does not appear to us probable that the radial velocity of any solar star deduced from measures of one plate will be affected by over 0.1 km on account of screw-errors. The general accordance in the measures of the same plate by the two observers with the two different machines is confirmatory of this opinion.

We desire to emphasize the independent character of the measurements of a plate by the two observers. The selection of the lines of both comparison and stellar spectrum was made by the measurer without reference to his own or the other observer's

measures of other plates of the same star, and no comparisons were made until all the reductions were completed. There has been no attempt to secure conformity in the habit of the two observers in making settings. Thus, for instance, A. is accustomed to make his settings on the inner tips of the comparison lines, while F. has followed the practice of setting upon a certain point estimated as one-third of the length of the comparison line from the inner tip. Of course, the corrections for curvature[1] have been different for these two distances from the center of the star spectrum, which distance is the argument in our table of curvature corrections. But if any serious error had entered in the habit of either observer in regard to the point at which the setting is made, it would appear as a persistent systematic difference in the results, which we cannot discover.

It is not to be doubted that the observers' mode of setting may change with time and experience, but we have not found any such changes, of magnitude sufficient to be regarded as real, during our work since the Bruce spectrograph has been in operation. The differences that occur in the results of the independent measurements of the same plate by F. and by A., tabulated below, are perhaps disappointingly large; and the deviations of the separate plates from the mean may be larger than would be anticipated by those who have not directly engaged in line-of-sight work; but they seem in any event to be of an accidental character. It is perhaps safe to say that at present, with a stable spectrograph, errors due to personality are quite as much to be feared as errors due to instrumental sources.

We have elsewhere[2] discussed the effect of our assumption of solar wave-lengths for spark lines. If arc and spark wave-length are absolutely identical, we might expect that a systematic correction to our results would be required to compensate for the displacement of lines due to pressure in the stellar atmosphere. There are not at present sufficient observational data available for making such corrections (to the solar wave-lengths),

[1] These are applied to the mean of the settings on each comparison line, and do not further appear in the reductions.

[2] *Loc. cit.*, p. 155.

and furthermore our control plates on the Moon and the planets do not reveal such a systematic difference from the computed velocities.

The excellent character of many lines in the stellar spectra, which are made up of two or more lines too close to one another to be separated with the dispersion employed, has necessitated the use of a number of blends. In all such cases we have followed the usual practice of assigning weights to the component lines according to the intensities given them in Rowland's table, and taking the weighted mean of their wave-lengths as the wave-length of the compound line. The maximum distance between two lines for which blending is admissible depends upon the practical separating power of the instrument. On lantern-slide plates of the solar spectrum the least distance at which two lines can be seen as distinct is about 0.15 tenth-meter. The less fine-grained plates used for stellar spectra do not admit of the resolution of such close lines, and with them the limit is usually from 0.25 to 0.30 tenth-meter, according to the quality of the plate and the character of the lines involved. In addition to the matter of separation, it is, of course, necessary to determine the intensity which a line must have in order to exert an appreciable influence upon a closely adjacent line. The answer to such a question is naturally somewhat difficult; an examination, however, of solar and planetary plates has led us to the conclusion that, upon a plate of quality suitable for good measurement, a line of intensity 0 on Rowland's scale is appreciable, while fainter lines may, in general, be neglected. A few exceptions are found to this rule, but these occur mainly in cases of uncommonly strong lines, when the influence of the faint component appears to be negligible.

The arbitrary character of the assumption of equal intensities for the same lines in the Sun, and in stars which are classed broadly under the solar type, finds its justification in the agreement of the values deduced from lines whose wave-lengths have been obtained under this assumption with those derived from uncompounded lines whose wave-lengths are subject to no uncertainty due to this cause; and also in the accord of the com-

pounded lines with the same lines in planetary and lunar spectra where the above method of blending is evidently rigorously correct. In the case of the stars discussed in this paper the wave-lengths of the blended lines, found in the way described above, have proved quite satisfactory for all except *a Persei*. The spectrum of this star, however, is of a less developed type than that of the other stars, its most striking characteristic being the prominence of the "enhanced" metallic lines. When such lines, accordingly, have formed the most important part in the composition of the blends used for the other stars, the wave-length of the enhanced line alone has in each case been used, and the results obtained have shown this to be unquestionably the correct procedure.

The table which follows contains the wave-lengths of all of the blended lines which we have employed two or more times in the course of our reductions, together with the elements to which they are due. Those which have been employed but once are indicated, where they occur in the tables of detailed reductions, by the letter B, following the wave-length of the line.

Elements	Wave Length	Elements	Wave-Length	Elements	Wave-Length
Ni; Ti, Cr	4399.903	*Ni; Fe*	4400.701	*—; Cr; Fe*	4526.644
V; Fe; V	4408.549	*Fe; Ag*	76.214	*Ti; —*	27.518
Ti; Fe	27.420	*—, Fe; Fe*	82.376	*Ti-Co; —*	34.168
Fe; Ti	34.021	*Ni; Fe,—*	90.600	*Ni; Fe; Ni*	47.196
Ca; Fe	35.184	*Cr; Zr*	97.040	*Ni, Ti; Fe*	60.225
Mn; Ca	50.030	*Cr, Mn; Ti*	4501.422	*Cr; Co Fe*	65.750
Ti; V, Zr; Mn	57.656	*--; Fe*	15.475	*Ca; Co, Fe*	81.034
Ni; Fe; Fe, Cr	59.304	*Fe?; -- Ti*	22.853	*Cr; Fe*	4611.455
V; Mn; -	60.400				

JOURNAL OF OBSERVATIONS.

The table which follows contains the observational data for all of the plates discussed in this article. As usual the series letters A, B, and B′ refer to the three cameras already described earlier in this paper. The middle of the star exposure is given in Greenwich Mean Time, and the hour angle at the end of the exposure is added to indicate the position of the telescope at the time.

The exposure of the plate to the comparison spectrum has commonly been made at the beginning and end of the star

Object	Series and Number	Date	Middle of Exposure	Duration	Hour Angle at End	Slit-Width
		1902	h m	m	h m	mm
Moon	A 350	July 16	14 28	40	E 0 20	0.030
ϵ *Pegasi*	A 364	July 31	19 15	62	W 0 45	0.032
β *Ophiuchi*	B 378	Aug. 7	15 06	77	W 1 15	0.038
ϵ *Pegasi*	B 379	Aug. 7	16 37	70	E 1 20	0.038
γ *Piscium*	B 381	Aug. 7	19 23	120	W 0 20	0.038
α *Persei*	B 382	Aug. 7	20 53	30	E 3 00	0.020
γ *Aquilae*	B 398	Aug. 27	17 02	100	W 2 40	0.038
Moon	B 401	Aug. 27	15 47	37	E 3 25	0.038
α *Boötis*	A 373	Sept. 6	7 43	15 ±	W 4 50	0.025
Moon	B 408	Sept. 13	10 13	35	W 1 32	0.038
γ *Piscium*	B 415	Oct. 8	14 38	153	E 0 05	0.038
γ *Aquilae*	B 417	Oct. 9	14 12	125	W 2 55	0.038
ϵ *Pegasi*	B 418	Oct. 9	16 14	93	W 2 40	0.038
α *Arietis*	B 420	Oct. 9	19 08	40	W 0 45	0.038
Moon	B 423	Oct. 15	15 44	30	E 0 46	0.028
γ *Cephei*	B 428	Oct. 16	17 44	70	W 2 25	0.038
α *Arietis*	B 430	Oct. 29	13 23	50	E 3 35	0.038
α *Persei*	B 431	Oct. 29	14 21	40	E 4 00	0.038
γ *Piscium*	B 436	Oct. 30	14 39	120	W 1 05	0.038
Moon	B 444	Nov. 6	13 38	40	W 3 15	0.032
ι *Aurigae*	B 446	Nov. 6	15 31	75	E 3 35	0.032
β *Leporis*	B 449	Nov. 6	19 55	90	W 0 25	0.038
α *Persei*	B 458	Nov. 19	13 27	33	E 3 35	0.038
γ *Cephei*	B 464	Nov. 27	10 35	75	W 4 45	0.038
α *Arietis*	B 465	Nov. 27	18 35	52	W 3 30	0.038
β *Geminorum*	B 477	Dec. 31	20 13	20	W 1 30	0.038
Mars	B 478	Dec. 31	20 56	32	E 2 25	0.038
		1903				
ϵ *Leonis*	B 483	Jan. 8	21 56	88	W 2 20	0.038
Moon	A 387	Jan. 16	14 28	25	E 0 30	0.038
α *Crateris*	B 491	Feb. 4	19 55	148	W 1 23	0.045
α *Boötis*	B 492	Feb. 4	21 39	20	E 1 25	0.045
β *Geminorum*	A 398	Feb. 5	19 31	18	W 3 05	0.038
Mars	A 400	Feb. 5	22 15	50	W 0 50	0.038
α *Boötis*	B 497	March 24	22 48	13	W 2 55	0.038
γ *Cephei*	A 424	April 3	15 41	68	W 11 28	0.038
β *Geminorum*	A 426	April 8	15 14	27	W 3 00	0.038
ϵ *Leonis*	A 427	April 8	16 11	70	W 2 20	0.038
Moon	A 429	April 8	18 35	25	W 3 05	0.038
α *Boötis*	A 433	April 8	21 46	16	W 2 50	0.030
α *Crateris*	A 438	April 16	17 24	132	W 3 18	0.038
ϵ *Leonis*	A 443	April 22	14 48	52	W 1 40	0.030
α *Crateris*	A 444	April 22	16 56	157	W 3 25	0.042
Mars	A 445	April 22	18 46	35	W 3 10	0.030
β *Ophiuchi*	A 451	April 30	19 55	95	E 0 20	0.038
α *Boötis*	B′ 499	May 6	14 44	20	E 2 15	0.038
γ *Cephei*	B′ 501	May 6	17 42	100	E 8 00	0.038
Mars	B′ 502	May 6	19 20	50	W 5 00	0.038
Venus	B′ 505	June 16	14 11	9	W 5 10	0.020
β *Ophiuchi*	B′ 508	June 26	15 37	83	E 1 00	0.040
γ *Aquilae*	B′ 509	June 26	17 25	110	E 0 55	0.040

Series and Number	Comparison Beginning	Comparison End	Temperature Beginning i	Temperature Beginning o	Temperature End i	Temperature End o	Seeing	Observer	Remarks
A 350	*Ti* 5	*Ti* 5	23.4	23.9	23.6	24.2	2; 1	A	Light cirrus
A 364	*Ti* 45	*Ti* 45	25.5	25.4	25.5	25.4	3; 3	A	clouds.
B 378	*Ti* 30	*Ti* 30	21.5	21.5	21.5	21.5	3-4; 3-4	F	
B 379	*Ti* 20	*Ti* 33	21.5	21.4	21.4	21.1	4; 3-4	F	
B 381	*Ti* 24	*Ti* 28	21.5	21.5	21.5	21.5	4; 4-3	F	
B 382	*Ti* 80	*Ti* 19			21.4	21.1	4; 4	F	
B 398	*Ti*50, *Cr*90	*Ti*40, *Cr*90	22.2	22.2	22.2	22.2	3; 3-4	A	Image very
B 401	*Ti* 40	*Ti* 45	22.2	22.4	....	....	3; 3	A	unsteady.
A 373		*Ti* 115	20.3	20.2	....	....	3-0; 2	A	Frequent
B 408	*Ti* 40	*Ti* 40	12.7	12.7	12.7	12.4	3; 1	A	clouds.
B 415	*Ti* 45	*Ti* 35	20.1	20.0	20.1	20.3	3; 2	A	
B 417	*Ti* 20 at 13:35	*Ti* 40	12.4	12.2	12.4	12.3	2; 2	F	
B 418	*Ti* 30	*Ti* 15	12.4	11.8	12.4	12.0	2; 2-3	F	
B 420	*Ti* 20	*Ti* 40	12.4	12.0	12.3	11.8	2; 2	F	
B 423	*Ti* 30	*Ti* 40	15.4	15.4	15.4	15.3	3; ...	A	
B 428	*Ti* 20	*Ti* 20	10.3	10.0	10.2	9.9	2-3; 2-3	F	
B 430	*Ti* 40	*Ti* 35	7.4	7.6	7.5	7.6	2; 2-1	A	
B 431	*Ti* 40	*Ti* 25	7.5	7.6	....	....	2; 2	A	
B 436	*Ti* 30	*Ti* 30	12.8	12.4	12.8	12.5	4; 4	F	
B 444	*Ti* 60		7.1	7.1	7.1	6.9	3; ...	F	
B 446	*Ti* 30	*Ti* 30	7.2	7.1	7.3	7.1	4; 4	F	
B 449	*Ti* 25	*Ti* 25	7.2	7.3	7.2	7.3	4; 4	A	
B 458	*Ti* 35	*Ti* 25	9.8	9.9	9.8	9.8	3; 3-4	A	Heavy dew on
B 464	*Ti* 25	*Ti* 30	− 0.2	0.0	− 0.1	+ 0.4	4; 3	F A	object-glass.
B 465	*Ti* 25		− 0.1	− 0.6	− 0.1	− 0.4	4; 4-3	F	Star's light
B 477	*Ti* 20	*Ti* 18	− 1.2	− 1.2	− 1.2	− 1.3	4; 2	A	cut down by
B 478	*Ti* 24	*Ti* 16	− 1.3	− 1.4	− 1.3	− 1.2	4; 1	A	snow clouds after 15:10.
B 483	*Ti* 25	*Ti* 25	−10.2	−10.4	−10.2	−10.3	3; 3	A	
A 387	*Ti* 20	*Ti* 20	− 0.3	− 0.1	− 0.4	− 0.8	3; ...	A	
B 491	*Ti* 15	*Ti* 15	− 4.8	− 4.8	− 4.8	....	3; 2	F	
B 492	*Ti* 15	*Ti* 15	− 5.0	....	− 4.9	− 4.6	3: 2	F	
A 398	*Ti* 25	*Ti* 25	− 6.1	− 6.0	− 6.1	− 6.3	3; 4	A	
A 400	*Ti* 23	*Ti* 20	− 6.1	− 6.4	− 6.1	− 6.6	3; 3	A	
B 497	*Ti* 40		0.0	0.0	....	....	2; 2	A	
A 424	*Ti* 30	*Ti* 30	1.3	1.2	1.4	1.4	3; 3	F	
A 426	*Ti* 30	*Ti* 25	14.7	14.8	14.8	14.8	3; 1	A	
A 427	*Ti* 25	*Ti* 25	14.8	14.8	14.8	14.8	3; 1-2	A	
A 429	*Ti* 25	*Ti* 25	14.8	14.9	14.8	14.5	3; ...	A	
A 433	*Ti* 32	*Ti* 30	14.8	14.9	14.8	14.8	3; 2	A	
A 438	*Ti* 20	*Ti* 25	8.3	8.0	8.2	7.8	4; 3	A	
A 443	*Ti* 30	*Ti* 35	9.7	10.5	10.2	9.7	4; 4+	F	
A 444	*Ti* 25	*Ti* 30	9.9	10.4	9.9	10.1	3; 4	F	
A 445	*Ti* 25	*Ti* 35	9.9	10.0	9.9	9.3	3; 4	F	
A 451	*Ti* 25	*Ti* 25	4.5	4.4	4.5	4.0	3-0; 3	A	Cloudy from
B' 499		*Ti* 120	13.1	12.6	13.1	12.9	3; 3	F	13:00-
B' 501	*Cr*45, *Ti*40	*Ti*55, *Cr*40	13.1	13.0	13.1	12.3	2-3; 2-3	F	13:35.
B' 502	*Cr* 60, *Ti* 80		....	....	13.2	12.4	2-1; 2	F	
B' 505	*Ti* 180, *Cr* 150		19.5	19.6	....	....	3; 2	A	
B' 508	*Ti* 50	*Ti* 85	23.3	22.9	23.1	22.5	2; 3	F	
B' 509	*Ti* 45	*Ti* 65	23.1	24.5	23.1	23.3	3; 3-4	F	

exposure, and the length in seconds is given by the number following the symbol of the element employed. The temperature conditions during the star exposure are indicated in the column headed "Temperature." The letter *i* refers to the thermometer within the prism box, *o* to the thermometer in the large outside temperature case. The readings are given only for the beginning and end of the exposure, as these indicate in the case of all the plates considered here the maximum range of temperature through which the air within the prism-box has passed. The last column of the table gives the estimated transparency of the sky and the steadiness and quality of the star's image, in the order named, and on a scale of 0 to 5, 5 denoting perfect conditions in each case.

The kind of photographic plate employed is not stated in the journal of observations. Previous to December 1902, Seed's "Gilt Edge 27," were used almost exclusively. Since that time the majority of photographs have been made with Seed's double-coated non-halation plates, with the outer emulsion also of the "Gilt Edge 27" quality. The principal developers used have been rodinol and edinol.

We are indebted to Mr. F. R. Sullivan, engineer in charge of the telescope, for much efficient assistance in the labor of guiding during the exposures entered in the list.

DETAILED REDUCTIONS.

The tables of detailed reductions which follow are divided into two parts, the first containing the results given by the plates of the Moon and the planets which have been taken as a check upon the behavior of the spectrograph, and the second the results for the stars whose discussion forms the subject-matter of this article. We have endeavored, so far as possible, to make the form of tabulation the same for both.

The time and hour angle given for each plate refer to the middle of the exposure. The former is expressed in Greenwich Mean Time. The first column of the table gives the wave-length employed for the stellar line, and following it in parallel columns is the velocity found for that line by each observer. The

mean for each observer is given to hundredths of a kilometer in order to avoid error in the formation of the final mean, but the latter is given only to tenths of a kilometer. On account of the great amount of repetition involved, it has not seemed desirable to give the wave-lengths of all the comparison lines used in the reduction of the plates, but the number of lines so used, as well as the wave-lengths of the standard lines employed in the determination of the constants of the reduction formula, is found at the foot of the table for each plate.

All of the velocities given, as well as the mean errors, are, of course, expressed in kilometers per second.

CONTROL PLATES OF THE MOON AND THE PLANETS.

The series of Moon and planet plates given below covers practically the whole interval over which the stellar results given in this paper extend. The first five plates of the Moon are summarized in a short table, the details of their reduction being given in Vol. II of the *Publications of the Yerkes Observatory*.

The theoretical velocities both of the Moon and of the planets have been computed with the aid of the formulæ given in so convenient a form by Professor Campbell.

One plate of *Mars*, B′502, has been excluded from the list given, as it was found on development that the glass was broken diagonally across, the photographic film alone holding the fragments together.

MOON.

Series and Number	Date	Hour Angle	Measured by	Number of Lines	Mean	Computed Velocity	O.−C.	Quality of Plate
A 350	1902, July 16, 14^h 28^m	E 0^h 42^m	A.	13	+0.4	+0.4	0.0	Fair
B 401	Aug. 27, 21 47	E 3 44	A.	14	−1.1	−1.5	+0.4	Fair
B 408	Sept. 13, 16 13	W 1 14	A.	12	+0.1	+0.2	−0.1	Good
B 423	Oct. 15, 15 44	E 1 01	A.	13	−0.6	−0.4	−0.2	Good
B 444	Nov. 6, 13 38	W 2 54	A.	14	+0.7	+0.6	+0.1	Good

MARS—B 478.

1902, December 31, $20^h\ 56^m$. Hour angle, E $2^h\ 41^m$. Measured by A.

Planet spectrum a trifle weak; comparison spectrum weak.

Line, Wave-Length in Sun	Velocity	Line, Wave-Length in Sun	Velocity
4468.663	−17.11	4565.750	−17.07
82.376	14.94	81.634	17.27
4501.422	17.92	84.018	17.66
28.798	16.49	4611.455	19.44
48.938	18.06		
54.211	15.74		
63.939	18.33		(11)
Number of comparison lines ..	9		
Ti standards used	4468.663 4534.139 4617.452	Mean Computed velocity	−17.3 −17.1

MOON—A 387.

1903, January 16, $20^h\ 40^m$. Hour angle, E $0^h\ 42^m$.

Moon spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	+0.14	−1.56	4494.738	−3.60	−1.27
42.510	−1.15	−1.82	97.046	−3.60	−1.53
43.976		−1.62	4501.422	−2.13	−1.33
47.892	+0.94	0.00	22.853		+0.27
56.030	−1.95		28.798	+0.33	−0.73
57.656	−2.49	+0.87	34.168		−1.19
68.663	−0.40		54.211	−1.51	
76.214	−0.07				
82.376		−0.80		(12)	(12)
No. of compar. lines	11	11	Means	−1.29	−0.89
Ti standards used	4427.266 4481.438 4555.662	4427.266 4481.438 4527.490	Mean Computed velocity	−1.1 −0.6	

MARS—A 400.

1903, February 15, $22^h\ 15^m$. Hour angle, W $0^h\ 25^m$.

Planet spectrum fair; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−15.85	−16.46	4494.738	−16.21	−15.61
42.510	14.31	14.65	4501.422	14.32	16.12
43.976	12.69	13.97	28.798	14.37	15.03
47.892	13.48	14.56	34.168		15.21
56.030	14.47		54.211	19.62	
57.656	15.20	15.40			
68.663	13.76	12.55			
76.214	10.38				
82.376		13.98			
90.900		16.09		(12)	(12)
No. of compar. lines	13	9	Means	−14.56	−14.97
Ti standards used	4427.266	4427.266	Mean	−14.8	
	4481.438	4481.438	Computed velocity	−15.5	
	4555.662	4527.490			

MOON—A 429.

1903, April 8, $18^h\ 35^m$. Hour angle W $2^h\ 53^m$. Measured by A.

Moon spectrum fair; comparison spectrum good.

Line, Wave-Length in Sun	Velocity	Line, Wave-Length in Sun	Velocity
4427.420	+2.91	4482.376	+2.74
42.510	2.16	97.046	2.00
47.892	2.97	4501.422	0.60
56.030	1.55	17.702	3.19
57.656	2.35	28.798	0.20
68.663	0.00		
76.214	0.54		(12)
Number of comparison lines...	10		
Ti standards used.....	4427.266	Mean	+1.8
	4481.438	Computed velocity	+1.3
	4527.490		

MARS — A 445.

1903, April 22, $18^h\ 46^m$. Hour angle W $2^h\ 52^m$.

Planet spectrum fair; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+4.13	+3.39	4476.214	+7.70	+6.10
35.184	5.81		82.376	7.29	6.49
41.881		6.21	94.738	3.94	
42.510	5.87	5.87	97.046	1.53	3.20
43.976		7.02	4501.422		3.73
47.892	5.19	4.18	15.475	3.92	6.57
56.030	5.18	4.78	26.644		6.82
57.656		5.45	28.798	6.42	4.37
59.304	5.31		54.211	7.04	
68.663	2.64	3.22		(14)	(15)
No. of compar. lines	14	11	Means	+5.14	+5.16
Ti standards used	4427.266	4427.266			
	4490.318	4481.438	Mean	+5.2	
	4555.662	4527.490	Computed velocity	+5.2	

VENUS — B 505.

1903, June 16, $14^h\ 11^m$. Hour angle W $5^h\ 6^m$. Measured by A.

Planet spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity	Line, Wave-Length in Sun	Velocity
4427.420	−13.39	4482.376	−11.44
42.510	11.53	94.738	14.01
43.976	11.32	97.046	13.07
47.892	11.58	4501.422	11.09
56.030	12.57	20.644	13.99
57.656	13.31	28.798	14.05
68.663	14.55		
76.214	11.72		(14)
Number of comparison lines	10		
Ti standards used	4427.266	Mean	−12.8
	4481.438	Computed velocity	−13.2
	4527.490		

The mean value of O.−C. for the above eleven plates taken with regard to sign is +0.10 km. A quantity of this size might easily be due to accidental causes, so that the conclusion may

be drawn that the control plates show no evidence of the existence of any systematic error in the spectrograph.

STANDARD VELOCITY STARS.

The following tables give the results of our measures upon the plates of standard velocity stars. The ten stars of the principal list are arranged in order of right ascension, followed by three stars which we have observed in the supplementary list.

In addition to the data given for the control plates, we append, for the star plates, the mean error ϵ of the determination of the velocity from a single line for each observer, and the mean error ϵ_0 of the final velocity of the star deduced by each observer from one plate, viz.:

$$\epsilon = \pm\sqrt{\frac{\Sigma v^2}{n-1}}, \quad \text{and} \quad \epsilon_0 = \pm\sqrt{\frac{\Sigma v^2}{n(n-1)}}.$$

The reduction of the star's velocity to the Sun is given in its two parts, V_a denoting the correction due to the Earth's orbital motion, and V_d the correction due to the Earth's diurnal rotation. Schlesinger's tables of star constants for the determination of V_a have been employed throughout.

Reference should be made to the fact that all of the measurable plates obtained are included in the reductions given below, and that none have been omitted because of inferior quality or lack of agreement with other plates.

α *ARIETIS* — B 420.

1902, October 9, $19^h\ 08^m$. Hour angle W $0^h\ 25$

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−23.03	−23.37	4482.376	−21.34	−22.82
28.711		21.53	94.738	23.68	24.02
41.881		23.70	97.046	22.40	23.74
42.510	23.42	22.89	4501.422	24.25	23.58
43.976		23.62	12.906	21.33	
47.892	22.38		26.644		23.12
56.030	24.09	22.01	28.798	22.12	23.64
60.460		23.20	48.938	23.40	
68.663	21.20		54.211	24.29	
76.214	23.65	21.78		(14)	(15)
No. of compar. lines	12	11	Means	−22.90	−23.07
ϵ	±1.10	±0.78	Mean	−22.98	
ϵ_0	±0.29	±0.21	V_a	+9.70	
Ti standards used	4427.266	4427.266	V_d	−0.03	
	4481.438	4481.438	Reduction to Sun	+ 9.67	
	4527.490	4527.490	Radial velocity	−13.3	

α *ARIETIS* — B 430.

1902, October 29, $13^h\ 23^m$. Hour angle E $4^h\ 0^m$.

Star spectrum very good; comparison spectrum very good.

4427.420	−13.82	−13.75	4499.900	−16.49	
28.711		15.23	94.738	15.41	−16.28
42.510	12.02	13.64	97.046	11.14	11.80
43.976		13.77	4501.422		15.59
47.892	12.81	13.14	12.906	16.15	16.01
57.656	12.65	11.70	15.475	14.67	
60.460	14.05	16.13	17.702		15.46
68.663	10.06	12.41	26.644	13.38	
76.214	14.40	16.48	28.798	11.98	12.12
82.376	13.65	12.38			
				(15)	(16)
No. of compar. lines	12	11	Means	−13.51	−14.12
ϵ	±1.80	±1.74	Mean	−13.82	
ϵ_0	±0.46	±0.44	V_a	−0.14	
Ti standards used	4427.266	4427.266	V_d	+0.27	
	4481.438	4481.438	Reduction to Sun	+ 0.13	
	4527.490	4527.490	Radial velocity	−13.7	

α *ARIETIS* — B 465.

1902, November 27, $18^h\ 35^m$. Hour angle W $3^h\ 4^m$.

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+0.07	+1.08	4482.376	+2.27	+2.21
28.711		−0.20	94.738	−2.13	−0.87
41.881		−0.47	97.046	+0.73	+1.00
42.510	+1.69	+2.23	4501.422	−0.80	+0.60
43.976		−0.74	12.906	+2.53	+1.26
47.892	−0.13	0.00	28.798	+2.05	+0.86
56.030	+1.35	+0.54	54.211	+0.07	
57.656	+0.81	+2.35			
60.460		−0.87			
68.663	+1.68	+2.42			
76.214	+0.94	+0.40		(14)	(17)
No. of compar. lines	13	11	Means	+0.80	+0.69
ϵ	±1.29	±1.14	Mean	+ 0.74	
ϵ_0	±0.34	±0.28	V_a	−14.58	
Ti standards used	4427.266	4427.266	V_d	− 0.23	
	4481.438	4481.438	Reduction to Sun	−14.81	
	4555.662	4527.490	Radial velocity	−14.1	

α *PERSEI* — B 382.

1902, August 7, $20^h\ 53^m$. Hour angle E $3^h\ 15^m$.

Star spectrum fair but unsymmetrical; comparison spectrum fair.

4443.976		−24.36	4515.508	−25.16	−25.76
47.892		27.71	20.397	26.54	
59.301	−26.02		22.802	26.19	24.40
66.727	25.04	26.52	28.798	27.41	28.01
68.663	25.50	24.56	54.211	26.53	25.61
76.214	27.47	28.07	58.827		23.74
89.351	28.99		63.939		23.65
91.570	24.97		72.156		25.18
94.738	31.02	28.49			
4501.445	26.24	27.18			
08.455	27.93	28.20		(14)	(15)
No. of compar. lines	12	15	Means	−26.79	−26.10
ϵ	±1.68	±1.74	Mean	−26.44	
ϵ_0	±0.44	±0.45	V_a	+24.48	
Ti standards used	4455.485	4443.976	V_d	+ 0.17	
	4512.906	4501.445	Reduction to Sun	+24.65	
	4555.662	4563.939	Radial velocity	− 1.8	

α *PERSEI* — B 431.

1902, October 29, $14^h\ 21^m$. Hour angle E $4^h\ 20^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4443.976		−11.27	4508.455	−12.17	−11.24
44.728		12.68	15.508	11.69	10.82
47.892		10.85	20.397	12.88	13.80
59.301	−14.32		22.802	11.74	
66.727	14.23	15.30	28.798	13.18	13.91
68.663	11.94	11.07	34.139		10.71
76.214	10.45	12.06	39.759		13.74
94.738	12.61	13.61	54.211	11.39	13.10
4501.445	12.59	13.19		(12)	(15)
No. of compar. lines	12	12	Means	−12.43	−12.49
ϵ	±1.13	±1.44	Mean	−12.46	
ϵ_0	±0.32	±0.37	V_a	+10.60	
Ti standards used	4457.600	4449.313	V_d	+ 0.20	
	4512.906	4501.445	Reduction to Sun	+10.80	
	4555.662	4555.662	Radial velocity	− 1.7	

α *PERSEI* — B 458.

1902, November 19, $13^h\ 27^m$. Hour angle E $3^h\ 51^m$.

Star spectrum good; comparison spectrum slightly strong.

4427.420		−2.51	4515.508	−4.25	−2.92
43.976		2.36	20.397	6.63	6.24
44.728		6.61	22.802	3.45	
47.892		7.15	28.798	7.02	6.69
59.301	−2.02		34.139		3.37
66.727		4.23	41.690		2.51
68.663	4.50	4.23	54.211	6.52	2.70
76.214	7.17		58.827	4.41	5.99
80.351	3.81		63.939	4.73	3.68
91.570	2.80		72.156	4.98	
94.738		4.20			
4501.445	5.46	3.40			
08.455	4.99	6.98		(15)	(17)
No. of compar. lines	11	13	Means	−4.85	−4.46
ϵ	±1.47	±1.75	Mean	−4.66	
ϵ_0	±0.37	±0.42	V_a	+1.48	
Ti standards used	4457.600	4427.266	V_d	+0.19	
	4512.906	4481.438	Reduction to Sun	+1.67	
	4563.939	4555.662	Radial velocity	−3.0	

β LEPORIS — B 449.

1902, November 6, $19^h\ 55^m$. Hour angle E $0^h\ 20^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−24.04	−26.75	4501.422	−24.58	−22.98
42.510		23.56	08.455	25.41	25.21
43.976		23.55	15.475	25.30	26.83
47.892	22.92	22.65	25.285 B	26.50	
56.030		26.18	26.644		26.23
57.656	25.70	24.55	28.798	22.84	23.64
68.663	23.89	23.82	46.129	23.74	
76.214	24.19	23.45	54.211	24.56	
82.376	23.48	23.75			
94.738		26.49			
97.046	21.40	22.87		(14)	(16)
No. of compar. lines	11	11	Means	−24.18	−24.53
ϵ	±1.32	±1.50	Mean	−24.36	
ϵ_0	±0.35	±0.37	V_a	+11.95	
Ti standards used	4427.266	4427.266	V_d	+ 0.03	
	4481.438	4481.438	Reduction to Sun	+11.98	
	4555.662	4527.490	Radial velocity	−12.4	

β GEMINORUM — B 477.

1902, December 31, $20^h\ 13^m$. Hour angle W $1^h\ 20^m$.

Star spectrum good; comparison spectrum weak.

4427.420	−3.12		4522.853	−1.86	
42.510	3.44	−2.02	28.798	2.45	−2.32
43.976		1.15	34.168	2.18	1.92
47.892		1.75	47.196	2.51	
56.030		2.49	48.938	5.08	
57.656	2.09	1.68	54.211	3.23	2.30
60.460		1.01	63.939	3.28	3.55
68.663	2.89	2.48	71.275	5.90	2.36
76.214		2.55			
82.376		3.14			
97.046	3.87	4.33			
4501.422	5.33	3.86		(14)	(16)
No. of compar. lines	12	10	Means	−3.37	−2.43
ϵ	±1.28	±0.92	Mean	−2.90	
ϵ_0	±0.34	±0.23	V_a	+6.30	
Ti standards used	4427.266	4443.976	V_d	−0.10	
	4501.445	4501.445	Reduction to Sun	+6.20	
	4563.939	4563.939	Radial velocity	+3.3	

β GEMINORUM—A 398.

1903, February 5, $19^h\ 31^m$. Hour angle W $2^h\ 56^m$

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4399.903		+13.83	4460.460		+15.60
4406.810		16.47	68.663	+14.29	16.64
07.851 B		17.14	76.214	14.27	
08.549		16.39	82.376	15.45	16.66
11.240		14.55	94.738		15.68
27.420	+15.58	17.95	97.046	15.60	14.94
42.510	16.60	16.33	4501.422	14.45	15.39
43.976		14.85	12.906	15.08	17.61
47.892	16.45	14.83	22.853	17.30	
56.030		17.83	28.798	17.28	16.03
59.304	15.80			(12)	(18)
No. of compar. lines	10	13	Means	+15.68	+16.04
ϵ	±1.07	±1.18	Mean	+15.86	
ϵ_0	±0.31	±0.28	V_a	−12.16	
Ti standards used	4427.266	4399.935	V_d	− 0.21	
	4481.438	4457.600	Reduction to Sun	−12.37	
	4527.490	4512.906	Radial velocity	+ 3.5	

β GEMINORUM—A 426.

1903, April 8, $15^h\ 14^m$. Hour angle W $2^h\ 46^m$.

Star spectrum very good; comparison spectrum very good.

4427.420	+ 31.77	+ 32.23	4468.663	+ 31.74	+ 32.74
28.711		33.53	76.214	33.90	32.00
35.184	33.47		82.376	35.19	34.39
41.881		31.80	97.046	33.41	34.07
42.510	32.94	33 95	4501.422	31.11	31.84
43.976	33.40	33.47	22.853	34.74	
47.892	32.09	33.23	28.798	32.24	34.04
56.030	32.64	31.96	47.196	32.97	
57.656		35.32	54.211	34.24	
				(15)	(14)
No. of compar. lines	15	10	Means	+ 33.06	+ 33.19
ϵ	± 1.16	± 1.10	Mean	+ 33.12	
ϵ_0	± 0.30	± 0.30	V_a	− 29.42	
Ti standards used	4427.266	4427.266	V_d	− 0.20	
	4481.438	4481.438	Reduction to Sun	− 29.62	
	4552.632	4527.490	Radial velocity	+ 3.5	

α *CRATERIS*—B 491.

1903, February 4, 19 h 55 m. Hour angle W 0 h 9 m.

Star spectrum too weak; comparison spectrum strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4457.656		+ 30.49	4565.750		+ 31.59
68.663		32.49	71.275		31.10
76.214		31.30	81.634		30.18
97.046		29.15	84.018		29.83
4500.451		29.99	90.126	+ 29.33	30.58
01.422		32.52	91.614 B		29.52
15.475		31.62	4611.455	27.50	30.04
28.798	+ 31.45	33.85	25.227	33.78	
46.129	26.64		26.358	27.67	
54.211	29.96		54.743 B	31.44	
60.225		31.70	73.370 B	27.08	
63.939	29.56			(11)	(16)
No. of compar. lines	10	11	Means	+ 29.44	+ 31.00
ϵ	± 2.18	± 1.26		Mean + 30.22	
ϵ_0	± 0.66	± 0.32	V_a	+ 16.66	
Ti standards used	4527.490	4457.600	V_d	− 0.01	
	4617.452	4527.490	Reduction to Sun		+ 16.65
	4682.088	4617.452	Radial velocity		+ 46.9

α *CRATERIS*—A 438.

1903, April 16, 17 h 24 m. Hour angle W 2 h 12 m.

Star spectrum weak; comparison spectrum strong.

4427.420	+ 61.84		4497.046	+ 60.48	+ 60.21
41.881		+ 61.23	4501.422		60.62
42.510		62.91	15.475	59.23	61.69
47.892		62.08	22.853	66.83	
57.656	60.61	61.42	28.798	63.36	62.78
60.460		63.40	47.196	62.44	
68.663		63.01	48.938		62.89
76.214		63.58	54.211	65.71	61.44
82.376	61.75		63.939		60.58
85.846		61.83	65.750		62.12
94.738	57.50				
				(10)	(16)
No. of compar. lines	9	13	Means	+ 61.08	+ 61.99
ϵ	± 2.82	± 1.04		Mean + 61.08	
ϵ_0	± 0.89	± 0.26	V_a	− 14.52	
Ti standards used	4427.266	4449.313	V_d	− 0.18	
	4496.318	4496.318	Reduction to Sun		− 14.70
	4555.662	4555.662	Radial velocity		+ 47.3

α *CRATERIS*—A 444.

1903, April 22, 16 h 56 m. Hour angle W 2 h 6 m.

Star spectrum weak; comparison spectrum strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+69.22		4512.906	+63.53	+65.32
35.184	65.65		28.798	65.68	62.18
45.641		+64.14	44.845 B		62.09
47.892	69.16	65.86	47.196	61.72	
60.460		64.07	63.939		64.91
68.663	67.77	65.42	71.275		62.01
76.214	65.26	63.32	81.634		64.08
82.376	64.29	64.96	86.191 B	67.86	
90.900		63.36	4606.404		64.48
97.046	66.01		11.455		64.37
4501.422	63.94	62.81		(12)	(16)
No. of compar. lines	12	14	Means	+65.84	+64.02
ϵ	± 2.32	± 1.15	Mean	+64.93	
ϵ_0	± 0.67	± 0.30	V_a	−16.73	
Ti standards used	4427.266	4427.266	V_d	− 0.17	
	4512.906	4512.906	Reduction to Sun	−16.90	
	4590.126	4617.452	Radial velocity	+48.0	

α *BOÖTIS*—A 373

1902, September 6, 13 h 43 m. Hour angle W 4 h 38 m.

Star spectrum excellent; comparison spectrum excellent.

4427.420	+10.63	+10.84	4482.376	+11.84	+10.90
28.711		11.92	94.738	10.67	11.81
34.021	13.59		97.046	11.47	10.40
35.851		11.15	4501.422		10.39
42.510	12.28	11.95	12.906	12.03	10.76
43.976	10.32	11.88	15.475		11.69
47.892	12.20	11.93	28.798	11.06	11.26
50.030	11.17	11.24	48.938	13.45	
57.050	11.30		54.211	12.77	
66.701		11.61			
68.663	13.62				
75.026		10.40			
76.214	12.06	11.59		(16)	(17)
No. of compar. lines	17	11	Means	+11.90	+11.28
ϵ	± 1.05	± 0.57	Mean	+11.59	
ϵ_0	± 0.26	± 0.14	V_a	−15.79	
Ti standards used	4427.266	4427.266	V_d	− 0.30	
	4496.318	4495.975	Reduction to Sun	−16.09	
	4552.632	4527.490	Radial velocity	− 4.5	

α *BOÖTIS*—B 492

1903, February 4, $21^h\ 39^m$. Hour angle E $1^h\ 35^m$.

Star spectrum good; comparison spectrum weak.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−31.02	−29.60	4501.422	−27.31	−26.64
41.881		30.58	12.906	25.98	26.31
42.510	27.61	27.47	26.644		29.41
43.976		28.95	28.798	29.93	30.26
47.892	27.10		46.129	30.60	30.08
56.030	29.06	27.72	48.024		29.66
57.656	29 33	28.72	54.211	29.89	
68.663	26.84	26.64	63.939	29.63	
76.214	28.01	28.61	65.750		28.76
82.376		27.30		(13)	(16)
No. of compar. lines	13	12	Means	−28.64	−28.54
ϵ	±1.59	±1.38	Mean	−28.59	
ϵ_0	±0.44	±0.34	V_a	+24.03	
Ti standards used	4427.266	4427.266	V_d	− 0.13	
	4501.445	4501.445	Reduction to Sun	+24.16	
	4563.939	4563.939	Radial velocity	− 4.4	

α *BOÖTIS*—B 497.

1903, March 24, $22^h\ 48^m$. Hour angle W $2^h\ 49^m$

Star spectrum very good; comparison spectrum very good.

4427.420	−13.82	−13.81	4512.906	−12.82	−14.09
28.711	13.00		18.198		14.93
41.881		14.78	28.798	13.18	13.97
42.510	13.16	12.29	34.953	14.74	
47.892	14.56	13.55	46.129	16.49	
56.030	15.75	14.53	48.024	13.19	
57.656	14.80	12.44	48.938	11.40	
68.663	13.02	12.41	54.211	14.09	
76.214	14.40	13.53	71.275	15.54	
82.376	13.38	13.05			
94.738	15.14	14.08			
97.046	13.54	12.27			
4501.422	16.79	14.45		(20)	(15)
No. of compar. lines	17	12	Means	−14.14	−13.65
ϵ	±1.35	±0.96	Mean	−13.90	
ϵ_0	±0.30	±0.25	V_a	+8.85	
Ti standards used	4427.266	4427.266	V_d	−0.22	
	4481.438	4481.438	Reduction to Sun	+ 8.63	
	4555.662	4527.490	Radial velocity	− 5.3	

α *BOÖTIS*—A 433.

1903, April 8, $21^h\ 46^m$. Hour angle W $2^h\ 42^m$.

Star spectrum excellent; comparison spectrum excellent.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−7.25	−7.45	4501.422	−7.73	−7.46
28.711		9.14	12.063		8.51
41.881		8.24	12.906	3.79	
42.510	7.42	7.36	22.853	4.38	
43.976	6.61	6.88	28.798	4.37	6.89
47.892	8.43	7.21	46.129	8.11	
56.030	8.01	7.27	48.024	5.34	
57.656	7.13	7.27	48.938	8.11	
68.663	5.03	5.90	54.211	6.06	
76.214	6.83	7.84			
82.376	7.09	7.16			
94.738	9.01	7.74			
97.046	7.93	6.47		(19)	(16)
No. of compar. lines	15	12	Means	−6.77	−7.42
ϵ	±1.53	±0.96	Mean	−7.10	
ϵ_0	±0.35	±0.25	V_a	+2.43	
Ti standards used	4427.266	4427.266	V_d	−0.21	
	4496.318	4481.438	Reduction to Sun	+2.22	
	4555.662	4527.490	Radial velocity	−4.9	

α *BOÖTIS*—B′ 499.

1903, May 6, $14^h\ 44^m$. Hour angle E $2^h\ 25^m$.

Star spectrum excellent; comparison spectrum slightly weak.

4427.420	+3.12	+2.51	4501.422	+2.46	+4.66
41.881		2.77	12.906	5.98	
42.510	5.94	4.72	26.644	5.23	3.91
43.976	6.55	5.40	28.798	6.16	4.70
47.892	3.71	3.37	34.953	2.84	
56.030	4.10	3.70	46.129	3.89	
57.656	3.90	4.10	48.024	4.48	
60.460		4.91	48.938	3.43	
68.663	6.04	4.70	54.211	1.71	
76.214	3.48	2.08	63.939	4.73	
82.376	5.02	4.42	71.275	3.15	
94.738		2.04		(20)	(15)
No. of compar. lines	18	10	Means	30	+3.93
ϵ	±1.37	±1.00			+4.12
ϵ_0	±0.31	±0.26	V		
Ti standards used	4427.266	4427.266	V		
	4501.445	4481.438	Reduction to S		-9.17
	4572.156	4527.490	Radial velocity		5.1

β OPHIUCHI—B 378.

1902, August 7, $15^h\ 6^m$. Hour angle W $0^h\ 37^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+ 8.53	+ 8.53	4501.422		+ 8.33
42.510	10.80	10.12	12.906	+ 8.31	9.83
43.976		10.19	17.702		11.55
47.892	10.85	9.24	26.644	9.21	
56.030		11.17	27.518	10.53	
57.656	9.82	9.55	28.798	6.95	10.13
59.922		10.35	46.129	10.42	
68.663	10.67	7.52	48.024	9.76	
76.214	8.04	11.12	48.938	9.29	
82.376	9.43	8.36	54.211	8.49	
94.738		8.01			
97.046	8.94	10.34		(16)	(16)
No. of compar. lines	11	11	Means	+ 9.38	+ 9.65
ϵ	±1.13	±1.21	Mean	+9.52	
ϵ_0	±0.28	±0.30	V_a	−20.20	
Ti standards used	4427.266	4427.266	V_d	− 0.05	
	4481.438	4481.438	Reduction to Sun	−20.25	
	4555.662	4527.490	Radial velocity	−10.7	

β OPHIUCHI—A 451.

1903, April 30, $19^h\ 55^m$. Hour angle E $1^h\ 8^m$.

Star spectrum good; comparison spectrum good.

4427.420	−29.26	−30.21	4468.663	−29.46	−28.85
35.184	28.34		76.214	30.22	29.48
41.881		31.53	82.376	28.50	29.03
42.510	32.87	28.35	94.738	35.89	30.36
43.976		30.10	96.222 B	27.77	
47.892	31.14	28.58	97.046	27.67	29.61
56.030	29.94	31.36	4501.422	30.77	30.44
57.656	31.35	30.14	28.798	30.46	31.05
60.460	30.39	28.37		(15)	(15)
No. of compar. lines	12	11	Means	−30.27	−29.83
ϵ	±2.11	±1.05	Mean	−30.05	
ϵ_0	±0.54	±0.27	V_a	+18.46	
ndards used	4427.266	4427.266	V_d	+ 0.10	
	4481.438	4481.438	Reduction to Sun	+18.56	
	4527.490	4527.490	Radial velocity	−11.5	

β OPHIUCHI — B′ 508.

1903, June 26, $15^h\ 32^m$. Hour angle E $1^h\ 42^m$.

Star spectrum good; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−9.28	−7.45	4501.422		−6.06
43.976		6.55	12.063		7.44
47.892	5.93	6.00	12.906	−6.64	7.31
56.030	8.41	6.46	17.702		4.45
57.656	6.53	7.27	27.518	5.03	
59.922		6.45	28.798	6.62	6.22
68.663	3.62	4.29	46.129	7.12	
76.214	7.91	6.90	48.024	4.55	
82.376	6.15	6.36	48.938	7.67	
94.738	7.60	5.87	54.221	7.18	
97.046	9.27			(16)	(15)
No. of compar. lines	14	12	Means	−6.84	−6.34
ϵ	±1.58	±0.96	Mean	−6.59	
ϵ_0	±0.39	±0.25	V_a	−4.58	
Ti standards used	4427.266	4427.266	V_d	+0.15	
	4481.438	4481.438	Reduction to Sun	−4.43	
	4555.662	4527.490	Radial velocity	−11.0	

γ AQUILAE — B 398.

1902, August 27, $17^h\ 2^m$. Hour angle W $1^h\ 50^m$.

Star spectrum weak; comparison spectrum too strong.

4456.030	+12.38		4528.798	+13.97	
57.656	14.53		46.129	14.57	+14.51
68.663	15.10	+14.43	48.024	15.62	
76.214	12.19	11.19	48.938		10.09
82.376		10.04	54.211	12.25	
85.846		10.83	60.225		13.55
97.046	13.34	14.27	65.750		14.25
4500.451		14.39	74.899		13.44
01.422	12.32		4603.126		12.31
12.006	13.22	11.50	11.455		13.07
15.475	13.28				
27.518	14.57	14.50		(13)	(15)
No. of compar. lines	13	15	Means	+13.64	+12.82
ϵ	±1.17	±1.70	Mean	+13.23	
ϵ_0	±0.32	±0.44	V_a	−14.49	
Ti standards used	4155.485	4481.438	V_d	− 0.16	
	4497.023[1]	4548.938	Reduction to Sun	−14.65	
	4555.662	4617.452	Radial velocity	− 1.4	

[1] *Cr.*

γ *AQUILAE* — B 417.

1902, October 9, $14^h\ 12^m$. Hour angle W $1^h\ 52^m$.

Star spectrum slightly weak; comparison spectrum good.

Line, Wave Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+22.28		4501.422		+20.05
28.711		+21.53	12.906	+22.39	22.46
34.021	24.01		28.798	23.64	22.98
41.881		21.67	46.129		23.61
42.510	23.08	21.60	47.196	23.14	
47.892	23.12	22.99	48.938		21.75
56.030	22.61	21.33	56.202 B		22.18
68.663	26.64		60.225		21.77
76.214	22.24	22.91	63.939		21.42
82.376	23.15	23.15	65.750		21.28
97.046	23.80	23.60			
				(12)	(17)
No. of compar. lines	13	14	Means	+23.34	+22.13
ϵ	±1.19	±0.96	Mean	+22.74	
ϵ_0	±0.34	±0.23	V_a	−24.85	
Ti standards used	4427.266	4427.266	V_d	− 0.16	
	4481.438	4481.438	Reduction to Sun	−25.01	
	4544.864	4555.662	Radial velocity	− 2.3	

γ *AQUILAE* — B′ 509.

1903, June 26, $17^h\ 25^m$. Hour angle E $1^h\ 50^m$.

Star spectrum good; comparison spectrum good.

4427.420	−11.85		4512.906	− 13.56	12.76
29.366	12.58		17.702		12.34
42.510	9.99		26.644		13.58
43.976	12.02		28.798	11.32	12.05
47.892	12.40		46.129	12.99	12.33
56.030	14.06		48.024	13.71	12.99
57.656	11.50		48.938	12.20	12.85
68.663	8.33	−11.00	54.211	12.90	
76.214	13.80	12.66	63.939	12.02	10.51
82.376	14.58	13.31	71.275	15.68	14.76
97.046	11.80	11.94	77.356		14.54
4501.422	12.92	13.79			
12.063		−15.02			
				(20)	(16)
No. of compar. lines	20	13	Means	−12.51	−12.90
ϵ	±1.57	±1.28	Mean	−12.70	
ϵ_0	±0.34	±0.32	V_a	+10.70	
Ti standards used	4427.266	4465.975	V_d	+ 0.15	
	4496.318	4527.490	Reduction to Sun	+10.85	
	4590.126	4590.126	Radial velocity	− 1.8	

ϵ *PEGASI*—A 364.

1902, July 31, $19^h\ 15^m$. Hour angle W $0^h\ 14^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−3.52		4497.046	−4.53	−5.20
34.021	4.67		4500.451		2.27
42.510	2.57		01.422	2.93	4.26
45.641		−4.38	05.003		3.86
47.892	2.97		12.906	3.26	3.06
49.313		4.78	27.518		5.30
56.030	5.99		28.798	3.64	3.11
57.656	4.91		46.129	5.28	5.14
59.922	4.24		48.938	6.00	5.47
68.663	1.14	2.35	65.750		5.45
76.214	4.02	5.76	74.899		2.43
82.376		2.81	90.126		3.85
88.363 B	4.21				
				(16)	(17)
No. of compar. lines	17	14	Means	−3.99	−4.09
ϵ	±1.28	±1.22	Mean	−4.04	
ϵ_0	±0.32	±0.30	V_a	+10.17	
Ti standards used	4427.266	4449.313	V_d	− 0.02	
	4481.438	4512.906	Reduction to Sun	+10.15	
	4544.864	4590.126	Radial velocity	+ 6.1	

ϵ *PEGASI*—B 379.

1902, August 7, $16^h\ 37^m$. Hour angle E $1^h\ 55^m$.

Star spectrum good; comparison spectrum good.

4427.420	+0.47		4512.906	−2.26	−2.46
42.510	+0.67	+0.74	15.475	−3.85	−2.46
47.892	−0.34	−1.08	28.978	−1.26	
56.030	−1.61		34.168		−1.39
57.656	−1.88		46.129	−2.37	−1.58
68.663	0.00	−0.13	48.024	−0.33	
76.214	−1.68	−1.41	48.938	−2.24	−0.73
82.376	−3.01	+0.27	63.939	−0.20	−0.85
97.046		−0.53	65.750		−2.43
97.842	−0.20		71.275	−0.98	−2.76
4501.422	−1.73	−0.47		(18)	(15)
No. of compar. lines	17	13	Means	−1.27	−1.15
ϵ	±1.24	±1.06	Mean	−1.21	
ϵ_0	±0.29	±0.27	V_a	+7.17	
Ti standards used	4427.266	4443.976	V_d	+0.16	
	4481.438	4512.906	Reduction to Sun	+7.33	
	4544.864	4572.156	Radial velocity	+6.1	

ϵ *PEGASI*—B 418.

1902, October 9, $16^h\ 14^m$. Hour angle W $1^h\ 54^m$.

Star spectrum a trifle faint; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4442.510		+27.88	4527.518	+27.82	+26.82
57.656	+26.64		28.798	26.29	
59.922	26.09		34.168		27.44
60.460		26.62	44.077 B		24.81
68.663	28.72	28.58	46.129	25.65	27.64
76.214	27.87	28.74	47.196	25.51	
90.900		25.36	48.024	27.23	
95.664 B		25.15	48.938	26.96	27.29
4501.422		24.98	65.750		26.14
12.906	24.85	25.52		(11)	(14)
No. of compar. lines	12	15	Means	+26.69	+26.64
ϵ	±1.16	±1.34	Mean	+26.66	
ϵ_0	±0.34	±0.36	V_a	−19.98	
Ti standards used	4457.600	4449.313	V_d	−0.16	
	4512.906	4512.906	Reduction to Sun	−20.14	
	4552.632	4555.662	Radial velocity	+ 6.5	

γ *PISCIUM*—B 381.

1902, August 7, $19^h\ 23^m$. Hour angle E $0^h\ 40^m$.

Star spectrum weak; comparison spectrum fair.

4427.420	−28.85		4497.046	−28.07	−29.54
42.510	25.25		4501.422	26.64	27.04
43.976		−29.22	08.455		28.40
47.892	26.90	27.30	12.906	26.58	
56.030	26.98	27.59	15.475	29.55	
57.656	30.94	29.67	26.644		27.69
68.663	27.24	29.26	28.798	28.34	27.75
76.214	25.80	29.95	46.129	26.91	
82.376	27.23	27.43	48.024	27.88	28.02
94.738	27.62	29.02	54.211		29.30
				(16)	(15)
No. of compar. lines	13	11	Means	−27.55	−28.48
ϵ	±1.41	±0.99	Mean	−28.02	
ϵ_0	±0.35	±0.26	V_a	+16.59	
Ti standards used	4427.266	4449.313	V_d	+ 0.06	
	4481.438	4501.445	Reduction to Sun	+16.65	
	4544.864	4552.632	Radial velocity	−11.4	

γ *PISCIUM*—B 415.

1902, October 8, $14^h\ 38^m$. Hour angle E $1^h\ 21^m$.

Star spectrum weak; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4441.881		+0.14	4526.644	+2.52	+2.98
42.510		+3.44	28.798	+6.42	+4.17
47.892		+0.88	46.129	+1.91	+4.35
56.030		−1.21	48.938		+1.38
57.656	+2.15		54.211	+1.32	
59.304	+1.82		58.827	+0.07	
68.663	+4.16	+2.15	65.750		−1.12
76.214	+2.34	+2.21	71.275		−0.98
82.376	+2.14	+2.94	72.156	−2.56	−2.16
94.738	+1.33				
97.046	+4.00	+3.87	81.634		+3.73
4501.422	+3.20	+4.06	90.126	−1.31	
4512.906	+3.12			(16)	(17)
No. of compar. lines	12	13	Means	+2.20	+1.81
ϵ	±1.94	±2.19	Mean	+2.00	
ϵ_0	±0.48	±0.53	V_a	−12.85	
Ti standard used	4457.600	4449.313	V_d	+ 0.12	
	4527.490	4527.490	Reduction to Sun	−12.73	
	4590.126	4590.126	Radial velocity	−10.7	

γ *PISCIUM*—B 436.

1902, October 30, $14^h\ 39^m$. Hour angle W $0^h\ 5^m$.

Star spectrum very good; comparison spectrum a trifle strong.

4427.420	+10.50	+11.38	4468.663		+12.48
33.390		12.37	76.214	+11.12	10.79
34.021	11.36		82.376	12.18	11.11
35.851	10.00	8.86	94.738	9.54	9.54
42.510	13.03	12.49	97.046	11.14	10.94
43.976		11.27	4501.422	12.92	11.79
47.892	11.80	11.93	08.455		11.11
54.093 B	9.69		12.906	11.23	
56.030		11.57	15.475	11.62	
57.656	10.56	10.70	28.798	11.59	12.05
66.701	11.00			(16)	(16)
No. of compar. lines	12	12	Means	+11.20	+11.27
ϵ	±1.01	±1.00	Mean	+11.24	
ϵ_0	±0.25	±0.25	V_a	−21.94	
Ti standard used	4427.266	4427.266	V_d	− 0.01	
	4481.438	4481.438	Reduction to Sun	−21.95	
	4527.490	4527.490	Radial velocity	−10.7	

SUPPLEMENTARY STARS.

ι *AURIGAE*—B 446.

1902, November 6, $15^h\ 30^m$. Hour angle E $4^h\ 12^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+2.30		4497.046	+3.40	+4.40
29.366		+2.23	4501.422	2.86	3.40
42.510	5.80	5.06	12.906	2.86	3.52
47.892		4.04	18.198		4.45
56.030	4.17	2.09	26.644	1.32	
57.656	2.35		28.798	3.58	4.30
60.460		4.03	46.129	2.70	
68.663	8.25		48.024	4.81	
76.214	3.08	2.61			
82.376	3.55	3.75		(14)	(12)

No. of compar. lines	12	13	Means	+3.64 +3.66
ϵ	±1.73	±0.93	Mean	+3.65
ϵ_0	±0.46	±0.27	V_a	+15.07
Ti standards used	4427.266	4427.266	V_d	+ 0.26
	4481.438	4481.438	Reduction to Sun	+15.33
	4544.864	4527.490	Radial velocity	+19.0

ε *LEONIS*—B 483.

1903, January 8, $21^h\ 56^m$. Hour angle W $1^h\ 36^m$.

Star spectrum good; comparison spectrum fair.

4427.420	−9.96	−11.65	4494.738	−12.07	−10.94
35.851	14.20	12.03	97.046	10.67	
41.881		8.71	4501.422	9.33	8.33
42.510		9.38	08.455	10.04	10.31
43.976	8.17		15.475	9.83	
47.892	10.38	11.32	28.798	10.06	9.54
57.656	8.14	8.01	46.129		10.88
68.663	9.73	9.26	48.938		11.60
76.214	10.79	10.45	54.211		8.23
82.376	9.43	9.30		(14)	(16)

No. of compar. lines	12	11	Means	−10.20 −10.00
ϵ	±1.53	±1.32	Mean	−10.10
ϵ_0	±0.41	±0.33	V_a	+15.57
Ti standards used	4427.266	4427.266	V_d	− 0.13
	4481.438	4481.438	Reduction to Sun	+15.44
	4527.490	4555.662	Radial velocity	+ 5.3

ϵ *LEONIS* — A 427.

1903, April 8, $16^h\,11^m$. Hour angle W $1^h\,45^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4399.903		+29.92	4476.214		+28.68
4427.420	+28.51	30.61	82.376	+26.22	27.43
28.711		28.24	97.046	29.67	30.67
41.881		27.34	4501.422	30.64	31.77
42.510		30.17	08.455	31.73	
43.976	29.28	30.10	15.475	30.48	
47.892	31.08	31.41	28.798	32.38	30.59
57.656	26.10	27.18	54.211	32.06	
60.460		28.18			
68.663	29.45	29.52		(13)	(15)
No. of compar. lines	13	11	Means	+29.69	+29.45
ϵ	±2.01	±1.51	Mean	+29.57	
ϵ_0	±0.56	±0.39	V_a	−24.79	
Ti standards used	4427.266	4399.935	V_d	− 0.14	
	4481.438	4457.600	Reduction to Sun	−24.93	
	4552.632	4527.490	Radial velocity	+ 4.6	

ϵ *LEONIS* — A 443.

1903, April 22, $14^h\,48^m$. Hour angle W $1^h\,14^m$.

Star spectrum good; comparison spectrum good.

4427.420	+32.85	+32.04	4482.376	+35.99	+35.59
42.510		33.28	94.738		32.35
43.976		36.03	97.046	37.07	
47.892	33.23	35.73	4501.422		33.44
56.030	35.19	35.13	08.455	32.66	34.85
57.656	33.50	33.77	15.475	33.07	
59.304	34.16		28.798	34.56	32.25
60.460		32.34	47.196	33.77	
68.663	34.22	35.23	54.211	34.63	
76.214	34.24	36.05		(14)	(14)
No. of compar. lines	14	11	Means	+34.22	+34.15
ϵ	±1.23	±1.53	Mean	+34.18	
ϵ_0	±0.33	±0.41	V_a	−27.67	
Ti standards used	4427.266	4427.266	V_d	− 0.10	
	4481.438	4481.438	Reduction to Sun	−27.77	
	4555.602	4527.490	Radial velocity	+ 6.4	

γ CEPHEI—B 428.

1902, October 16, $17^h\ 44^m$. Hour angle, W $1^h\ 50^m$.

Star spectrum weak; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4456.030		−49.05	4526.644	−46.57	
57.656	−46.55	49.51	27.518	46.23	
60.460		49.28	28.798	46.68	−46.95
68.663	48.85	49.32	46.129	43.99	47.69
76.214	49.71	49.19	48.024		48.66
82.376	47.70	48.50	48.938		47.59
97.046	48.01		54.211	50.96	
4501.422		49.76	65.750		49.32
12.906	51.03		71.275		48.61
17.702		46.80	72.156		48.20
				(11)	(15)
No. of compar. lines	9	11	Means	−47.87	−48.56
ϵ	±2.20	±0.93	Mean	−48.22	
ϵ_0	±0.66	±0.24	V_a	+7.32	
Ti standards used	4457.600	4455.485	V_d	−0.04	
	4512.906	4501.445	Reduction to Sun	+7.28	
	4552.632	4572.156	Radial velocity	−40.9	

γ CEPHEI—A 424.

1903, April 3, $15^h\ 41^m$ Hour angle W $10^h\ 54^m$

Star spectrum good; comparison spectrum a trifle strong.

4400.577 B	−31.89		4459.304	−30.53	
08.549	33.05		60.460		−32.67
27.420	32.31	−31.97	68.663	30.26	32.41
28.711		30.74	76.214	32.36	32.96
35.184	32.46		82.376	30.91	30.84
42.510		33.14	94.738	32.29	31.43
43.976		33.87	97.046		32.54
47.892	32.63	30.60	4528.798	32.91	32.65
56.030		33.17	47.196	31.00	
57.656	31.75	31.01	54.211	28.51	
				(14)	(14)
No. of compar. lines	13	10	Means	−31.63	−32.14
ϵ	±1.26	±1.05	Mean	−31.88	
ϵ_0	±0.34	±0.28	V_a	−9.23	
Ti standards used	4399.935	4427.266	V_d	−0.02	
	4481.438	4481.438	Reduction to Sun	−9.25	
	4555.662	4527.490	Radial velocity	−41.1	

γ *CEPHEI*—B′ 501.

1903, May 6, $17^h\ 42^m$. Hour angle E $8^h\ 50^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−38.06	−39.42	4482.376	−40.27	−39.00
28.711		38.59	94.738		37.56
42.510	37.12	38.20	97.046	38.47	39.01
43.976		38.67	4501.422	41.63	
47.892	39.64	39.50	17.702		38.16
56.030	39.16	39.43	22.853	39.91	
57.656	37.60	38.01	28.798	37.41	37.75
60.460		38.25	46.129	39.37	
68.663	36.30	37.84	48.938	38.10	
76.214	37.86	38.32	54.211	38.45	
				(15)	(15)
No. of compar. lines	13	12	Means	−38.62	−38.51
ϵ	±1.38	±0.64	Mean	−38.56	
ϵ_0	±0.36	±0.16	V_a	−3.06	
Ti standards used	4427.266	4427.266	V_d	+0.06	
	4481.438	4481.438	Reduction to Sun	− 3.00	
	4555.662	4527.490	Radial velocity	−41.6	

SUMMARY.

The results of the above detailed reductions are summarized in the following table. The values obtained by the two observers are placed in parallel columns, followed by the difference between the two for each plate. In the last column is given the mean of both determinations, and, finally, at the foot of the summary for each star are found the results derived from all of the plates.

In the final determinations given, those for β *Leporis* and ι *Aurigae* are, of course, entitled to low weight, since but one plate has been measured in the case of each star. Among the other stars of the list, the result obtained for α *Crateris* is probably subject to the greatest uncertainty: the low altitude and photographic faintness of the star have made it a difficult object, and all of the plates secured have been too weak for the most accurate measurement. A few of the plates of γ *Aquilae* and γ *Piscium* are also somewhat weak, but in a less degree.

The excellent agreement of the values of the two observers for each plate of ϵ *Leonis*, but unusually large discordance in the

results given by the different plates, might lead one to suspect that the range of 1.8 km. may be real, but a much larger number of plates would be required before a definite conclusion could be reached on the subject. The range in the values obtained for *a Persei* we consider to be due to the character of its spectrum.

a ARIETIS.

Series and Number	Date	F.	A.	F.-A.	Mean
B 420	1902, October 9	−13.23	−13.40	+0.17	−13.3
B 430	October 29	−13.38	−13.99	+0.61	13.7
B 465	November 27	−14.01	−14.12	+0.11	14.1
	1902.84	−13.5	−13.9	+0.30	−13.7

a PERSEI.

B 382	1902, August 7	−2.14	−1.45	−0.69	−1.8
B 431	October 29	−1.63	−1.69	+0.06	1.7
B 458	November 19	−3.18	−2.79	−0.39	3.0
	1902.77	−2.3	−2.0	−0.34	−2.1

β LEPORIS.

B 449	1902, November 6	−12.20	−12.55	+0.35	−12.4
	1902.85	−12.2	−12.6	+0.35	−12.4

β GEMINORUM.

B 477	1902, December 31	+2.83	+3.77	−0.94	+3.3
A 398	1903, February 5	+3.31	+3.67	−0.36	3.5
A 426	April 8	+3.44	+3.57	−0.13	3.5
	1903.12	+3.2	+3.7	−0.48	+3.4

a CRATERIS.

B 491	1903, February 4	+46.09	+47.65	−1.56	+46.9
A 438	April 16	+47.28	+47.29	−0.01	47.3
A 444	April 22	+48.94	+47.12	+1.82	48.0
	1903.23	+47.4	+47.4	+0.08	+47.4

α BOÖTIS.

Series and Number	Date	F.	A.	F.-A.	Mean
A 373	1902, September 6	−4.19	−4.81	+0.62	−4.5
B 492	1903, February 4	−4.48	−4.38	−0.10	4.4
B 497	March 24	−5.51	−5.02	−0.49	5.3
A 433	April 8	−4.55	−5.20	+0.65	4.9
B′ 499	May 6	−4.87	−5.24	+0.37	5.1
	1903.12	−4.7	−4.9	+0.22	−4.8

β OPHIUCHI.

B 378	1902, August 7	−10.87	−10.60	−0.27	−10.7
A 451	1903, April 30	−11.71	−11.27	−0.44	11.5
B′ 508	June 26	−11.27	−10.77	−0.50	11.0
	1903.14	−11.3	−10.9	−0.40	−11.1

γ AQUILAE.

B 398	1902, August 27	−1.01	−1.83	+0.82	−1.4
B 417	October 19	−1.67	−2.88	+1.21	2.3
B′ 509	1903, June 26	−1.66	−2.05	+0.39	1.8
	1902.98	−1.4	−2.2	+0.81	−1.8

ε PEGASI.

A 364	1902, July 31	+6.16	+6.06	+0.10	+6.1
B 379	August 7	+6.06	+6.18	−0.12	6.1
B 418	October 9	+6.55	+6.50	+0.05	6.5
	1902.65	+6.2	+6.2	+0.01	+6.2

γ PISCIUM.

B 381	1902, August 7	−10.90	−11.83	+0.93	−11.4
B 415	October 8	−10.53	−10.92	+0.39	10.7
B 436	October 30	−10.75	−10.68	−0.07	10.7
	1902.73	−10.7	−11.1	+0.42	−10.9

SUPPLEMENTARY STARS.

ι AURIGAE.

Series and Number	Date	F.	A.	F.-A.	Mean
B 446	1902, November 16	+18.97	+18.99	−0.02	+19.0
	1902.85	+19.0	+19.0	−0.02	+19.0

ε LEONIS.

B 483	1903, January 8	+5.24	+5.44	−0.20	+5.3
A 427	April 8	+4.76	+4.52	+0.24	4.6
A 443	April 22	+6.45	+6.38	+0.07	6.4
	1903.20	+5.5	+5.4	+0.04	+5.5

γ CEPHEI.

B 428	1902, October 16	−40.59	−41.28	+0.69	−40.9
A 424	1903, April 3	−40.88	−41.39	+0.51	41.1
B′ 501	May 6	−41.62	−41.51	−0.11	41.6
	1903.13	−41.0	−41.4	+0.36	−41.2

An examination of the above results for systematic differences between the two observers indicates a slight tendency for F.—A. to be positive. A general mean for all of the stellar plates (37) gives F.—A. a little less than +0.10 kilometers. It is, however, extremely doubtful whether this is to be considered as other than accidental, as it is chiefly due to the abnormally large positive differences given by two somewhat inferior plates of *γ Aquilae*. Accordingly it would seem that the personality errors are pretty evenly balanced in this series of measures; particularly when the possibility is considered of errors due to the regular difference in the corrections applied for curvature.

No evidence is shown in the results given above of any dependence of the value obtained upon the position of the spectrograph in reference to the pier of the telescope. Of other instrumental causes which might give rise to errors, temperature changes may be regarded as eliminated, as a brief examination

of the Journal of Observations will show. Possible errors arising from changes in the electrical conditions of the comparison spectrum apparatus may also be disregarded, as this has remained without essential modification during the entire interval covered by the observations.

Spectrographic determinations of the radial velocities of four of the stars in the principal list, and of one of the supplementary list, have been published since Vogel and Scheiner obtained their results in 1889–90. These are collected in the following table. For convenience in comparison, the value we have found for each star in this paper is repeated immediately below its name.

Star	Observer	Velocity	Epoch	No. of Plates	Range	Reference
		km			km	
α Arietis	Campbell	−14.1	1896.8	4	0.6	ASTROPHYSICAL JOURNAL, **8**, 150, 1838
−13.7	Adams	−13.7	1901.9	1	...	ASTROPHYSICAL JOURNAL, **15**, 24, 1902
	Newall	−14.3	1902.8	3	2.8	*Monthly Notices*, **63**, 298, 1903
α Persei	Campbell	− 2.4	1897.8	4	1.7	ASTROPHYSICAL JOURNAL, **8**, 150, 1898
−2.1	Vogel	− 3.2	1901.0	13	3.3	ASTROPHYSICAL JOURNAL, **13**, 322, 1901
	Newall	− 2.6	1902.8	14	5.7	*Monthly Notices*, **63**, 298, 1903
α Boötis	Frost and Adams	− 4.3	1902.3	8	1.8	*Publications of the Yerkes Observatory*, **2**, 35, 1903
−4.8	Newall	− 5.8	1903.4	5	2.7	*Monthly Notices*, **63**, 298, 1903
ε Pegasi	Campbell	+ 5.7	1897.8	4	1.2	ASTROPHYSICAL JOURNAL, **8**, 150, 1898
+6.2						
ε Leonis	Wright	+ 5.1	1899.4	7	2.7	ASTROPHYSICAL JOURNAL, **11**, 414, 1900
+5.5	Adams	+ 4.0	1900.8	3	0.9	ASTROPHYSICAL JOURNAL, **15**, 25, 1902

To us a surprising difference in the above comparison is that of 0.5 km between our earlier measures of eight plates of *α Boötis* and the five plates of this paper. We attach greater weight to the present series, on account of the improved method of insuring the uniform illumination of the collimator lens by the light from the spark. But the range in velocity observed in either series is larger than might be expected for a star having such well-defined lines.

There would appear to be a slight tendency toward a systematic difference between our results and those of other observers, in the direction of a larger positive, or smaller negative, value for our velocities. The amount is so small, however, and the quantity of material with which comparison may be made is at present so meager, that no certain inference can now be drawn. An

investigation of this and of many other matters of interest will be possible as the results of other participators in the co-operation are published. It is known that, owing to changes being made in their spectrographs, at least two of those planning to co-operate have been unable to carry out the program during the past year, but expect to do so during the present year. A general comparison of approximately contemporaneous results by all participators will therefore not be available until later, but we have thought that the prompt publication of our results would nevertheless be of service. We shall continue our observations this year, following the program as closely as possible.

YERKES OBSERVATORY,
August 27, 1903.

ON THE SPECTRA OF IMPERFECT GRATINGS.

By A. A. MICHELSON.

IT will be convenient to consider separately gratings which act by opacity and those depending on retardation. The simplest case of the former may be represented by the expression

$$y = a \cos ks + b \sin ks \ ,^1 \qquad (1)$$

in which y represents the reflection (or transmission) factor at any point s of a line perpendicular to the rulings, and $k = 2\pi/\sigma$, where σ is the constant grating-space.

If a plane wave-train of frequency $n/2\pi$ fall normally on such a grating, the effect in a direction making an angle θ with the normal will be

$$\int y ds \cos (nt - \vartheta)$$

where

$$\vartheta = \frac{2\pi}{\lambda} s \sin \theta \ .$$

The intensity of the diffracted light is

$$I = \left[\int y ds \cos \vartheta\right]^2 + \left[\int y ds \sin \vartheta\right]^2 \qquad (2)$$

or

$$I = a^2 \left[\int \cos ks \cos \vartheta \, ds\right]^2 + b^2 \left[\int \sin ks \sin \vartheta \, ds\right]^2 .$$

The integrals, taken from $-\infty$ to ∞, have sensible values only at $\sin \theta = \pm \frac{\lambda}{\sigma}$, and the intensity in these directions is

$$I = a^2 + b^2 \ .$$

If the grating cannot be represented by a simple sine curve, let

$$y = \Sigma a_m \cos mks + \Sigma b_m \sin mks \ .$$

This value substituted in (2) gives

$$I_m = a_m^2 + b_m^2$$

in directions such that $\sin \theta = \pm \frac{m\lambda}{\sigma}$.

[1] To avoid negative values, a constant should be added, but this affects only the central image and not the lateral spectra.

As an illustration suppose $y = 1$ from $s = 0$ to $s = \frac{1}{2}\sigma$, and $y = 0$ from $s = \frac{1}{2}\sigma$ to $s = \sigma$.

The value of the coefficients in the series for y will then be

$$a_m = 0\ ,\quad b_0 = \frac{\pi}{2}\ ,\quad b_1 = 1\ ,\quad b_2 = 0\ ,\quad b_3 = \frac{1}{3}\ ,\quad \text{etc.}$$

The intensities of the spectra will be in the proportion of $\frac{\pi^2}{4}$ for the central image and 1, 0, $\frac{1}{3^2}$, 0, $\frac{1}{5^2}$, etc., for the spectra in their order, in agreement with the results obtained by Lord Rayleigh.[1]

In the case of a reflection grating the retardation is

$$\vartheta = \frac{2\pi}{\lambda}[s \sin\theta - y\,(1 + \cos\theta)]\ ,*$$

and the effect in a direction θ will be

$$\int ds \cos(nt - \vartheta)\ . \tag{3}$$

The intensity will be

$$I = \left[\int ds \cos\vartheta\right]^2 + \left[\int ds \sin\vartheta\right]^2 . \tag{4}$$

Putting

$$\cos\frac{2\pi}{\lambda}\left(y \,.\, 2\cos^2\frac{\theta}{2}\right) = \Sigma a_m \cos mks\ ,$$

and

$$\sin\frac{2\pi}{\lambda}\left(y \,.\, 2\cos^2\frac{\theta}{2}\right) = \Sigma b_n \sin mks\ ,$$

also

$$\frac{2\pi}{\lambda}\left(m\frac{\lambda}{\sigma} - \sin\theta\right) = p\ ,$$

and

$$\frac{2\pi}{\lambda}\left(m\frac{\lambda}{\sigma} + \sin\theta\right) = p'\ ,$$

$$\sqrt{I} = \sum a_m \frac{\sin ps}{p} + \sum a_m \frac{\sin p's}{p'} + \sum b_m \frac{\sin ps}{p} - \sum b_m \frac{\sin p's}{p'}\ .$$

[1] *Lord Rayleigh Scientific Papers*, Vol. III.

* For a transmission grating $\vartheta = \frac{2\pi}{\lambda}\,[sn \sin\theta + y\,(1 - n\cos\theta)]$.

The limits being infinite, I has finite values only at $p = 0$ and at $p' = 0$, or for $\sin\theta = m\lambda/\sigma$ and $\sin\theta = -m\lambda/\sigma$. For the former

$$I = (a_m + b_m)^2$$

and for the latter $$I' = (a_m - b_m)^2 \ . \tag{5}$$

If $a_m = b_m$, all the negative spectra vanish. If of the positive spectra all are to vanish except the mth, then

$$\cos\frac{2\pi}{\lambda}\left(y \cdot 2\cos^2\frac{\theta}{2}\right) = \cos mks \ ,$$

$$\sin\frac{2\pi}{\lambda}\left(y \cdot 2\cos^2\frac{\theta}{2}\right) = \sin mks \ ,$$

which gives

$$y = (x + n\sigma)\tan\frac{1}{2}\theta \ . \tag{6}$$

The section of the grating surface is that represented in Fig. 1. It is essentially the arrangement of plates in a reflecting echelon.

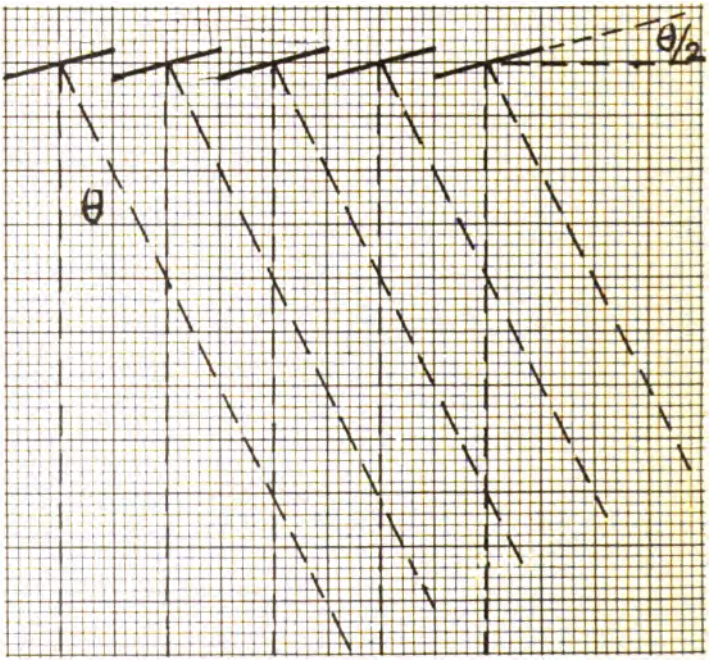


FIG. 1.

In actual gratings the ruled space is limited, and the spectrum of a homogeneous source will not be a line, but an image of finite width. It is proposed to find the distribution of light in this image for any given grating. This distribution can be affected only by the imperfections of the ruling (*e. g.* changes in the strength of the lines or in their spacing), but not by the character of the furrow. We may therefore let

$$y = \phi(s) \cos mks + \psi(s) \sin mks = f(s) \ .$$

By Fourier's formula

$$f(s) = \int_{-\infty}^{\infty} (C \cos \alpha s d\alpha + S \sin \alpha s d\alpha) \ ,$$

where
$$C = \int_{-\infty}^{\infty} f(\lambda) \cos \alpha\lambda d\lambda \ , \tag{7}$$

and
$$S = \int_{-\infty}^{\infty} f(\lambda) \sin \alpha\lambda d\lambda \ .$$

A single element of $f(s)$ is therefore

$$C \cos \alpha s + S \sin \alpha s \ , \tag{7'}$$

which gives a spectrum in a direction

$$\sin \theta = \pm c\alpha \quad \text{whose intensity is} \quad I = C^2 + S^2 \ . \tag{8}$$

If the spacing is exact, $\psi = C\phi$, whence

$$I = \left[\int \phi(\lambda) \cos p\lambda d\lambda\right]^2 + \left[\int \phi(\lambda) \sin p\lambda d\lambda\right]^2 ,$$

where $p = mk \pm \alpha$. The spectrum images are always symmetrical.

If the spacing is not exact, but the strength of the rulings is constant, the usual case in actual gratings,

$$f(s) = \cos (ks - \omega) \ ,$$

$$\phi = \cos \omega \ , \quad \psi = \sin \omega \ , \quad C = \int \cos (\omega - p\lambda) d\lambda \ ,$$

$$S = \int \sin (\omega - p\lambda) d\lambda \ ;$$

hence

$$I = \left[\int \cos (\omega - p\lambda) d\lambda\right]^2 + \left[\int \sin (\omega - p\lambda) d\lambda\right]^2 . \tag{9}$$

The spectral images are generally unsymmetrical.

One form of spacing error which doubtless occurs in practice is that caused by the sudden release of strain in some part of

the course, thus producing a "slip" in all the subsequent rulings. Such a case may be represented by putting $\omega = -a$ from $-l$ to 0 and $\omega = a$ from 0 to l. These values substituted in (9) give

$$C = \frac{1}{p}(\sin a - \sin(a - pl)) \quad . \tag{10}$$

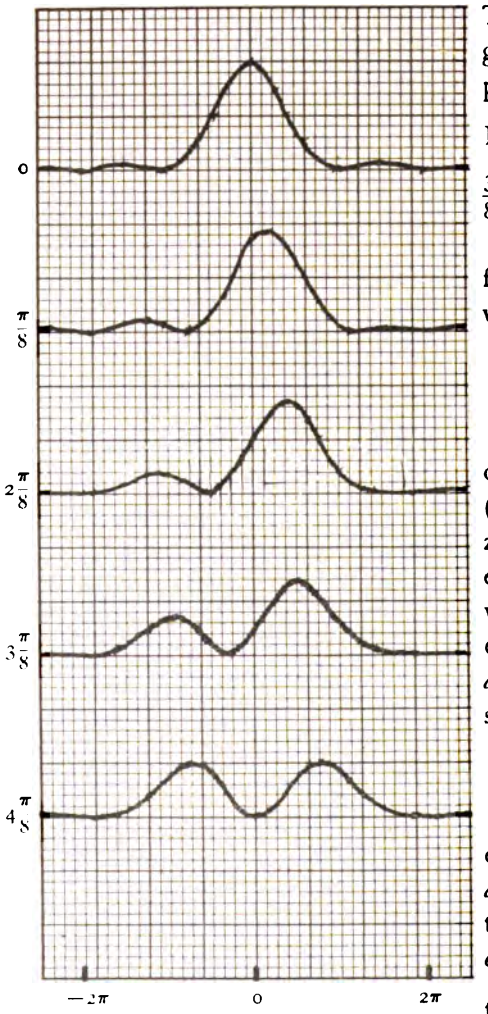


FIG. 2.

The intensity curve which is given by squaring this expression is represented in Fig. 2 for $a = 0$, $\frac{1}{8}\pi$, $\frac{2}{8}\pi$, $\frac{3}{8}\pi$, and $\frac{4}{8}\pi$.

The minima are readily found by putting $C = 0$, which gives

$$pl = 2\left(a - \frac{2n+1}{2}\pi\right) \quad . \tag{11}$$

The maxima may be obtained by differentiating (10) for p and equating to zero. It can be shown, however, that the result is given with an error less than 0.002π from $a = \pi/2$ to $a = -\pi/2$ by the expression

$$p_1 l = \frac{3}{2}(a - n\pi) \quad . \tag{12}$$

If h is the actual linear displacement of the rulings, $a = 2\pi h/\sigma$. Substituting this value in (12), restoring $a = k - p$ and remembering that $\sin\theta = \frac{\lambda}{2\pi}a$, we have

$$\sin\theta = \frac{\lambda}{\sigma} - \frac{3}{2}\frac{\lambda}{l}\frac{h}{\sigma} + \frac{3}{4}\frac{n\lambda}{l} \quad .$$

For $h = 0$ and $n = 0$, $\sin \theta_0 = \lambda/\sigma$, whence

$$\sin \theta - \sin \theta_0 = \cos \theta \partial\theta = \frac{\lambda}{\sigma}\left(\frac{3h}{2l} - \frac{3}{4}\frac{n\sigma}{l}\right),$$

$$\frac{\partial\lambda}{\lambda} = \frac{\partial\theta}{\tan\theta} = \frac{3}{2}\frac{h}{l} - \frac{3}{4}\frac{n\sigma}{l}.$$

If the observations are made in the mth spectrum, σ must be replaced by σ/m. Putting also $2l = N\sigma$ we have

$$\frac{\partial\lambda}{\lambda} = \frac{3}{N}\left(\frac{h}{\sigma} - \frac{n}{2m}\right).$$

Treating (11) in a similar way,

$$\frac{\partial_1\lambda}{\lambda} = \frac{4}{N}\left(\frac{h}{\sigma} - \frac{2n+1}{4m}\right).$$

These expressions represent the percentage error due to setting on the maximum and the minimum respectively. The graphs of these errors are represented in Fig. 3 by the systems of straight lines.

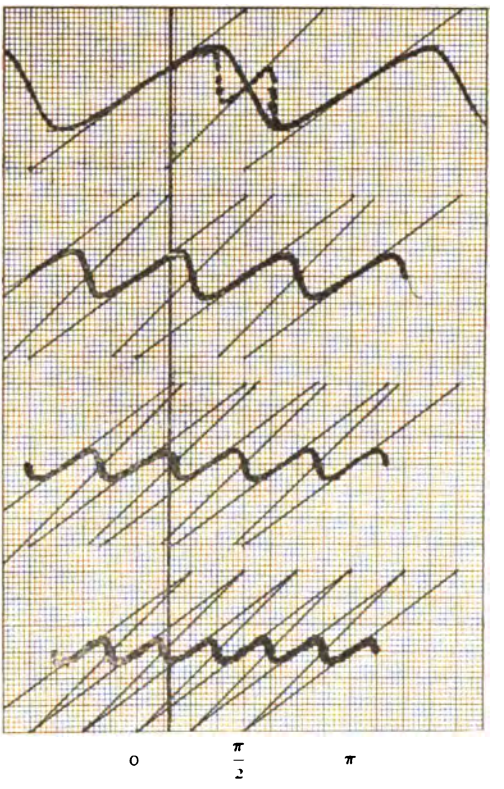


FIG. 3.

The actual phase which may be selected for measurement will, however, depend largely on the observer. When h is small, this will be the brightest part of the line, and the error will be represented by the lines of lesser inclination; while if h is nearly one-fourth of the grating space, the minimum would be selected, and for values between, the measurements would be uncertain, with a tendency to setting on the "center of gravity."

This surmise is amply verified by observation. For this

purpose a grating was mounted on a spectrometer, and a pair of plane-parallel glasses was placed in front of the collimator lens. One of the plates could be rotated through a measured angle, thus introducing in the corresponding half of the field any required retardation. The results are shown in Fig. 4.

The curves from this and other observations are repeated in Fig. 3. If a vertical line, for instance that at $a = 0.15\pi$, be drawn through the curves, the intersections will correspond to errors of 4.0, 3.5, 2.0 and -1.5 twentieths respectively. The constant error introduced in comparing the second with the

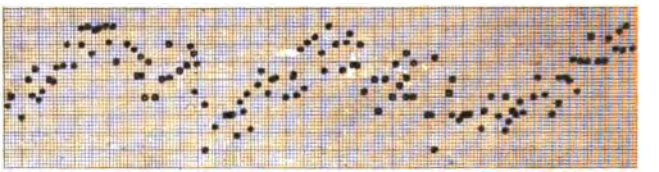


FIG. 4.

third spectrum will be 0.1, while between the third and fourth the error will amount to 0.17. If $a = \frac{1}{3}\pi$, corresponding to $h = \frac{1}{6}\sigma$, the error introduced in comparing the first spectrum with the second will be 0.5. The absolute error depends on the number of lines in the grating. If this be 100,000, these errors will be 1, 1.7, and 5 parts in a million — quantities of the same order as those given by Perot and Fabry,[1] indicating a systematic error in Rowland's table of wave-lengths.

If the ruling is affected by a periodic error, the spectral lines are accompanied by ghosts. The elementary explanation has long been known, but, so far as I know, no general theory has been proposed which furnishes a relation between the character of the periodic error and the intensity of the ghosts.

The result in its widest generality would be obtained by substituting for ϕ and ψ any periodic functions; but it will serve our purpose if ϕ is even and ψ is odd.

[1] *Comptes Rendus*, 133, 153, 1901.

Putting $\phi(s)=\Sigma a_q \cos qs$ and $\psi(s)=\Sigma b_q \sin qs$ in (7), we obtain

$$C=\int a_q \cos(q-p)\lambda d\lambda + \int b_q \sin(q-p)\lambda d\lambda ,$$
$$+\int a_q \cos(q+p)\lambda d\lambda - \int b_q \sin(q+p)\lambda d\lambda ,$$
$$S=0 .$$

If the number of periods is large, the value of C will be appreciable only in the vicinity of $p=q$ and $p=-q$, and in these directions the intensity will be

$$I_q = C_q^2 = (a_q + b_q)^2 \quad \text{and} \quad I_q' = (a_q - b_q)^2$$

or

$$I_q = \left[\int_0^{2\pi} \phi(s)\cos qs\, ds + \int_0^{2\pi} \psi(s)\sin qs\, ds\right]^2$$
$$I_q' = \left[\int_0^{2\pi} \phi(s)\cos qs\, ds - \int_0^{2\pi} \psi(s)\cos qs\, ds\right]^2 . \qquad (13)$$

If $b_q = 0$, the ghosts are symmetrical about the principal line. If $a_q = b_q$, the ghosts on the side of $p=-q$ vanish.

If the only error is that due to unequal spacing, let $\phi(s)=\cos\omega$ and $\psi(s)=\sin\omega$, then, if $q=\frac{2\pi m}{S}$, where S is the period of the error, and $m=1, 2, 3 \ldots$

$$a_q = \int_0^S \cos\omega \cos qs\, ds , \quad b_q = \int_0^S \sin\omega \sin qs\, ds ,$$

$$I_q = \left[\int_0^S \cos(\omega - qs)\, ds\right]^2 , \quad I_q' = \left[\int_0^S \cos(\omega + qs)\, ds\right]^2 .$$

As an illustration take $\omega = hs$ from 0 to S; then

$$I_m = \left(\frac{\sin \frac{hS}{2} - \pi m}{\frac{hS}{2} - \pi m}\right)^2 \quad \text{and} \quad I_m' = \left(\frac{\sin \frac{hS}{2} + \pi m}{\frac{hS}{2} + \pi m}\right)^2 .$$

The expression (4) represents the intensity of the reflected light, whether the surface be furrowed or not. For normal incidence, therefore, the intensity of the diffraction image is [1]

$$I = \left[\int ds \cos(2my\cos\theta - ms\sin\theta)\right]^2 +$$
$$\left[\int ds \sin(2my\cos\theta - ms\sin\theta)\right]^2 .$$

[1] A more general expression including both absorption and retardation is $I = C^2 + S^2$, where

$$C = \int f(s)\cos(\omega - ps)ds \quad \text{and} \quad S = \int f(s)\sin(\omega - ps)ds .$$

But if we replace $2my \cos\theta$ by ω and $m \sin\theta$ by p, this is the same as (9), save that p is now single-valued. Hence all the preceding results except those relating to the form of the furrow may be applied to the case of smooth surfaces such as mirrors and prisms.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO,
October 1903.

SOLAR PROMINENCES AND TERRESTRIAL MAGNETISM.

By A. L. CORTIE, S.J.

RECENT researches with regard to the relation which exists between Sun-spots and terrestrial magnetism, while confirming the general connection between the two classes of phenomena established by Mr. Ellis and Canon Spée, have emphasized the conclusion that this relation is not one of efficient cause and effect. Such is the outcome of the comparison of the magnetic storms recorded at Stonyhurst and the greater solar spots made by Father Sidgreaves, and the analysis of the Kew magnetograms for the period of eleven years recently concluded and published by Dr. Chree. The same conclusion was set forth in a former paper by the writer on "Minimum Sun-spots and Terrestrial Magnetism,"[1] which contained a detailed study of the individual Sun-spots and the magnetic curves for the period 1899–1901. There is an undoubted general connection, not a mere series of coincidences, between Sun-spots and magnetic storms, but the supposed direct action of Sun-spots is in individual cases so capricious and irregular that it is doubtful whether they can be regarded as even instrumental causes of the magnetic storms. The existence rather of a common cause is indicated by the observations, which acts sometimes on the Sun causing spots, sometimes on the magnets, and sometimes on both Sun and magnets.

But the question has been raised more than once, and recently in a paper on "The Relation between Solar Prominences and Terrestrial Magnetism" by Sir Norman and Dr. Lockyer,[2] whether solar prominences may not supply the places of spots in those cases in which great or active magnetic storms occur without the presence of any spot. The curves presented in the paper show, as was to be expected, that at times of Sun-spot maximum activity the disturbance is general, and extends even

[1] ASTROPHYSICAL JOURNAL, 16, 203–210, 1902. [2] *Proc. R. S.*, 71, 244, 1903.

to prominences in high solar latitudes, the curve of annual frequency for such prominences bearing a very striking similarity to that of great magnetic disturbances; that is, disturbances in which the range of the declination magnet is greater than 60′, as deduced from Mr. Ellis's observations. But we would submit that this fact does not show that solar prominences, and in particular those in high solar latitudes, possess any special virtue or efficacy in the causation of magnetic storms not possessed by Sun-spots, and does not warrant, of itself, the conclusion that prominences may cause magnetic storms in the absence of Sun-spots. Rather is it not only one manifestation of the increased solar external activity which is displayed in greater and more numerous spots, and increase in number, latitude, and extent of both prominences and coronal streamers?

Observations of the prominences and chromosphere were taken at Stonyhurst during the years 1880–92, and in the *Observatory* for March 1893 will be found the curve of profile area of the prominences, compared with the curve of Sun-spot surface deduced from the Greenwich records for the whole period of the observations. The accord of the two curves is very striking, and corroborates what was known before, that the activity of the prominences is in direct proportion to the activity of Sun-spots. Now, both Canon Spée and Mr. Ellis have shown the complete similarity of the curves for Sun-spot frequency and magnetic declination over long periods. Mr. Ellis, too, has demonstrated that the Sun-spot curve is also identical with the curve of frequency of magnetic storms. Hence it follows that the accord of the prominence curves and the magnetic curves is no proof of the causality of prominences with regard to magnetic storms. If it were possible to make a curve of isolated solar prominences and then show that such a curve, to the exclusion of the Sun-spot curve, corresponded to the curve of magnetic storm frequency, then the argument would have a greater force. But, except that they occur in latitude in which Sun-spots are not found, solar prominences are not phenomena isolated from spots and faculæ. Nay more, it is only one class of solar prominences that are even separated in latitude from Sun-spots, namely, the relatively

quiet, as distinguished from the eruptive, prominences. For, with regard to this latter class, they are almost entirely confined to the Sun-spot zones, and as a general rule accompany outbursts of spots or faculæ. To take one period as an instance, namely, that treated of by Professor Wolfer in the *Publicationen der Sternwarte des Eidg. Polytechnikums zu Zürich* (Band III). Of 315 metallic or eruptive prominences included in the lists discussed for the years 1893–95, 274, or 86 per cent., were connected with spots; 27, or 9 per cent., with faculæ; while only 14, or 5 per cent., were independent of spots and faculæ. Father Fenyi, too, in the publications from the Haynald Observatory at Kalocsa, calls attention to the same fact, though the converse is not true, as spots and faculæ have frequently no accompanying eruptive prominences.

Nevertheless, to quote from Sir Norman and Dr. Lockyer's paper: "The magnitude of magnetic storms appears to vary according to the particular position as to latitude of the prominence on the Sun's disk. The nearer the poles (either north or south) the prominence occurs, the greater the magnetic storm, and these are the regions where no spots exist." As a contribution to the elucidation of this point, it was determined to study in detail all the more noteworthy prominences of some selected period, and to see whether they exercised any effect upon the magnets independently of the spots and faculæ. Obviously a minimum period of solar activity is the best for this purpose, first because in a maximum period it is impossible to unravel the various outbursts of spots, faculæ, and prominences, and apportion to them their proper magnetic storms; and, secondly, because, if in a time of minimum Sun-spots an extraordinary prominence appears, and prominences are to be supposed, for the sake of argument, to be the cause of magnetic storms, it ought to be accompanied by at least an active movement of the magnets. It was intended to study the four years 1887–90, for which an excellent series of observations with detailed descriptions of the greater outbursts has been published by Father Fenyi at Kalocsa, but as the work of comparison is rather laborious, the results for only two years have been gathered together in the following table,

which contains details with regard to forty-eight prominences. The principle of selection has been to admit such as were especially noteworthy either on account of their eruptive nature, or the displacements of their spectrum lines, or were of a height equal to or greater than 100″. Their character as eruptive or non-eruptive is indicated in the third column, their maximum height or displacement and their heliographic co-ordinates being given in other columns of the table. In the columns devoted to the readings of the curves of the declination magnet, the first gives the maximum diurnal range for the date on which the prominence was observed, the second the intensity of the storm, if any, observed within three days before or after the prominence was seen, and the third the date on which the magnetic storm occurred. In Mr. Ellis's discussion of Sun-spot frequency and magnetic storms an active storm on the declination magnet is one in which the range exceeds 30′ and is less than 60′, while a great storm is one in which the range exceeds 60′. We have used the corresponding numbers, 3, 4, etc., as indications of intensity. In the last column are given the Greenwich numbers for such spot-groups as have been identified as connected with any of the prominences.

Of these forty-eight prominences twenty-nine were either immediately associated with spots and faculæ, or occurred in the latitudes frequented by Sun-spots. This number includes all the so-called metallic prominences. During the year 1887 there were only two noteworthy Sun-spot groups on the Sun; those numbered thirty-two and thirty-four in the Stonyhurst series. The first was born on the visible disk on May 14, and after five rotations disappeared, also while on the visible disk, on August 4, but in the next rotation there was an outburst in such close proximity to its position as evidently to have formed part of the same disturbance. This one disturbance covers the groups numbered 1978, 1987, 1992, 1998, 2006, and 2010 at Greenwich. Its mean longitude was 92° and its mean latitude −8°. At its first appearance it crossed the western limb on May 22, and the table shows that there was a fine metallic prominence seen on the limb, which also showed a displacement corresponding to no less

NOTEWORTHY PROMINENCES, 1887–88.

No.	Date	Character	Maximum Height	Maximum Displacement	Heliographic Longitude	Heliographic Latitude	Magnetic Declination			Greenwich Nos. Spots or Faculæ
							Diurnal Range	Storm		
								Intensity	Date	
	1887			km						
1	May 22	M.	49″	426	90°	− 8°W	10′0	0	.....	1978
2	June 6	H.	...	135	75	−12 E	7.0	0	.....	1987 Faculæ
3	8	M.	24	...	48	− 1 E	3.1	0	.....	1988
4	23	H.	137	...	30	+31 W	5.9	3	June 20	.....
5	25	H.	20	70	180	− 7 E	7.5	0	.....	1991?
6	26	M.	22	...	169	− 8 E	8.0	0	.....	1991
7	27	H.	105	...	335	−11 W	8.0	0	.....	.....
8	July 1	H.	165	320	275	− 6 W	8.8	0	.....	Faculæ?
9	9	M.	51	85	175	−13 W	15.8	0	.....	1991
10	16	H.	113	...	265	−47 E	13.7	0	.....	.....
11	16	H.	122	...	85	+ 8 W	13.7	0	.....	Faculæ?
12	18	M.	...	...	58	+ 2 W	20.7	0	.....	1994
13	19	H.	125	...	45	+22 W	11.0	0	.....	.....
14	20	H.	119	...	210	−17 E	9.2	0	.....	.....
15	29	H.	246	...	270	+47 W	9.2	3	Aug. 1	.....
16	30	M.	...	...	259	−12 W	9.0	3	Aug. 2	1995 Faculæ
17	30	M.	...	...	79	− 6 E	9.0	3	Aug. 2	1999
18	30	H.	92	284	280	−10 W	9.0	3	Aug. 2	1995
19	Aug. 12	M.	...	...	267	− 6 E	6.3	0	.....	2001
20	25	M.	...	...	97	− 7 E	12.8	0	.....	2006 Faculæ
21	29	M.	...	...	223	+ 1 W	31.3	3	Aug. 28	2005
22	29	H.	106	...	40	+21 E	31.3	3	Aug. 29	.....
23	Sept. 15	M.	...	...	0	− 7 W	16.1	0	.....	Faculæ
24	18	H.	...	184	320	−45 W	11.0	0	.....	.....
25	Oct. 12	H.	108	...	0	+46 W	16.9	0	.....	.....
	1888									
26	Jan. 10	H.	...	85	76	− 8 E	3.0	3	Jan. 7	2029
27	Feb. 1	H.	...	412	326	−10 W	1.0	0	.....	.....
28	7	H.	126	...	68	+18 E	1.0	0	.....	.....
29	17	M.	...	133	295	− 2 E	....	0	.....	2035
30	March 6	H.	...	113	58	+11 E	9.0	3	March 9	Faculæ
31	7	H.	...	57	224	−49 W	15.6	3	March 9	.....
32	14	M.	24	...	311	− 8 E	3.1	0	.....	2039
33	20	M.	...	150	235	− 9 E	3.0	0	.....	2041
34	April 6	H.	100	...	179	−17 W	13.3	3	April 4	Faculæ?
35	16	H.	101	...	238	−37 E	12.5	3	April 13	.....
36	29	M.	...	...	246	− 8 W	11.0	0	.....	2051
37	May 11	H.	...	102	266	− 6 E	0.0	3	May 7	2052?
38	June 7	H.	...	120	271	+58 E	11.2	4	June 3	.....
39	July 13	H.	127	...	265	−22 W	9.5	0	.....	2059 Faculæ
40	28	H.	...	145	314	− 7 E	15.5	0	.....	2061
41	Aug. 22	H.	100	...	325	+20 E	6.5	0	.....	.....
42	Sept. 5	M.	151	...	147	−10 E	2.0	0	.....	2070
43	6	M.	158	297	147	−10 E	2.0	0	.....	2070
44	10	M.	38	...	272	− 8 W	4.0	0	.....	2066
45	Oct. 13	H.	101	...	197	−47 W	14.9	0	.....	.....
46	25	H.	152	...	220	+34 E	1.5	0	.....	.....
47	Nov. 26	M.	27	...	159	− 9 E	16.1	0	.....	2076
48	30	H	120	...	104	−23 E	11.8	0	.....	.....

a velocity than 426 km per second in the line of sight. The maximum diurnal range on the day was 10′, and there was no disturbance of the magnets that could be classed as active. The other prominences of the table connected with this spot-group are No. 2, seen on its second entrance on the disk, again without magnetic storm, and No. 20, also without storm of any kind, the maximum diurnal range being 7′.0 and 12′.8 respectively. The first magnetic storm which occurs at any date near to that on which a prominence was seen was on June 20, a fine isolated prominence being observed on the twenty-third. If this is to be considered as a case of connection of isolated prominences and magnetic storm, the next big prominence, possibly connected with a facula, but having no connection with spots, beyond its occurrence in the spot-zones, is directly opposed to any such connection. It was a splendid prominence which changed its form very rapidly. In eleven minutes it rose from 60″ to 163″, with a corresponding line-of-sight displacement of the C line indicating a velocity of 320 km per second. In about seventeen minutes after attaining its maximum height it had practically disappeared. More than this, as the table shows, there were many fine prominences observed by Father Fenyi in July, and Canon Spée's tables deduced from observations at Rome and Palermo also show an increase in number of prominences during this month. Yet the magnets were very quiet, and remarkably so as the end of the month approached. However, at the end of the month, on the twenty-ninth and thirtieth, four fine prominences were observed, three connected with Sun-spot groups, one of which, that numbered eighteen, was active all day, and one an isolated hydrogen prominence in higher latitudes. The magnets were actively disturbed on August 1 and 2. Is the connection here, if any, with the spots or with the one isolated prominence? Again we have storms on the magnets on August 28 and 29, and on the twenty-ninth two fine prominences are observed, one connected with a Sun-spot, the other isolated. Which is to have the credit of the magnetic storm? On January 7 there was a magnetic storm, and on January 10 a fine prominence connected with group 2029 of the Greenwich

series, while the magnetic storm of March 9 may be credited to the isolated prominence of March 7, or to that connected with the faculæ of March 6. The most telling cases, however, in favor of isolated prominences affecting the magnets are those numbered 35 and 38 in the list. On April 16 the spotted area of the Sun was extremely small, and so too was the area covered by faculæ; therefore the storm of the thirteenth may possibly be connected with the prominence of the sixteenth. Similar remarks apply to the prominence of June 7, on which day there were no spots on the Sun, and a moderate area of faculæ, and the magnetic storm of June 3, allowing four days' grace, for a prominence which at the time of the storm must have been on the Sun's invisible hemisphere. But the most active prominence of the year was observed on September 5 and 6, in connection, however, with a spot-group of moderate size, No. 2070 in the Greenwich series, and which, according to the Stonyhurst records, was one of three distinct formations occurring in the same position. Its presence is also shown in the table in No. 47, connected with spot-group 2076. There is not the slightest corresponding movement of any sort on the magnets in connection with the big September prominences, the curves being straight lines, for several days before and after, and only a slight movement in connection with the November prominence. Finally on October 13 and 25, and November 30, 1888, there occur cases of isolated prominences higher than that of April 16, with which we connected the magnetic storm of April 13. The magnets, however, are quiet, the greatest range being 16'.1 on November 26. It follows, therefore, that in the period discussed, which, though limited in extent, contains many fine prominences, no conclusion can be drawn as to the relationship of prominences and magnetic storms, seeing that in the cases of both eruptive and non-eruptive prominences, though some few seem to be connected with magnetic storms, others equally high and equally active are totally unconnected with any extraordinary movements of the needles.

STONYHURST COLLEGE OBSERVATORY,
September 1903.

THE SPECTRUM OF LIGHTNING.

By PHILIP FOX.

SPECTRA of lightning flashes were photographed on the nights of July 16, 17, August 3, and October 6 by means of an objective-prism spectroscope. The camera lens was of 35mm aperture and 274mm focal length. The 30° flint-glass prism was from a large spectroscope loaned to the Observatory by the Massachusetts Institute of Technology. Of the dozen plates showing spectra the best was obtained on August 3 at 9^h 10^m in the evening, and shows three flashes. They are reproduced herewith (Plate IX). The original negative is on a Cramer Isochromatic plate.

Vogel and Lohse,[1] and Schuster[2] identified certain lines in the lightning spectrum with lines of the spark spectrum of air. Following the suggestion of their work, the spectroscope was provided with a slit and collimator, and the air spectrum, obtained by passing a spark between silver terminals, was photographed. The agreement in position of the lines is shown in Fig. 1. The wave-lengths of the lines, with their identification, are given in the table, which also shows the wave-lengths of lines as determined by Vogel and Lohse, Schuster, and E. C. Pickering.[3] Lieutenant Herschel[4] also identified some of the lightning lines, the principal one probably being that at λ5003 of the present determination.

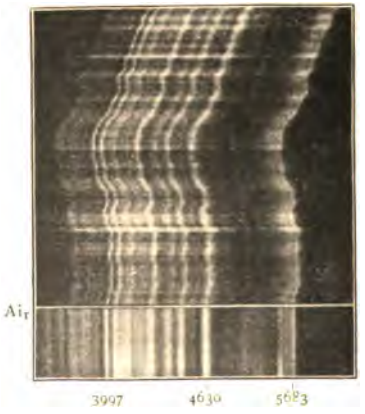


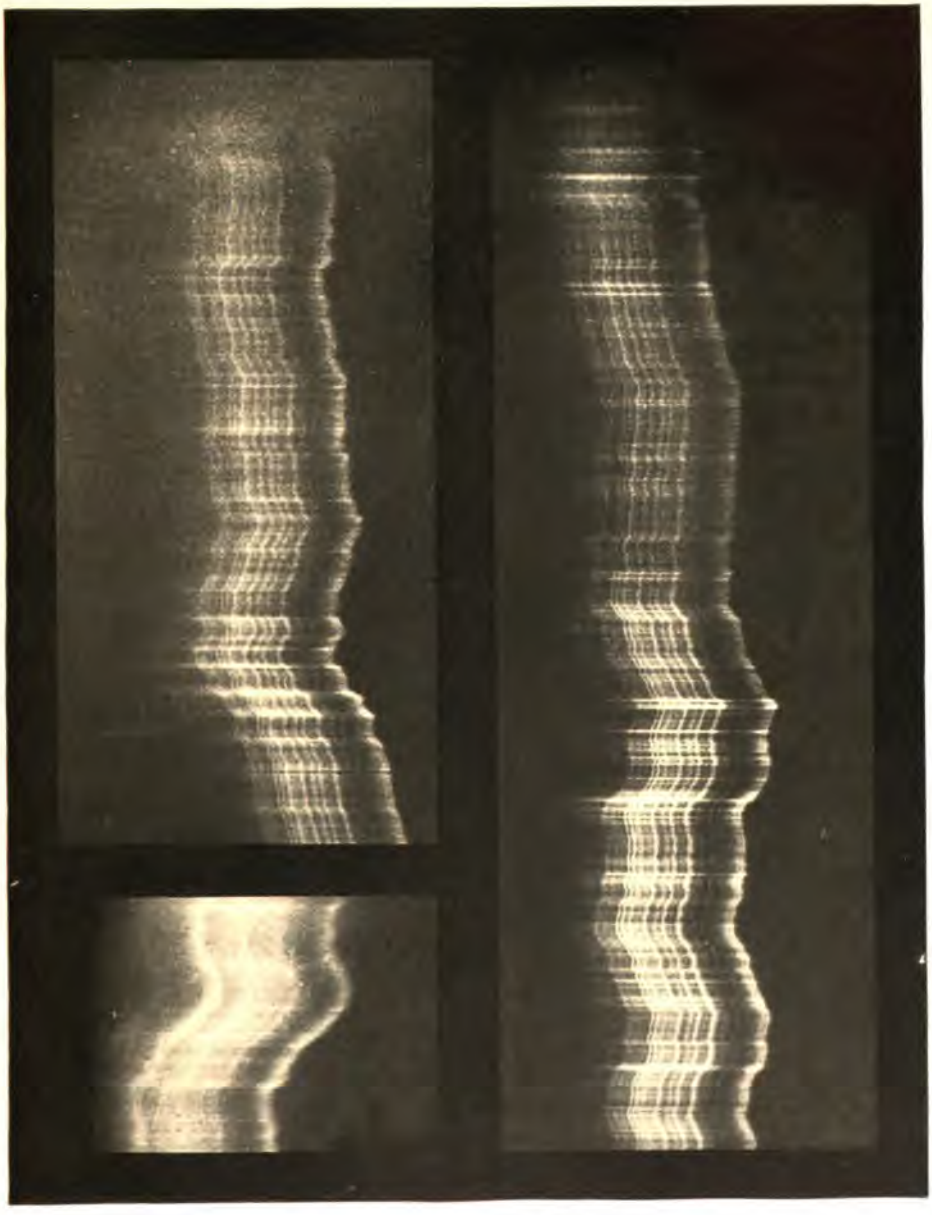
FIG. 1.—Spectrum of lightning and of air.

[1] VOGEL, *Pogg. Ann.*, 143, 653–654, 1871.

[2] *Phil. Mag.*, (5) 7, 316–321, 1879.

[3] *Harvard College Observatory Circular* No. 62. [4] *Proc. R. S.*, 17, 61, 1869.

PLATE IX.



Spectrum of Lightning Flashes.

TABLE.

Measured Wave-Length		Character	Air Spectrum (Neovius)	Vogel	Schuster	Pickering
High Power	Low Power					
			3839.8			
			3843.1			
3848	3838	Faint, broad	3845.3			
			3850.6			
			3857.2			
			3882.6			3881
3898	3890	Faint, double,	3912.2			
	3915	at times	3919.2			
			3945.3			
3950	3943	Very faint	3947.5			
			3954.6			
			3956.1			3956 } *He*
3997	3997	Sharp	3995.2			3998 }
4041.5 s	4041,5 s	Sharp	4041.5			4046
			4072.4			
			4076.3			
4074	4077	Poor } coalesce at times	4097.3			
4106	4105	Fair } coalesce at times	4103.4			4102 *Hδ*
			4105.2			
			4119.4			
			4152.0			4147
4143 v. edge		Broad,	4153.7			
4165 center	4154	double at	4176.7			
4183 r. edge	4183	times	4180.3			
			4185.8			4187
			4228.9			
4236	4238	Strong	4237.0			4222
			4242.3			4263
4349	4359	Broad, fair	4349.4			4341
			4415.0			
4439	4439	Strong	4417.3			
			4447.3			
4529	4535	Broad, hazy	4530.3			4519
			4596.6			
	4603. v.edge		to	4583 } band		
4630.7 s	4630.7 center	Broad, strong	4601.9	4673 }		
	4660. r. edge		maximum			
			4650.0			4643
4790	4786	Very faint	4788.5			4754
4858	4842	Very faint	4861.0	4860		4861 *Hβ*
			5002.7	5002	5002	4940
5003.7 s	5003.7 s	Good	5005.7			5022
5175	5156	Very faint	5180	5184	5181	5173
			includes			
			5453.8		5260	
			5462.8			
5306 v. edge		Broad band,	5479.8	5341	5334	
5600 max.		red edge	5490.6			5595
5683 r. edge		very strong	5535.2		5592	
			5667.1			
			5679.8		5681	

The letter s indicates the lines used as standards in deriving the formula for wave-lengths. The higher power used in the microscope magnified about fifteen times, the lower about three times.

In studying the spectra some curious facts were observed. The relative intensity of some lines with regard to their fellows is found to vary in different parts of the spectrum. In the first flash, the line at λ4349 diminishes rapidly in relative intensity from the cloud to the ground. The line, a combination of λ4074 and λ4106, at the top of the flash is one of the heaviest lines, being much stronger than its two neighbors to the left, λ4041.5 and λ3997. Toward the ground, however, it has lost greatly in relative intensity, having been surpassed by both λ4041.5 and λ3997. These two have increased from very faint lines to rank among the strongest.

In comparing the first flash with the second, it is found that in the second these same changes do not exist. The only case of change in relative intensity is shown by the line at λ4439, which increases slightly toward the ground.

The third flash, which was at the very edge of the plate, shows the line at λ3848 very strong, while in the other flashes it is comparatively faint. This flash shows a line far to the violet of λ3848.

This work has been conducted under the direction of Mr. Hale. The writer is indebted to Mr. Ellerman for assistance in the laboratory.

YERKES OBSERVATORY,
October 12, 1903.

Minor Contributions and Notes.

PHOTOGRAPHIC SPECTRUM OF *NOVA GEMINORUM.*[1]

At the suggestion of Director Campbell, I obtained the spectrum of the *Nova* with the Crossley reflector, using the small slitless spectrograph. Six negatives were obtained on the night of April 2, with exposures ranging from 10 seconds to 19 minutes. Thirty seconds showed the stronger lines very faintly, and five minutes gave a good negative. The accompanying reproductions, Figs. 2 and 3, are chiefly from the negative having an exposure of ten minutes, although all the plates were made use of in determining the relative intensities. Fig. 2 shows the spectrum as a negative.

The plates used were ordinary commercial dry plates, sensitive only to blue and violet radiations. The recorded spectrum of *Nova Geminorum* consists of bright lines and bands superposed on a continuous spectrum, and extends from $H\beta$ to $\lambda 335$. The general appearance of the spectrum resembles somewhat the April 1901 spectrum of *Nova Persei* obtained by Campbell and Wright with the Mills spectrograph, in the region where the two instruments give comparable results. As no observations of the ultra-violet spectrum of *Nova Persei* were secured before September 1901, when the star had become a nebula, we do not know the early history of the lines at $\lambda 339$ and $\lambda 346$, photographed by Mr. Stebbins. These lines are certainly not yet developed in the case of *Nova Geminorum*, although there is a decided strengthening in the spectrum at about this point. There is very little similarity in the spectra of *Nova Persei* in September 1901 and *Nova Geminorum* in the region above $H\delta$. Lines occupying approximately the positions of $H\epsilon$ and $H\zeta$, as well as those at $\lambda\lambda 339$ and 346, are the strongest lines in *Nova Persei*, but are indicated in *Nova Geminorum* only by slight strengthenings in the continuous spectrum. $H\beta$ and $H\delta$ are strong in the recent *Nova*, but very weak in the later spectrum of *Nova Persei*. The chief nebular line at $\lambda 501$ is not shown in the spectrum of *Nova Geminorum*, but was conspicuous in *Nova Persei*.

It is altogether probable that these differences between the spectrum of *Nova Geminorum* and the later spectrum of *Nova Persei* are

[1] From *Lick Observatory Bulletin* No. 37.

due to the different stages of development of the two stars. As *Nova Geminorum* assumes the nebular state, we may expect its ultra-violet spectrum to conform more and more to that shown by *Nova Persei*.

Following are the positions of the lines and maxima in the spectrum determined from three of the plates:

1903, April 2				
No. 8 5 m.	No. 9 10 m.	No. 10 19 m.		Description
λ	λ	λ		
486	486	486	*Hβ*	Strong. Narrow
462	462	463		Very strong. Broad
445	446	446		Faint
434	434	434	*Hγ*	Very strong
410	410	410	*Hδ*	" "
397	397	397	*Hε*	Moderately strong
389	389	390	*Hζ*	Faint
	384			Max. of band *Hη*
374	374	372		" " *Hκ*
350	350	352		" "
335	335	333		End of spectrum

Spectrograms were secured also on April 3, 5, 6, and 8. A comparison of those taken on the 2d and 8th show changes in the character of the spectrum, in the interval of six days. The most noticeable change is in the ultra-violet, where the continuous spectrum has become weaker and the bands at λλ 350, 374, and 384, more conspicuous in consequence. There are indications also of the development of the lines at λλ 339 and 346. Below *Hδ* there seems to be little or no change in the continuous spectrum. *Hβ* has become weaker, however, and there are traces of radiations in the region of the chief nebular line.

The spectrum was examined visually on all the nights when spectrograms were secured. The *Hα* line was always very conspicuous. A brightening was observed in the yellow at about the position of the sodium lines.

In this connection it should be said that the dispersion with this spectrograph is so small, the linear distance between *Hβ* and λ 335 being only 3½ mm, that close lines cannot be separated with it.

Between April 1 and 8 the *Nova* decreased slightly in brightness. On the latter date it was recorded as fully as bright as the 8.6 magnitude star preceding, with which it was compared each night.

C. D. PERRINE.

APRIL 15, 1903.

PLATE X.

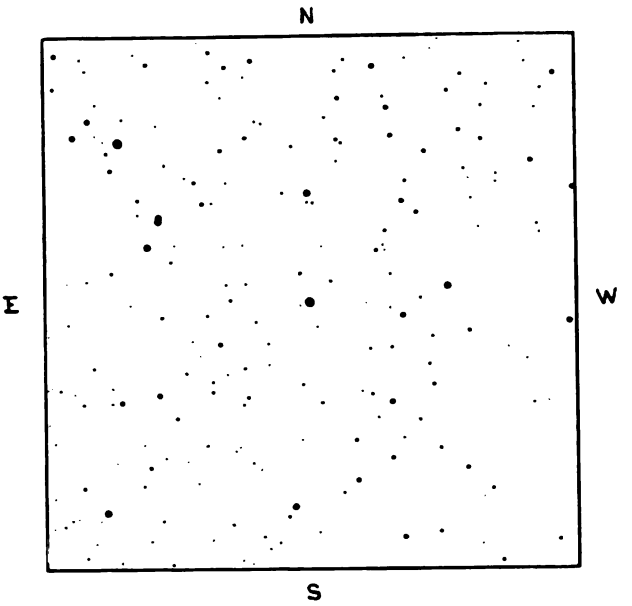


FIG. 1.— Region about *Nova Geminorum*.

FIG. 2.— The Photographic Spectrum of *Nova Geminorum* (Negative). April 2, 1903.

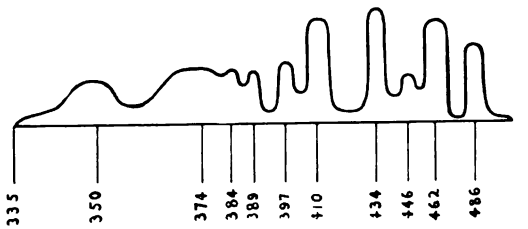


FIG. 3.— Intensity Curve.

THE SPECTRUM OF *NOVA GEMINORUM*.[1]

THE first opportunity to observe the spectrum of *Nova Geminorum* was on the evening of April 1, when its magnitude was estimated at 8.5. Only visual observations were attempted, by means of Spectrograph I, consisting of a single light flint prism in connection with the collimator and camera sections of the Mills Spectrograph.[2] These visual observations and later photographs showed that the spectrum was intermediate between that of *Nova Persei* in April 1901 and in the following July. It consists mainly of isolated bright bands, many of which coincide approximately with the lines of hydrogen. The brightest band was coincident with $H\beta$. To the less refrangible side of it there was another strong band, and a stretch of faint continuous spectrum extending just beyond the position of the D lines. There were several faint maxima near the red end of this continuous spectrum. On another night we found that one of these coincided approximately with the D lines; and the bright $H\alpha$, overlooked on the first evening, was easily visible.

To the violet of the $H\beta$ band there appeared another band, fainter and broader, and farther up there was a faint band about the position of $H\gamma$. Nothing could be seen above this.

Following is the list of good photographs secured to date:

Number	Date, G. M. T.	Exposure	Kind of Plate
2717*A*.....	April 2, $17^h\ 18^m$	$4^h\ 0^m$	Cramer Crown
2818*D*.....	5, 17 26	3 46	Cramer Iso. Inst.
2722*B*.....	6, 17 21	3 55	Cramer Iso. Inst.
2731*A*.....	8, 17 33	4 40	Cramer Iso. Inst.
2737*D*.....	22, 17 12	3 0	Cramer Crown
2757*A*.....	May 4, 17 9	2 36	Cramer Crown

The first photograph was secured on the evening of April 2. The auxiliary correcting lens brought all light in the $H\gamma$ region in focus on the slit, and the adjustments of the instrument were such that the camera was in fair focus from $\lambda 5000$ to the ultra-violet. The plate substantiated our results of the previous night in the region common to both visual and photographic observation. Fig. 4 is a reproduction of this plate, and Fig. 5 is its intensity curve. Table I gives a description of the details of the bands. The wave-lengths are calculated by means of the Hartmann-Cornu formula, the constants of which are determined

[1] From *Lick Observatory Bulletin* No. 37. [2] See *L. O. Bulletin*, 1, No. 8, p. 46.

from certain lines in the comparison spectrum of iron. The wave-lengths are corrected for the orbital and the diurnal motions of the Earth. The effect of curvature of the lines is negligible. They are not corrected for the velocity in the line of sight of the star itself, for we have no means of determining that velocity with certainty. The spectrum seems at first sight to resemble that of *Nova Persei* in July 1901,[1] but there are great differences in the details. The several bands in the present case have not the uniformity of structure that characterized those of *Nova Persei*, nor are the maxima and minima by any means so sharp and definite. Indeed, it is quite possible that a number of the details shown in Fig. 5 and Table I are due to accidental arrangements of the silver grains.

More important still is the apparently total absence of certain bands which were present in the spectrum of *Nova Persei*. One of these is that corresponding to the nebular line at λ 3869, which in *Nova Persei* was considerably stronger than the *Hδ* band. No trace of it appears on the present plate.

The band farthest to the violet has its maximum at λ 3987. Here again is a decided difference: in *Nova Persei* this band was very strong, and was displaced to the violet of the corresponding hydrogen line *Hε* more than the other bands corresponding with hydrogen lines; and it was shown[2] that the preponderating radiation was not *Hε*, but a companion nebular line, previously unobserved. In *Nova Geminorum* this band is quite faint, and the maximum is 17 tenth-meters to the red of *Hε*.

In *Nova Persei* the *Hγ* band was superimposed on another corresponding to the nebular line λ 4363. In *Nova Geminorum*, though the corresponding band shows greater evidences of complexity than any other in the spectrum, λ 4363 would fall almost wholly outside of it, so that no radiations corresponding to this line exist with an intensity at all comparable with those due to *Hγ*.

There is in *Nova Geminorum* a band with maximum at λ 4625, which may perhaps be said to correspond with the λ 4643 band in *Nova Persei*, but there is none corresponding to the complex band λ 4669—λ 4746, which was apparently formed of two or more superposed bands. In *Nova Geminorum* the nebular line λ 4686 would fall just on the edge of the band that is present, and the helium line λ 4713 would lie entirely outside of it. No trace of the two nebular lines λ 4959 and λ 5007 can be seen on this plate. No doubt this is partially due to the lower sen-

[1] See *L. O. Bulletin*, 1, No. 8. [2] *L. O. Bulletin*, 1, No. 8, p. 54.

sitiveness of the plate at this region, and to the fact that much of the light is lost at the slit, for a later negative showed a very faint band at λ 5007 when the slit was properly focused for this light and an isochromatic plate used. It may be concluded, however, that with this exception the star showed no certain traces of the so-called "nebular" lines in the early part of April.

The positions of the bands are interesting. The strong band between *Hβ* and *Hα* has its maximum 18 tenth-meters to the violet of what we may call the corresponding band in the July spectrum of *Nova Persei*. The band corresponding to *Hε* has its maximum on the other hand 17 tenth-meters to the red of the position of the hydrogen line. Hartmann[1] states that on March 31 the middle of the *Hβ* and *Hγ* bands were both displaced 8 Ångström units toward the red from the positions of the corresponding hydrogen lines. Our plate shows no such displacement; *Hβ* appears only 1½ and *Hγ* about ½ of a tenth-meter too far to the red, and these amounts are hardly greater than the probable error. The discordance between Hartmann's results and ours may be explained, of course, by a real change in the spectrum between March 31 and April 2; but more probably it is due to the fact that Hartmann's plate was underexposed. The limits he assigns to the bands fall well within ours in each case, and in each of the bands the more intense part is toward the red of the middle. These two facts would certainly tend to make the middle of the band appear more to the red on his plate than on ours.

Messrs. Frost and Adams, discussing a plate taken March 28[2] describe *Hγ* as a very faint band merging into a brighter band which extends from λ 4347 to λ 4371. As indicated in Figs. 4 and 5, our plate shows some evidence of a composite character in what we have called the *Hγ* band, with much greater intensity near the red border than near the violet; but, as noted above, the middle of the whole band coincides very closely with the normal position for *Hγ*, so that we have thought it best to speak of it as a single band showing considerable detail in structure. In any case, however, the limit λ 4371 falls outside the limits of the band on our plate, in spite of the fact that this part of the spectrum is doubtless much stronger on our plate than on theirs. There seems to be some evidence, therefore, of a perceptible change in the *Hγ* region between March 28 and April 2.

[1] *Astronomische Nachrichten*, 161, 324, 1903.

[2] ASTROPHYSICAL JOURNAL, 17, 304, 1903.

A second photograph of the same range of spectrum was taken April 22. It is reproduced in Fig. 6, with the corresponding intensity curve shown in Fig. 7. Table II gives a description of the details. As in the case of the earlier negative, many of the details plotted in

TABLE I,

April 2 2717 *A*	Description	
λ 3974	Beginning	Faint band
3987	Maximum	
3992	End	
}	Faint continuous spectrum between these points	
4064	Slight increase in brightness	
4078	Beginning of stronger part of band	*Hδ* band
4081	Maximum	
4083	Minimum	
4094	Suspect a minimum	
4098	Maximum	
4100	Suspect a minimum	
4102	Maximum	
4103.0	Minimum rather sharp	
4107	Maximum	
4110.9	Minimum	
4116	Maximum	
4116.8	Minimum	
4120	Maximum	
4124	End of stronger part	
4144	End of weaker part	
4317	Beginning of bright band	*Hγ* band
4327	Maximum	
4331.7	Minimum	
4334	Maximum	
4336.8	Minimum	
4340	Maximum, not well defined	
4342.4	Minimum	
4347	Maximum	
4350.2	Minimum	
4358	Maximum	
4365	End of band	
}	Continuous spectrum between these points	
4449	Beginning	Faint band
4464	Maximum	
4492	End	
4570	Faint beginning of band	Strong band
4625	Principal maximum	
4646.2	Suspect narrow bright line or maximum	
4651.4	Suspect narrow bright line or maximum	
4658.9	Suspect narrow bright line or maximum	
4695	End of band	
4839	Beginning	*Hβ* band
4866	Maximum	
4887	End	

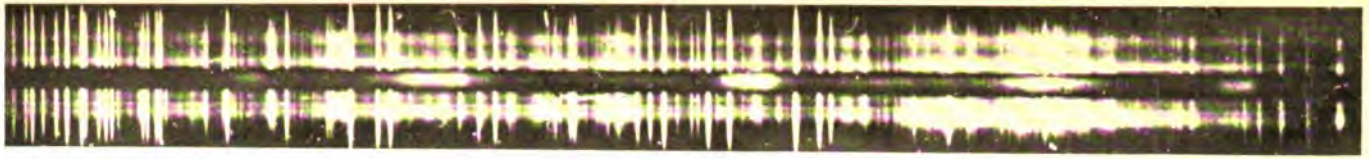


FIG. 4.—*Nova Geminorum*, enlarged ninefold. 1903, April 2.

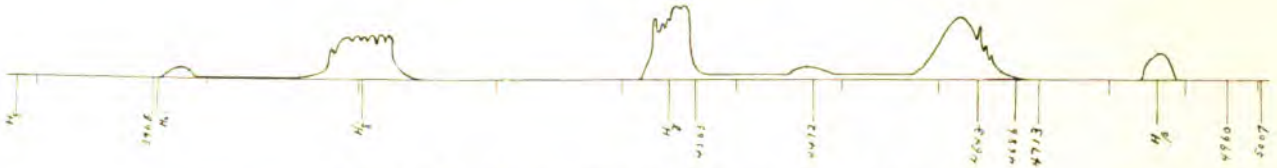


FIG. 5.—*Nova Geminorum*, Intensity Curve. 1903, April 2.

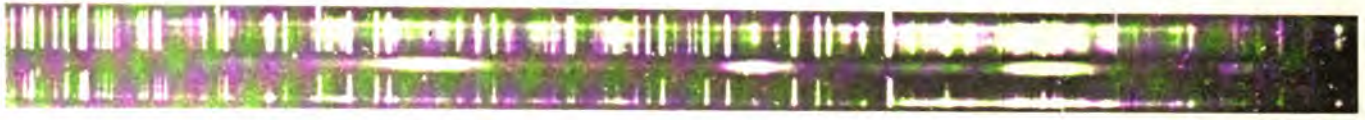


FIG. 6.—*Nova Geminorum*, enlarged ninefold. 1903, April 22.

FIG. 7.—*Nova Geminorum*, Intensity Curve. 1903, April 22.

Fig. 7 and tabulated in Table II may be false effects due to chance grouping of the silver grains, though in each case the plate was measured under low magnification to avoid as far as possible errors of this kind.

TABLE II.

April 22 λ 2727 *D*	Description	
3959	Beginning of very faint continuous spectrum	
3982	Very faint maximum	
4021	Beginning of faint part of band	*Hδ* band
4046	Maximum	
4052	Minimum	
4078	Suspect a minimum	
4080	Maximum	
4082.1	Minimum	
4084	Maximum	
4086.9	Minimum	
4090	Maximum	
4093.0	Minimum	
4095	Maximum	
4107	Very broad minimum	
4116	Broad maximum	
4126	End of brighter part	
4145	End of fainter part	
4322	Beginning of bright band	*Hγ* band
4326	Maximum	
4329.4	Minimum	
4334	Maximum	
4337.3	Minimum	
4341	Maximum	
4342.4	Minimum	
4346	Maximum	
4348.9	Minimum	
4352	Maximum	
4355.2	Minimum	
4358	Maximum	
4364	End of main part of band	
4378	Faint broad maximum	Very faint band in continuous spectrum, extending from λ 4392 to λ 4584
4382	End of stronger part of auxiliary band	
4392	Suspect very faint maximum	
4454	Beginning	
4536	Maximum	
4557	End	
4584	Beginning	Broad strong band
4630	Broad maximum	
4642	Broad minimum	
4656	Broad maximum	
4714	End	
4838	Beginning	*Hβ* band
4843	Suspect a minimum	
4877	Suspect a minimum	
4884	End	

Even a casual examination showed that pronounced changes had taken place in some of the bands. In regard to the *Hβ* band little can be said, for it is too weak on both plates to be sure of much structural detail. The middle of the strong band between *Hβ* and *Hγ* had been moved about 17 tenth-meters to the red, making it more nearly coincide with the band observed at about this place in *Nova Persei* in the summer of 1901. It was certainly changed in detail also, for it showed two pronounced broad maxima on the later plate, as against one such maximum and two or three suspected narrow maxima on the earlier date. The faint band to the violet of this was also

TABLE III

April 5 2718 *D*	April 6 2722 *B*	April 8 2731 *A*	Description	
4838	4836	....	Beginning of band	*Hγ* band
4843	....	....	Dark line?	
4852	....	....	Slight maximum	
4859.7	4859.9	....	Minimum (or dark line?)	
4866	4865	....	Slight maximum	
4870	....	....	Slight minimum or dark line	
4874	....	....	Slight minimum or dark line	
4877	4878	....	Maximum of band. Exact position uncertain	
4886	4886	....	End of band	
....	4987	....	First trace of very faint band	λ 5007
....	5015	....	Minimum?	
....	5022	....	Beginning of second portion	
....	5030	....	End of band	
....	....	5409	First certain trace of continuous spectrum	
....	....	5480	Very slight maximum	
....	....	5493	Well defined minimum	
....	....	5516	Slight maximum suspected	
....	....	5527	Minimum?	
....	....	5557	Maximum of band	
....	....	5579	Slight minimum	
....	....	5589	Slight maximum	
....	....	5634	Faintest portion between bands	
....	....	5653	Dark line suspected. May be chance arrangement of silver grains	
....	....	5671	Maximum of band	
....	....	5678	Minimum	
....	....	5687	Maximum	
....	....	5700	End of brighter portion of band	
....	....	5708	End of band	
....	....	5727	Beginning of band	
....	....	5739	Maximum of band	
....	....	5746	Slight maximum?	
....	....	5758	Slight maximum?	
....	....	5766	Slight maximum?	
....	....	5767	Rather well marked minimum	
....	....	5777	End of band	

changed, becoming much broader and displaced much more toward the red. This band is so faint and broad that determinations of its limits and of the position of the maximum are uncertain. The faint band near $H\epsilon$ appeared relatively still fainter than on the earlier plate, with the maximum perhaps more toward the violet; that is, more nearly coincident with $H\epsilon$. The $H\delta$ band showed some changes in structure, as indicated in the intensity-diagrams, and also a faint companion band to the violet.

The most striking change, however, was in the $H\gamma$ band, which showed a fairly strong companion band to the red. We at once suspected that this was the first appearance of the nebular line λ 4363, but the measurements locate it too far to the red of the proper position of that line. Still it is not impossible that the whole complex band is due to radiations physically connected with the nebular line and the hydrogen $H\gamma$. No further discussion of this plate seems necessary, except to say that the $H\gamma$ band seems to have grown fainter with respect to the $H\delta$ and the λ 4643 bands.

A third plate of the same region was obtained on May 4. Unfortunately, the star was by this time so far west at dark that the exposure was necessarily rather short, so that the negative is weak in spite of a somewhat wider slit than was used in the other cases. So far as it goes, however, it seems to show that no perceptible change had occurred since April 22.

Table III gives the results for the plates taken in the $H\beta$ and the D regions. These plates are Nos. 2718*D*, 2722*B*, and 2731*A*. The comparison lines used are iron and helium. In the plates for April 6 and April 8 there is evidence of slight flexure due to the necessity for carrying the exposures to so large an hour angle. On all these plates the bands show little certain evidences of structure, and some of the details plotted in the intensity curves may well be due to a fortuitous grouping of silver grains. Plates 2718*D* and 2722*D* show also traces of the band at λ 4643, but as this band is well seen on other plates, no attempt has been made to study its details on these plates. No trace is seen on Plate 2731*A* of any band at D_3 or beyond.

A comparison of these plates with those of similar regions of *Nova Persei* taken in July and August 1901, is rendered somewhat difficult, as far as the more minute details are concerned, by the comparatively diffuse and uncertain character of the structural details seen in the *Nova Geminorum* plates. In the case of $H\beta$ the maximum intensity is very clearly at the red edge of the band (λ 4878), while in *Nova Persei*

it lies at the violet edge (λ 4852). A very slight maximum is, however, to be seen at about λ 4852 in the plate of April 5. In the region from λ 5400–λ 5800 the similarity in general detail is more marked, but the band at λ 5671 is relatively much brighter in *Nova Geminorum* than in *Nova Persei* on August 11, 1901. The most striking differences in this portion of the spectrum are the faintness of the band at λ 5007 and the apparent lack of any band at D_3. λ 5007 was relatively faint in *Nova Persei* in February and March 1901, but in July and August was very much brighter than any of the bands between it and D_3. In *Nova Geminorum* it appears as a mere trace on one plate only, and in neither plate is there a trace of the band at λ 4959. Too much reliance cannot be placed on the apparent absence of a band at D_3, on account both of the faintness of the star and of the fact that the curve of sensitiveness of the plates used changes very rapidly at this point.

Figs. 8 and 9 give a view of the region covered in Plate 2731*A* and the corresponding intensity curve. Fig. 10 shows the *Hβ* band and the extremely faint trace of λ 5007. Figs. 11 and 12 give the intensity curves for Plates 2718*D* and 2722*B*.

It will be seen from Table III that in the plates for April 5 and April 6 there is a suspected dark line at approximately λ 4860. The mean corrected shift of this line, if real, is 1.7 t. m., corresponding to a velocity of −105 km per second. The observed appearance is too faint, however, to warrant placing much reliance upon the velocity thus obtained.

The spectrum was tested on April 4 for evidences of Zeeman effects, by rotating a Nicol prism in the eyepiece of the spectroscope. The forms of the broad bright bands appeared to remain unchanged in all positions of the prism. The same tests applied to *Nova Persei* in April 1901, gave similar results.

H. M. Reese.
H. D. Curtis.

May 9, 1903.

A LIST OF FIVE STARS WHOSE VELOCITIES IN THE LINE OF SIGHT ARE VARIABLE.[1]

The following five stars with variable radial velocities, discovered with the Mills spectrograph, are additional to the forty-two already

[1] *Lick Observatory Bulletin* No. 46.

PLATE XII.

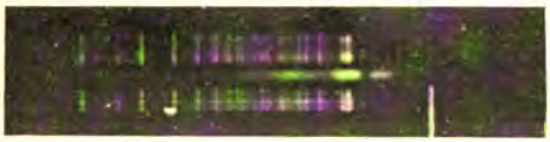


FIG. 8.—*Nova Geminorum*, Region near *D*, enlarged ninefold. 1903, April 8.

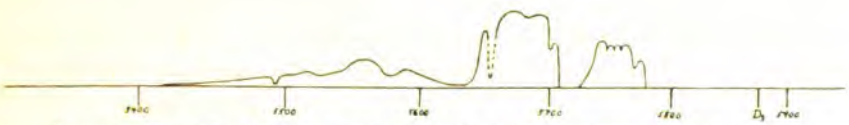


FIG. 9.—*Nova Geminorum*, Region near *D*, Intensity Curve. 1903, April 8.

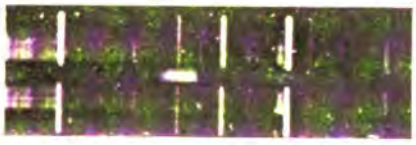


FIG. 10.—*Nova Geminorum*, $H\beta$ and λ 5007. 1903, April 8.

FIG. 11.

FIG. 12.

FIGS. 11 and 12.—*Nova Geminorum*, Intensity Curves for $H\beta$ Region. 1903, April 5 and 6.

announced. A number of the photographs and one of the measures on which these results depend are by Dr. Reese.

γ *Corvi* ($\alpha = 12^h\ 10^m.7$; $\delta = -16°\ 59'$).

Date	Velocity	Measured by
1902, December 30	+ 2 km	Curtis
1903, February 23	0	Curtis
May 6	−20	Curtis
May 11	− 6	Curtis
May 17	+ 4	Curtis

The variation was discovered from the third plate. The line at $H\gamma$ is very fair with proper exposure; those due to helium and magnesium are good.

η *Virginis* ($\alpha = 12^h\ 14^m.8$; $\delta = -0°\ 6'$).

Date	Velocity	Measured by
1903, May 17	+ 17 km	Curtis
May 24	+ 4	Curtis

$H\gamma$ is fair and λ4481 good. There are in addition scattered iron lines, narrow and quite good.

α *Draconis* ($\alpha = 14^h\ 1^m.7$; $\delta = +64°\ 51'$).

Date	Velocity	Measured by
1902, June 16	± 0 km	Stebbins
	+ 2	Curtis
1903, April 29	−43	Curtis
May 24	−42	Curtis

The variable velocity was discovered from the second plate. The measures depend on two lines only, a rather broad $H\gamma$ and a strong magnesium.

ϵ *Herculis* ($\alpha = 16^h\ 56^m.5$; $\delta = +30°\ 4'$).

Date	Velocity	Measured by
1903, May 24	−70 km	Curtis
May 27	−34	Curtis

In this star $H\gamma$ is very broad; the magnesium line a $\lambda 4481$ is also rather broad. The measures depend on the magnesium line alone.

δ *Aquilae* ($\alpha = 19^h\ 20^m.5$; $\delta = +2^\circ\ 55'$).

Date	Velocity	Measured by
1900, May 22	−25 km	Curtis
1902, July 31	−35	Reese
1903, May 12	− 2	Curtis
May 27	−32	Curtis

The iron lines are diffuse and broad; both $H\gamma$ and $\lambda 4481$ are broad and hard to measure. The first plate is a very poor one and the given velocity can be considered only as an approximation.

W. W. CAMPBELL.
HEBER D. CURTIS.

JUNE 11, 1903.

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THE VARIABLE STAR 1921 *W AURIGAE*.

By J. A. PARKHURST.

THE variability of this star was discovered by Madame Ceraski from photographs taken by Blajko at the Moscow Observatory.[1] It was found to be 8.9 magnitude (evidently near maximum) in March and April 1898, but in October of the same year it was invisible in a "7-foot" telescope. Blajko[2] has published a summary of photographic and visual observations from 1895 to 1900, deducing from them a period of 0.75 year. In column 295 of this summary the *minimum* attributed to the writer should read *maximum*. Hartwig[3] has published a note on the place of the variable, but his conclusion in regard to the period, "somewhat greater than a year," is not in accord with the results of this paper. Esch[4] has published a note containing a few observations in February and March 1902, which are in good agreement with the light-curve shown in Fig. 2. Provisional results from the observations of 1898 to 1900 have been published by the writer,[5] with charts and approximate magnitudes of the comparison stars.

[1] *Astronomische Nachrichten*, **148**, 15, 1898.
[2] *Ibid.*, **153**, 295, 1900.
[3] *Ibid.*, **149**, 6, 1899.
[4] *Ibid.*, **160**, 335, 1902.
[5] ASTROPHYSICAL JOURNAL, **12**, 54, 1900; **14**, 171, 1901; *Astronomical Journal*, **20**, 6, 1899; *Popular Astronomy*, **7**, 43, 1899; **8**, 461, 1900.

POSITION OF THE VARIABLE.

The variable was connected on three nights with the star *c*, which is *B.D.*+36°1141, and whose place is:

	R. A.	Dec.
	h m s	
From the Lund *A. G.* Catalogue	5 19 00.47	+36° 43′ 28″.7 (1875)
	5 20 41.84	+36 44 56.1 (1900)
Difference, *variable*−*c*	−0 33.28	−3 56.8
Place of the *variable*	5 20 08.56	+36 48 52.9 (1900)

As the last decimal place should be discarded, the result can be stated:

	h m s	
1921 W Aurigae	5 20 08.6	+36° 48′ 53″ (1900)
	5 17 06.0	+36 46 11 (1855)

This place agrees closely with that given by Hartwig in the reference above cited.

INSTRUMENTS.

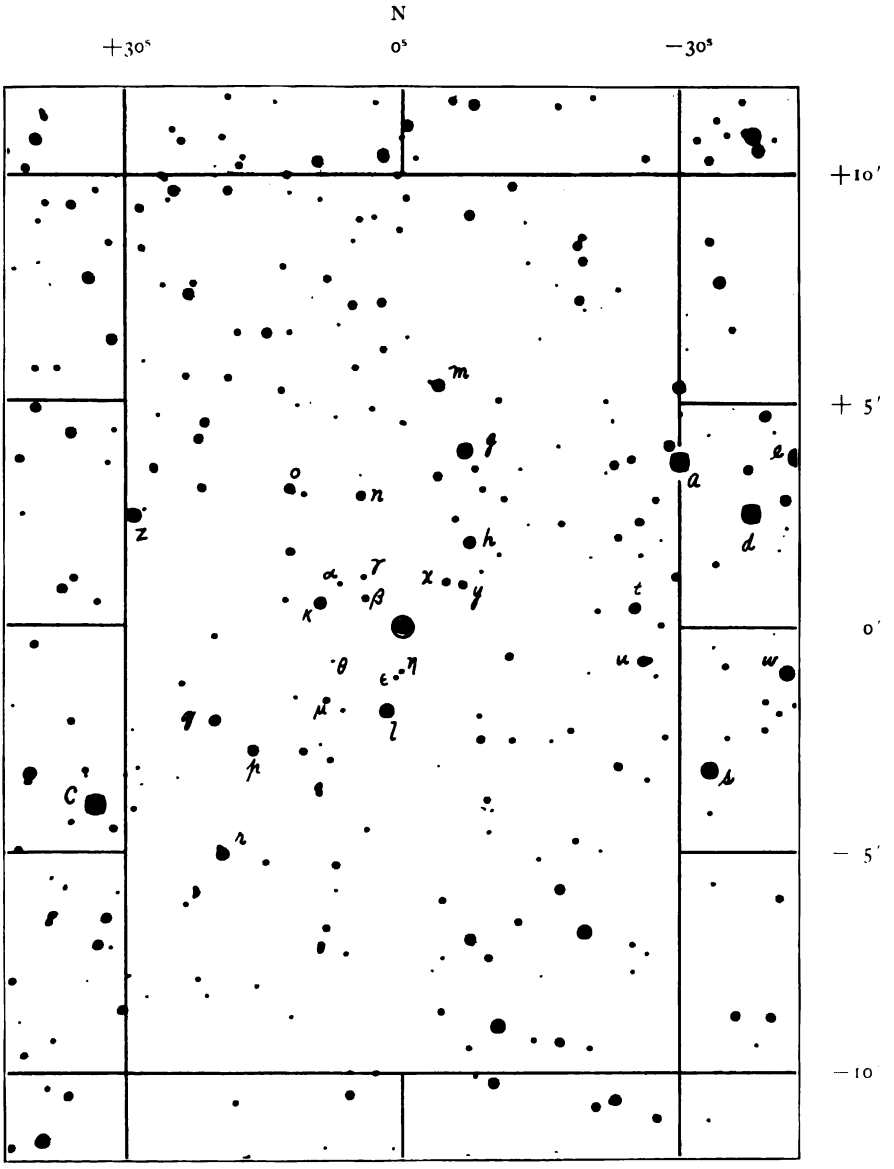
These are the same as described in the two preceding papers of this series,[1] a 157mm (6.2-inch) reflector, a 305mm (12-inch) refractor, and the 101cm (40-inch) refractor, the equalizing wedge photometer used in the photometric work being attached to each of these instruments. In the photometric measures of the brighter standard stars with the 12-inch telescope an absorption glass was interposed in the cone of rays from the objective. The adopted value of the absorption, 1.70 magnitudes, depends upon a series of laboratory measures and also on measures of *Pleiades* stars, but further measures may change it a few hundredths of a magnitude.

THE CHART.

Plate XIII shows the field 18′ × 24′ around the variable, on a scale of 10″ to the millimeter. The negative was taken with the 24-inch reflector January 11, 1902, exposure from $6^h\ 0^m$ to $6^h\ 55^m$, Central Standard Time, and shows stars to the sixteenth magnitude distinctly; but as the fainter stars were not used in the comparisons, the print for the cut was made to show stars only

[1] "The Variable Star *X Cephei*," ASTROPHYSICAL JOURNAL, 17, 48, 1903; "The Variable Star *V Lyrae*," *ibid.*, 18, 33, 1903.

PLATE XIII.



1921 *IV AURIGAE*

($5^{h}\ 20^{m}\ 820$; $+36^{\circ}\ 48'\ 53''$)

to the fifteenth magnitude. The strong color of the variable is shown by the fact that the photographic magnitude is about 10.9, the diameter of the disk being between that of *g* and *l*. The light-curve shows that the variable was near maximum, probably about 9.5 magnitude visually at this time.

COMPARISON STARS.

The three standard stars on which are based the measures of the comparison stars are as follows:

TABLE I.

Star	*B. D.* No.	Mag.	Position for 1855 R. A.	Dec.	Magnitudes Potsdam Color	Mag.	Harvard 45 Mag.
			h m s				
A....	+37° 1160	7.2	5 12 42.0	+37° 31′8	GW–	7.63	7.39
B....	+36 1122	6.8	5 14 59.4	+36 3.6	G–	6.60	6.74
C....	+36 1113	6.8	5 14 5.7	+36 15.1	GW	6.98	6.90
					Mean	7.07	7.01

Of the stars used for visual comparisons with the variable, the following are in the *Durchmusterung:*

TABLE II.

Star	No.	Mag.	1855 R. A.	Dec.
			h m s	
e	+36°1133	9.5	5 16 22.6	+36° 49′9
s	+36 1134	9.5	5 16 30.4	+36 42.9
a	+36 1138	9.5	5 16 35.8	+36 50.3
c.....	+36 1141	8.9	5 17 40.0	+36 42.6
b.....	+36 1145	9.2	5 18 13.9	+36 58.1
m'.....	+36 1147	9.2	5 18 19.2	+36 53.2
o'.....	+37 1200	8.5	5 18 23.5	+37 6.2

The date for the remaining comparison stars are given in the following table:

TABLE III.

COMPARISON STARS FOR *W AURIGAE*.

(IN ORDER OF RIGHT ASCENSION.)

Star	Co-ordinates from *V*		Light Scale	Magnitude		Star	Co-ordinates from *V*		Light Scale	Magnitude	
	R. A.	Dec.		Measured	From Curve		R. A.	Dec.		Measured	From Curve
	s		steps				s		steps		
f	−47	+5′ 42″	....	.....	.....	*l*	+ 2	− 1′ 51″	25.4	10.94	.....
e	−43	+3 48	33.8	.....	10.0	*β*	+ 4	+ 0 37	9.1	13.58	.....
w	−42	−0 57	20.4	.....	11.93	*γ*	+ 4	+ 1 6	4.9	14.25	.....
d	−38	+2 28	....	.....	.....	*n*	+ 4	+ 2 58	20.3	.....	11.94
s	−34	−3 5	31.3	.....	10.35	*a*	+ 7	+ 0 56	9.8	13.65	
a	−30	+3 19	33.3	10.35	.....	*θ*	+ 8	− 0 47	6.7	14.13	.....
t	−26	+0 26	19.3	.....	12.10	*μ*	+ 8	− 1 39	11.3	13.39	.....
u	−26	−0 43	18.3	.....	12.26	*k*	+ 9	+ 0 30	25.9	11.17	.....
h	− 7	+1 46	21.9	11.60	.....	*o*	+12	+ 2 58	....	.....	.....
g	− 7	+3 50	29.2	10.79	.....	*p*	+16	− 2 37	22.0	.....	11.73
y	− 7	+0 56	17.1	12.42	.....	*r*	+19	− 4 59	20.9	.....	11.89
x	− 5	+0 56	15.5	12.63	....	*q*	+20	− 2 1	22.3	.....	11.67
m	− 4	+5 20	22.9	.....	11.62	*z*	+29	+ 2 48	28.9	.....	10.73
η	− 0	−0 56	0.0	14.12	.....	*c*	+33	− 3 57	37.2	9.58	.....
ϵ	+ 1	−1 8	2.3	14.34	.....	*m′*	+73	+ 7 0	38.4	.....	9.37
						o′	+77	+20 0	42.9	.....	8.70

TABLE IV.

PHOTOMETRIC MEASURES OF COMPARISON STARS.

1902, January 24; 12-inch. Wedge V, seeing good, full Moon.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
Aa	23.7	24.2	23.3	23.8	23.0	23.0	23.50	7.63
Ca	15.3	15.0	16.0	15.0	15.8	15.8	15.48	6.98
Ba	14.5	13.2	13.8	13.2	11.2	12.2	13.02	6.60
a	34.1	35.0	35.0	35.2	33.4	35.0	34.45	10.66
g	37.8	30.2	38.2	34.8	34.2	34.0	36.37	10.87
h	43.3	42.0	41.3	42.2	42.2	43.2	42.37	11.54
k	38.2	38.9	39.1	39.8	39.1	39.9	39.17	11.18
l	38.2	38.2	37.0	40.2	39.7	40.8	39.02	11.17
v	29.5	28.8	28.7	....	....	....	29.00	9.95

Wedge constant, 0.110 mag.

TABLE IV.—*Continued.*

1902, February 7; 12 inch. Wedge V, seeing good.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
Aa.....	18.8	17.5	17.8	....	....	....	18.37	7.63
Ca.....	12.0	12.8	13.2	....	....	....	12.67	6.98
Ba	9.2	10.7	10.2	....	....	....	10.03	6.60
a.....	27.7	27.4	26.2	....	....	....	27.10	10.25
g.....	32.1	32.2	32.2	....	....	....	32.17	10.80
h.....	37.0	39 4	39.0	....	....	....	38.47	11.50
k.....	34.5	33.7	34.0	....	....	....	34.07	11.01
l.....	32.1	31.1	31.0	....	....	....	31.40	10.72

Wedge constant, 0.110 mag.

1903, January 1; 12-inch. Wedge V, seeing good.

A.....	14.7	12.0	12.8	....	....	....	13.17	7.63
Ba	19.0	21.2	19.8	....	....	....	20.00	6.60
C	7.3	4.1	4.7	....	....	....	6.16	6.98
Ca	21.4	23.1	22.0					
a.....	37.8	38.0	38.8	37.2	37.3	37.7	37.80	10.35
g.....	43.0	44.5	43.1	38.8	42.8	41.7	42.32	10.85
h.....	52.9	49.4	51.4	50.1	49.9	50.0	50.62	11.76
k.....	47.2	48.0	47.8	45.9	46.1	45.3	46.72	11.33
l.....	44.8	45.8	45.3	43.0	42.3	42.8	44.00	11.03

Wedge constant, 0.110 mag.

1903, March 21; 6-inch. Wedge V, seeing fair.

B.....	11.5	11.3	11.7	12.4	12.5	12.3	11.95	6.60
C.....	15.8	16.2	15.9	16.1	15.3	16.3	15.94	6.98
A.....	22.2	22.0	22.3	20.6	21.7	21.7	21.75	7.63
a.....	47.7	47.8	48.2	45.7	46.5	45.7	46.94	10.41

Wedge constant, 0.110 mag.

1903, March 22, 6-inch. Wedge V, seeing good.

B.....	14.0	13.8	13.4	13.6	15.0	14.7	14.08	6.60
C.....	17.0	15.7	16.2	17.7	18.5	18.2	17.22	6.98
A.....	23.2	22.6	23.2	21.5	22.7	21.5	22.45	7.63
a.....	45.5	45.5	44.7	43.3	43.7	43.0	44.28	9.97
g.....	49.6	49.4	49.8	51.2	50.9	51.2	50.35	10.04
c.....	40.8	39.0	39.7	40.9	40.9	40.4	40.78	9.58

Wedge constant, 0.110 mag.

TABLE IV.—*Continued.*

RESULTING MAGNITUDES, FROM MEASURES WITH 6- AND 12-INCH.

	1902 January 24	1902 February 7	1903 March 1	1903 March 21	1903 March 22	Means
c	.....	.....	.....	.....	9.58	9.58
a	10.66	10.25	10.35	10.41	9.97	10.35
h	11.54	11.50	11.76	.....	.....	11.60
g	10.87	10.80	10.85	.....	10.64	10.79
k	11.18	11.01	11.33	.....	.....	11.17
l	11.17	10.72	11.03	.....	.....	10.94

1900, October 23; 40-inch. Wedge II, seeing fair.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
k	15.9	16.7	17.2	19.0	17.9	16.8	17.25	11.17
h	21.0	19.9	21.8	21.9	22.7	22.0	21.55	11.60
g	13.2	15.0	15.9	14.0	15.5	14.0	14.60	10.79
l	17.1	17.8	17.0	19.8	19.2	19.9	18.47	10.94
a	40.2	41.1	39.9	37.3	37.9	37.5	38.98	13.63
γ	46.7	46.4	42.8	39.3	41.8	41.1	43.02	14.21
β	40.8	39.5	38.0	38.9	38.2	37.0	38.73	13.60
θ	42.7	40.9	40.0	42.2	44.1	42.8	42.12	14.06
μ	38.2	37.1	36.1	34.5	35.0	35.0	35.98	13.24
ϵ	43.8	46.0	44.2	45.9	46.4	45.0	45.22	14.44
η	42.5	43.7	43.4	41.8	43.5	41.0	42.65	14.11
x	30.0	29.4	30.1	30.0	31.8	32.1	30.58	12.54
y	25.2	28.8	25.8	28.0	27.0	26.0	26.80	12.05
δ	32.9	31.0	31.0	29.9	32.0	31.2	31.33	12.64
v	34.2	33.0	31.5	34.0	31.9	33.9	33.09	12.79

Wedge constant, 0.130 mag.

1902, February 4; 40-inch. Wedge V, seeing good.

k	10.0	10.0	9.2	11.1	10.9	11.8	10.50	11.17
l	10.8	11.3	11.1	10.2	12.2	11.3	11.15	10.94
ha	26.6	28.3	26.1	....	....	....	27.00	11.60
ga	18.3	19.9	18.9	....	....	....	19.03	10.79
μ	30.3	31.0	31.0	....	....	....	30.77	13.47
θ	37.7	38.7	37.3	...	....	...	37.90	14.26
ϵ	37.8	38.4	38.6	....	....	....	38.27	14.30
β	30.5	31.6	31.3	....	....	....	31.13	13.51
γ	37.3	38.0	38.3	....	....	....	37.87	14.26
α	33.6	32.6	32.3	....	....	....	32.83	13.70
x	20.9	22.0	23.0	....	....	....	21.97	12.51
y	22.0	21.0	20.8	....	....	....	21.27	12.43

Wedge constant, 0.110 mag.

TABLE IV.—*Continued.*

1902, February 6; 40-inch. Wedge V, seeing fair.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
h_a.....	35.2	35.0	35.1	33.8	34.0	33.0	34.35	**11.60**
k_a.....	27.5	26.3	26.0	....	....	....	26.60	**11.17**
l_a.....	29.4	29.2	29.3	28.0	27.1	27.2	28.37	**10.94**
y.....	27.8	29.0	28.2	....	....	....	28.33	12.78
x.....	29.1	28.6	29.1	....	....	....	28.93	12.85
β.....	35.2	35.8	36.8	....	....	....	35.93	13.62
γ.....	42.0	42.0	42.3	....	....	....	42.10	14.30
α.....	36.5	36.0	35.5	....	....	....	36.00	13.63
θ.....	40.5	40.0	39.5	....	....	....	40.00	14.07
μ.....	34.0	34.2	35.2	....	....	....	34.47	13.45
ϵ.....	41.2	43.0	41.8	....	...	....	42.00	14.29

Wedge constant, 0.110 mag.

RESULTING MAGNITUDES, FROM MEASURES WITH 40-INCH.

	1900 October 23	1902 February 4	1902 February 6	Means
x.....	12.55	12.51	12.85	12.63
y.....	12.05	12.43	12.78	12.42
α.....	13.63	13.70	13.63	13.65
β.....	13.60	13.51	13.62	13.58
γ.....	14.20	14.26	14.30	14.25
δ.....	12.65	.....	.....	12.65
ϵ.....	14.44	14.30	14.29	14.34
θ.....	14.06	14.20	14.14	14.13
η.....	14.12	.....	.....	14.12
μ.....	13.26	12.47	13.45	13.39

The photometric measures of the comparison stars are given in Table IV, which is arranged similarly to the corresponding table in the two previous papers. The stars used as standards are given first with their magnitudes in *bold-faced* type. The magnitudes for *A*, *B*, and *C* are taken from Table I, using the Potsdam values; to reduce to the Harvard scale subtract 0.06 from the numerical magnitude. The values for the fainter standards, lettered from *a* to *l*, are taken from the part of Table

TABLE V.

1921 W Aurigae.

(Comparisons of the Variable by Argelander's Method.)

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1898		C.S.T.	G. M. T. 2410000+			
1	Dec.	10	6	4634.5	40	6	*o'* 4 *v*, *c* 2 *v*, *v* 1 *a*
					80	6	*c* 1 *v*, *v* 1 *a*
2		12	18	4637.0	40	6	*c* 0-1 *v*, *v* 2 *a*
3		13	7	4637.5	40	6	*m'* 2 *v*, *v c*, *v* 1 *b*
4		15	8	4639.6	40	6	*m'* 1 *v*, *v* 2 *b*, *v* 0-1 *c*
5		17	7	4641.5	40	6	*m'* 1 *v*, *v c*, *v* 3-4 *a*
			8	4641.6	80	6	*v* 1 *c*
6		22	6	4646.5	40	6	*v* 2 *b*, *m'* 0-1 *v*, *v* 1 *c*, *v* 4 *a*
7		26	8	4650.6	40	6	*v* 2 *c*, *v m'*, *o* 5 *v*
8		28	8	4652.6	80	6	*c* 0-1 *v*
9		30	7	4654.5	40	6	*c* 1 *v*
	1899						
10	Jan.	5	6	4660.5	40	6	*v* 0-1 *c*
11		10	7	4665.5	150	6	*c* 2-3 *v*, *v* 3-4 *a*
					40	6	*c* 1-2 *v*, *v* 3 *a*
13		18	7	4673.5	150	6	*c* 4 *v*, *v* 1 *a*
					40	6	*c* 3 *v*, *v* 0-1 *a*
14		24	7	4679.5	40	6	*c* 4 *v*, *a* 2 *v*
					150	6	*c* 4-5 *v*, *a* 2 *v*, *v s*, *v* 3-4 *g*
15		28	7	4683.5	150	6	*c* 6 *v*, *a* 2 *v*, *s* 1 *v*, *v* 3 *g*
16	Feb.	1	7	4687.5	150	6	*a* 4 *v*, *e* 4 *v*, *s* 2 *v*, *v* 2 *g*, *v* 3 *l*
17		15	7	4701.5	150	6	*s* 5 *v*, *g* 2-3 *v*, *l* 1 *v*, *v* 0-1 *k*, *v* 4-5 *h*
18		28	8	4714.6	150	6	*h* 1-2 *v*, *v* 2 *n*, *v* 3 *y*
19	Mar.	6	8	4720.6	150	6	*h* 2 *v*, *v* 2 *y*
20		13	8	4727.6	150	6	*n* 2-3 *v*, *v* 2 *y*, *h* 4 *v*
21		28	8	4742.6	150	6	*y* 2-3 *v*
22	Apr.	4	8	4749.6	200	6	*y* 1-2 *x*, *x* 2 *v*
23		12	8	4757.6	150	6	*v* not seen
24		28	8	4773.6	150	6	*v* not seen
25	May	4	8	4779.6	150	6	*v* not seen
26	Oct.	30	8	4958.6	40	6	*a* 1 *v*, *v* 2 *s*, *v* 2-3 *g*
					150	6	*a* 0-1 *v*, *v* 2 *s*, *v* 3 *g*
27	Nov.	4	8	4963.6	150	6	*a* 1 *v*, *v* 1 *s*, *v* 3 *g*
28		20	7	4979.5	150	6	*g* 3 *v*, *v* 2 *l*
29		26	7	4985.5	150	6	*g* 5 *v*, *l* 0-1 *v*, *v* 1-2 *h*
30	Dec.	5	7	4994.5	150	6	*l* 3 *v*, *h* 1 *v*, *v* 1 *n*, *v* 4 *y*
31		19	7	5008.5	150	6	*h* 2-3 *v*, *v* 1 *n*
32		28	7	5017.5	150	6	*h* 6 *v*±, *v* 0-1 *n*, *v y*, *v* 1 *x*
	1900						
33	Jan.	4	7	5024.5	200	6	*y* 1-2 *v*, *v* 1 *x*
34		8	6	5028.5	350	40	*y* 2 *v*, *v* 1 *x*
35		25	8	5045.6	175	12	*x* 5-6 *v*
36		26	10	5046.7	350	40	*x* 2-3 *v*, *v* 2-3 β β α, α 4 γ, γ 4 ϵ, ϵ 2 η
37	Feb.	4	11	5055.7	350	40	*x* 3-4 *v*, *v* 3-4 α *x* 5 α, α 2 β, β 6 γ, γ 1 ϵ, ϵ 2 η

TABLE V.

1921 W Aurigae.

(Reduction of Observations.)

No.	Details in Steps	Means		Seeing	Remarks
		Steps	Magnitudes		
1	38.9, 35.2, 34.3 / 36.2, 34.3	35.7	9.74	good	
2	36.7, 35.3	36.0	9.68	fair, faint twilight	
3	36.4, 37.2, 36.6	36.7	9.59	good	
4	37.4, 37.6, 37.7	37.5	9.46	good	
5	37.4, 37.2, 36.8 / 38.2	37.6	9.64	good, Moon	
6	37.6, 37.9, 38.2, 37.3	37.2	9.50	good, Moon	
7	39.2, 38.4, 37.9	38.5	9.32	good, Moon	
8	36.7	36.7	9.59	poor	
9	36.2	36.2	9.67	fair	
10	37.7	37.7	9.43	fair to poor	
11	34.7, 36.8 / 35.7, 36.3	35.6	9.75	good	
13	33.7, 34.3 / 34.2, 33.8	34.0	9.97	good	
14	33.2, 31.3 / 32.7, 31.3, 31.3, 32.7	32.0	10.27	good, Moon	
15	31.2, 31.3, 30.3, 32.2	31.2	10.38	good	
16	29.3, 29.8, 29.3, 31.2	29.8	10.60	good	
17	26.3, 26.7, 24.4, 26.4, 26.4	26.0	11.13	good, Moon	
18	20.4, 22.3, 20.1	20.9	11.88	good	
19	19.9, 19.1	19.5	12.07	good	
20	17.8, 19.1, 17.9	18.2	12.27	good	
21	14.6	14.6	12.79	good	
22	13.5	13.5	12.94	fair to good	
23			<12.5	fair	limit x or y
24			<12.0	fair	limit n
25			<12.0	twilight	limit n and o
26	32.3, 33.3, 31.7 / 32.8, 33.3, 32.2	32.6	10.18	low, fair	
27	32.3, 32.3, 32.2	32.2	10.24	low, fair	
28	26.2, 28.4	27.3	10.96	poor	
29	24.2, 24.9, 22.4	23.8	11.47	fair	
30	22.4, 20.9, 21.3, 21.1	21.4	11.79	good	
31	19.4, 21.3	20.3	11.95	fair, Moon rising	
32	15.9, 20.8, 17.1, 16.5	17.5	12.37	good	
33	15.6, 16.5	15.8	12.58	small Moon	
34	15.1, 16.5, 15.8	15.8	12.61	Moon	
35	10.0	10.0	13.47	good	
36	13.5, 11.6	12.5	13.09		
37	13.5, 13.3	13.4	12.96		

TABLE V—*Continued.*

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1900		C.S.T.	G. M. T. 2410000+			
38	Feb.	18	10	5069.7	350	40	$v\ \alpha, v\ \beta, v$ 5-6 $\gamma, v\ 6\ \eta$ x 6-8 v, v 1-2 α (2d comparison)
39		22	9	5073.6	350	40	$\alpha\ 2\ v, \beta\ 2\ v, v\ 3\ \gamma, v\ 5\ \eta$ α 1 v, β 1 v, v 4 γ, μ 2 β, β 3 θ
40		24	10	5075.7	175	12	v not seen
41	Mar.	1	8	5080.6	350	40	α 1-2 v, β 1-2 v, v 4 γ, v 1 θ, v 5 η
42		21	9	5100.6	275	12	v 1 β, μ 4 v, v 1 θ
43		22	9	5101.6	350	40	μ 3 v, v 1 θ, v 3 γ, β 1 v
44	Apr.	4	9	5114.6	275	12	μ 2 v, v 2 β
45		18	8	5128.6	275	12	y 2 v, v 1 x, v 2 μ
46		19	8	5129.6	275	12	v 2 μ, x 1 v, y 2-3 v, v 3 β
47		27	10	5137.7	350	40	$v\ y, v$ 2 x, n 3 v
48		29	9	5139.6	150	6	v not seen
49	Aug.	30	19	5263.0	237	40	v 2 k, v 3 l
49*a*	Oct.	23	12	5316.8	237	40	
	1901						
50	Feb.	9	7	5425.5	150	6	v not seen
51		10	7	5426.5	150	6	v not seen
	1902						
52	Jan.	11	7	5761.5	...	24	
53		24	7	5774.6	67	12	
54	Feb.	6	10	5787.7	237	40	
	1903						
55	Jan.	1	8	6116.6	67	12	$v\ n, h$ 5 v, v 4 x
56	Mar.	21	11	6195.7	80	6	v not seen

IV headed "Resulting Magnitudes from Measures with 6- and 12- Inch." The subscript $_a$ indicates that the star was measured through the absorption glass, before mentioned, which cuts down the light 1.70 magnitudes (equals 15.45 scale-divisions with wedge V). For the fainter stars, from x down, the average deviation of the separate night's result from the mean of the three is 0.09 magnitude. If the star y, for which the deviation is anomalous, be omitted, the average deviation becomes 0.07 magnitude.

MAGNITUDE-CURVE.

This is given in Fig. 1, platted with photometric magnitudes as abscissæ and positions in the light-scale as ordinates; giving the points shown by the round dots. In this case the "curve"

TABLE V—*Continued.*

No.	Details in Steps	Means		Seeing	Remarks
		Steps	Magnitudes		
38	9.8, 9.1, 9.4, 6.0 / 8.5, 8.3	8.5	13.67	fair	
39	7.8, 7.1, 7.9, 5.0 / 8.8, 8.1, 8.9	7.6	13.82	good	
40		<11.5	<13.25	fair	limit $4 < x$
41	8.3, 7.6, 8.9, 7.6	7.5	13.83	good	
42	10.1, 7.3, 7.7	8.3	13.70	good	limit θ or $1 < \beta$
43	8.3, 7.7, 7.9	7.9	13.77	good	limit $4 < \eta$
44	8.3, 11.1	9.7	13.50	good, Moon	
45	15.1, 16.5, 13.3	14.9	12.76	fair, thick	
46	13.3, 14.5, 14.6, 12.1	13.6	12.94	good	limit β
47	17.1, 17.5, 17.3	17.3	12.39	fair	
48			<12.0		limit n
49	27.9, 28.4	28.1	10.83	twilight	
49*a*			12.79	fair	photometer
50		<20.4	<11.8	fair	limit $1.2 < h$
51		<19.9	<12.0	fair	limit n
52			<10.9		photo. 55 m. exp.
53			9.95	good, Moon	photometer
54			10.22	fair to good	photometer
55	20.3, 16.9, 19.5	18.9	12.18	good	
56		<21.9	<11.6	good, low	limit h

is a straight line. The average distance of the dots from the line is 0.10 magnitude; omitting η, the distance is 0.08. The value of one step is 0.14 magnitude.

VISUAL OBSERVATIONS.

The visual comparisons of the variable by Argelander's method are given in detail in Table V. The light-scale was formed from them in the usual manner, giving the quantities in the fourth column of Table III. Table V also contains the results of three sets of photometric measures of the variable (observations No. 49*a*, 53, and 54) and the magnitude determination from the photograph (No. 52).

LIGHT-CURVE.

In Fig. 2 the observed magnitudes from 1898 to 1903 are platted and the light-curve drawn through them; the parts of the curve covered by the observations being drawn full, while

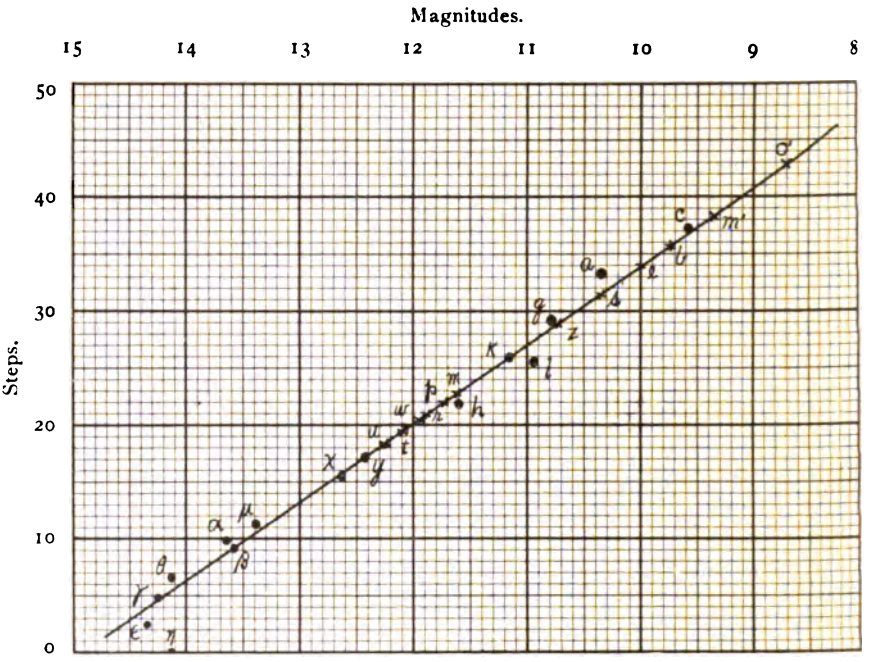


FIG. 1.— Magnitude Curve of *W Aurigae*.

the parts assumed, in absence of sufficiently numerous observations, are drawn with a broken line. The variations seem best represented by the elements of maximum —

1898, December 24 (J. D. 2414648) + 276 E (M-m=113)

Magnitudes: Max. 9.3, Min. 13.8.

In Table VI are collected the dates of maximum and minimum, together with a comparison with the observations given in Blajko's summary previously quoted, from the *Nachrichten*.

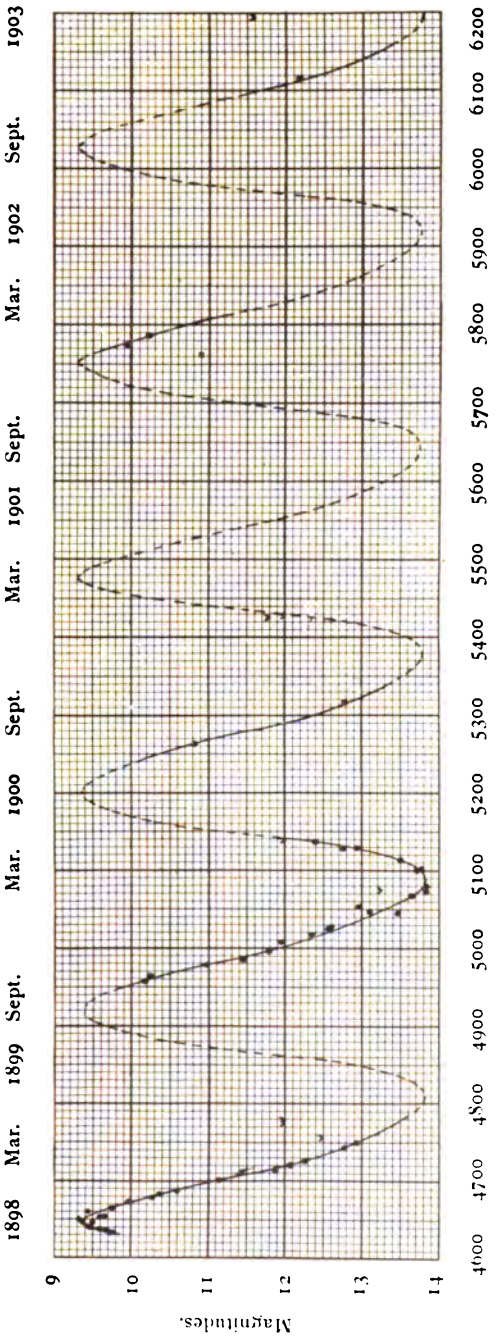


FIG. 2.—Light-Curve of *W Aurigae*.

TABLE VI.

MAXIMA.

Epoch	Calculated		Observations Blajko's Summary		Blajko's Maxima
	Julian Day	Calendar	Date	Magnitude	
	2410000+				
−5	3268	1895, Mar. 15	1895, Feb. 15 1895, Mar. 4	10.1 ph 9.3 ph	March or April
−4	3544	1895, Dec. 16	1895, Dec. 16 1895, Dec. 21	8.8 ph 8.8 ph	December
−3	3820	1896, Sept. 17			
−2	4096	1897, June 20			
−1	4372	1898, Mar. 23	1898, Apr. 10	9.2 vis	
0	4648	1898, Dec. 24	1898, October 1899, Feb. 21 1899, Mar. 31	< 12 vis 11 vis < 12 vis	December
1	4924	1899, Sept. 26	1899, October 1899, Dec. 28	10 vis 10 vis 11 vis 12 ± ph	
2	5200	1900, June 29	1900, April	12 vis	Second part of June
3	5476	1901, Apr. 1			March
4	5752	1902, Jan. 2			
5	6028	1902, Oct. 5			

MINIMA.

Epoch	Calculated		Observations Blajko's Summary	
	Julian Day	Calendar	Date	Magnitude
	2410000+			
−4	3431	1895, Aug. 25		
−3	3707	1896, May 27		
−2	3983	1897, Feb. 27	1897, Mar. 26	< 12 ph
−1	4259	1897, Nov. 30	1898, Jan. 16	< 12 ph
0	4535	1898, Sept. 2	1898, October	< 12 ph
1	4811	1899, June 5	1899, Apr. 4 1899, Apr. 6	< 12 ph < 12 ph
2	5087	1900, Mar. 8		
3	5363	1900, Dec. 9		
4	5639	1901, Sept. 11		
5	5915	1902, June 14		
6	6191	1903, Mar. 17		

In the column of "Observations," "ph" stands for "photographic" and "vis" for "visual." It will be seen that the above elements satisfy the observation between 1895 and 1903. The

calculated dates of maxima and minima are well represented by the light-curve, though the observations are not sufficiently numerous to enable us to detect variations of less than fifteen or twenty days with certainty. The magnitudes on the photographs near the maximum at epoch -4 do not agree very closely with my photograph taken near maximum at epoch $+4$ (indicated on the light curve, Fig. 2, by the small cross), but doubtless a large part of the divergence can be accounted for by the use of different photographic plates.

YERKES OBSERVATORY,
September 1903.

ON CERTAIN METHODS OF ECONOMIZING THE LIGHT IN SPECTRUM ANALYSIS.

By W. J. HUMPHREYS.

NO MATTER what the source of light nor what kind of spectrograph is used to analyze it, there are distinct advantages in obtaining its spectra as brilliant as possible. In this way lines are found that otherwise would escape detection, and photographs, which of course integrate the light during the time of exposure, are obtained more quickly, and therefore under more nearly uniform conditions. Moreover, such economy is desirable in obtaining spectrograms of very volatile substances, small amounts of any material, arcs under pressure, and other temporary sources of light. Again, it renders practical the use of instruments of correspondingly larger dispersive powers — an advantage of the utmost value in nearly every line of spectrum work.

The above are some of the reasons why a spectrograph should be equipped with light-saving devices, and in what follows I shall briefly describe a few of these, most of which have been fully tested experimentally.

I. ILLUMINATION OF THE SLIT.

Assuming that the source of light is not under the experimenter's control, or if so, that its intensity has been pushed to the practicable limit — in short, with a fixed source, — probably the most obvious method of increasing the brilliancy of its spectrum, as produced by a given instrument with a definite adjustment, is that of increasing the illumination of the slit.

If the source is so far away that it appears as a point, a star for instance, or line, like the disappearing crescent of the Sun at a total solar eclipse, then it is quite feasible, as is generally known, to discard the slit entirely and use an objective-prism or grating; but for obvious practical reasons it is the custom, when working with large instruments, to focus the light with convex

refractors or concave reflectors upon a suitable slit, though such a procedure is necessarily beset with difficulties. Large refractors have to be made of glass, and are therefore more or less opaque to the ultra-violet, while the light that gets through is always in some measure dispersed by chromatic aberration. Reflectors, on the other hand, are free from chromatic aberration, but even though perfectly figured, if they consist of speculum metal, return scarcely two-thirds of the incident visible light and barely two-fifths of the ultra-violet; while, if silver on glass is used, nearly all the ultra-violet is lost.[1] With such sources the only means, appliable to all types of spectrographs, that has occurred to me of increasing the illumination of the slit, when the size of the apparatus is fixed, is by the use of properly selected reflectors, silver for the visible light and magnalium, wherever practicable, for the ultra-violet. A special method adapted to concave gratings will be described further on.

When a flame, electric arc, or other near-by source is used, the problem is radically different from the preceding one, where it was supposed to be far off. Here, by merely placing the source in front of the slit, a fair amount of light is caused to reach the grating or other analyzer — an amount which, if the source is uniformly brilliant throughout, is independent of its distance so long as the cone of light passing through the slit is large enough to cover the analyzing surface; and to the end that this cone shall be of ample size it is customary to use an ordinary condensing lens, commonly consisting of quartz for gratings to secure the ultra-violet, with the slit and source at its conjugate foci. When the light is intense and constant, this method is sufficient for most purposes. It also has the great advantage, when prisms or flat gratings are used, of giving the distribution in the arc or other source, of the substances producing the spectra. It has, however, besides the loss of some light by reflection, the disadvantage of chromatic aberration, by virtue of which the relative intensities of the colors are not the same at the analyzer that they are in the source.

Of course, the sharper the image on the slit, as produced by

[1] HAGEN and RUBENS, *Annalen der Physik*, 8, 1–21, 1902.

any one color, the greater the amount of that kind of light that reaches the analyzer; but so long as the slit is kept in one of the foci, and the cone of light from it fully covers the analyzing surface, there is but little to be gained by altering the position, size, or focal length of the condensing lens. In the case of concave gratings, however, and in general where the distribution of the light in the source is not under consideration, it is easy to increase greatly the brilliancy of the spectrum by the use of suitable concave reflectors.

a) As the most general type of such reflector it is convenient to consider a portion of an ellipsoid of revolution. Let this be adjusted so that the center of the source of light, say sparks between metallic points, shall be in one focus, and the slit, which should be parallel to the line joining the spark terminals, in the other focus of the ellipsoid, whose inner surface is supposed to be a highly polished good reflector. The line joining the foci should, of course, if prolonged, intersect the center of the prism or grating. By this means the light that fills a large solid angle whose vertex is at the source, is converged upon the slit, and, by adapting the dimensions of the ellipsoid to the spectrograph in use, much of it is made to reach the analyzer. This method evidently would avoid chromatic aberration, as well as utilize a larger amount of light than could be secured by means of a lens.

b) If the foci of the ellipsoid are separated until the figure becomes a paraboloid of revolution, with the source centered in the focus, the rays of reflected light become approximately parallel and may be condensed upon the slit with a suitable lens; or by placing the slit in the focus and close to the source, but between it and the parabolic reflector, the parallel rays thus obtained would be adapted to an objective concave grating or other analyzer requiring the use of a collimator. Of course, the disturbing light from the source would have to be screened off from the instrument, and this could be done by a small opaque object suitably placed; or, among other ways, the source might be put quite to one side and, by the aid of a condenser and small optical flat mirror, a brilliant image formed on the slit.

But the ellipsoid and paraboloid, while simple enough in

theory, are necessarily difficult to make, and therefore expensive; still they are mentioned in this connection because of their decided value, if properly constructed, and, further, they naturally belong with a third method described below, which is exceedingly easy of construction, inexpensive, and efficient.

c) Again, considering the ellipsoid of revolution, let its foci be brought closer together, till they finally coincide, and the surface becomes spherical. In this case the source, which is at the center of the sphere, and its image are superimposed, and the condensing lens in front of the slit receives both the direct and the reflected light. Evidently the source could be placed

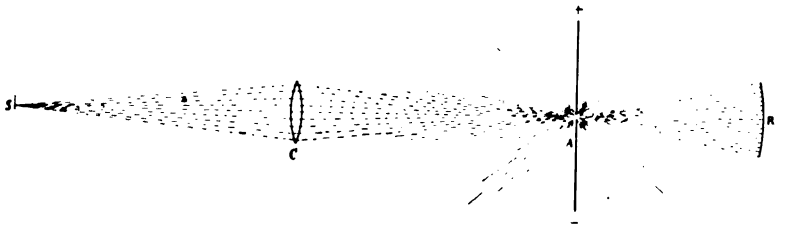


FIG. 1.

slightly to one side, and its reflected image alone received upon the slit. The particular advantage, that might occasionally be desirable, of this arrangement is the avoidance of chromatic aberration.

The method I have most frequently used is shown in Fig. 1, in which *A* is the source, *R* a spherical reflector, *C* a condensing lens, and *S* the slit.

In this way two distinct images are formed on the slit, one an inverted image produced by the direct light, the other an erect image given by the reflected light; and evidently they may be superimposed. But since the reflected light shines back through its own source, it must necessarily suffer greater or less absorption, so that the resulting brilliancy of the superimposed images is less than the sum of those of the direct and reflected images separately. Nevertheless, the combination gives spectra distinctly more brilliant than those produced by the direct light alone, and therefore much of the reflected light must somehow get through the source, and does so probably because the lumi-

nous vapors are not sufficiently dense or extensive to form a practically continuous intercepting layer.

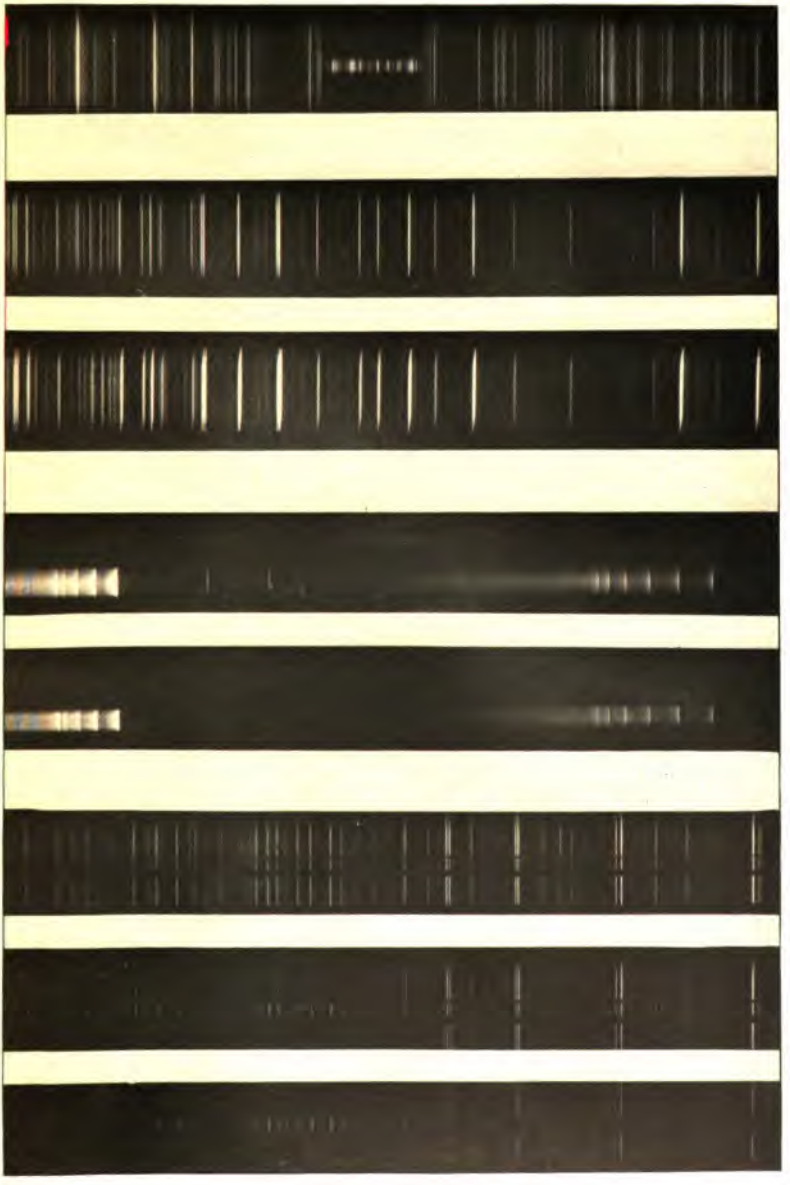
To demonstrate photographically the efficiency of the apparatus, I have taken advantage of the fact that silver is a poor reflector of ultra-violet light, but an excellent one of longer wave-lengths. The first three negatives shown in the plate were taken in the region of $\lambda 3200$ of the third order, which coincides with $\lambda 4800$ of the second order. The outer portions of I are due wholly to light reflected from a silver surface, while the middle strip was produced by light directly from the arc made to pass through a silver film, on a quartz disk of sufficient thickness to be nearly opaque to all luminous rays. The position of maximum transmission naturally coincides with that of minimum reflection, approximately $\lambda 3200$, and therefore the heavy lines on the outer portions of I belong to regions of longer wave-length.

The outer portions of II are also due to reflected light alone, but the middle strip was produced by undisturbed light directly from the arc. It will be noticed that, while the times of exposure were as nearly as possible the same and the source kept constant, the ultra-violet lines have very unequal intensities in the direct and reflected portions, and that those of longer wave-lengths differ but little from each other. Finally, the middle strip of III is due to direct light, while its outer portions were produced by the joint effect of the superimposed images, direct and reflected. The arc was practically constant and the times of exposure the same. Here it is seen that those ultra-violet lines which are scarcely at all reflected have nearly equal intensities in the outer and middle portions, while those of longer wave-lengths that are strongly reflected are much heavier in the outer parts. The conclusion, therefore, is that the vapors of the arc do not absorb all the reflected light, and that the method described does materially increase the brilliancy of arc-, and presumably of certain other spectra.

Incidentally it is seen that a silver reflector, or screen, can be used as an aid to differentiate between ultra-violet lines and those of longer wave-length.

In most of my work I have used silver-on-glass reflectors, but,

PLATE XIV.



while this is the best known reflector of visible light, it is, as shown by the plate, a very bad one for the ultra-violet, and therefore for this region some other material should be used, probably magnalium for the best results.

d) Of the many possible optical trains that might be used to illuminate the slit, there is one, other than those above described, with which I have worked that, for small images, yields good results with the concave grating. This consists of a spherical condensing lens used in the ordinary way, either with or without a reflector behind the source, in conjunction with a double concave cylindric lens of short focal length, placed close to the slit and with its axis at right angles to the slit's direction.

By focusing the light on the slit with the condensing lens, and then introducing the cylindric lens as above described, the image will be elongated in the direction of the slit, while its width at the center will remain unaltered; and the pencil of light that gets through, instead of covering and surrounding the grating with a circle, will cover it with a surface roughly rectangular in shape, the length of which is equal to the diameter of the original circle, while its width is much less.

The cylindric lens does not appreciably alter the amount of light that gets through the slit, but places a much larger percentage of it on the ruled surface, so that the final effect is to lengthen out the spectrum lines, owing to the elongated image, and, on account of the astigmatism of the grating, to increase their brilliancy.

These effects are shown in negatives IV and V, the cylindric lens being used in the latter case. The source was a round hole, about one millimeter in diameter, in a sheet of metal placed close to a large, and therefore approximately uniform, electric arc. The times of exposure were as nearly as possible the same, and the plates were developed together and for the same length of time. Many eye observations were taken, and several other comparison plates secured in the first three orders of the spectrum, and the results were everywhere the same—a slight lengthening of the lines, and a marked increase in their brilliancy.

II. VIRTUAL INCREASE OF THE SLIT-LENGTH.

One of the peculiarities of the concave grating is its astigmatism, by virtue of which a point source in the slit produces a line image on the focal curve. When the slit is parallel to the rulings on the grating, the several line-images due to its consecutive points are superimposed, and a relatively intense line is the result. The longer the slit, within certain limits, the more intense the resulting lines, but when it is increased beyond a definite length, which depends on the grating and its position, the lines produced by light coming from one end of it fail to overlap those due to light from the other end, and clearly increasing the slit-length beyond this does no good. However, there are many sources of light, such as end-on Plücker tubes, sparks between close terminals, and others, whose images as produced by an ordinary condensing lens limit the slit to much less than its maximum efficient length. Besides, the image may always be rendered small by means of a short-focus condensing lens, and in many cases the only objection to this is the loss of some light due to greater chromatic aberration, and the increased difficulty of keeping the image properly placed on the slit.

In practically all cases, then, the circle of light that covers the grating may be, and usually is, much greater than the area of the ruled surface, and thus only a small portion of the light that gets through the slit is of any service.

Clearly, however, if this waste light could be made to fall on the ruled surface in the direction which it would have if it came from some point close to and in line with the slit, the astigmatism would cause the resulting spectrum lines to overlap those due to the direct light from the slit, and the brilliancy of the spectrum would be increased.

a) As already explained, excellent results of this kind can be obtained with a concave cylindrical lens placed before the slit, but similar effects are possible with suitable apparatus properly located between the slit and grating. One of these, which, owing to the difficulty of its construction, will probably have only a theoretical interest, is a curved convex cylindrical lens—a portion of a ring—of proper focal length, and of such shape and

position that it will be normally intersected by each plane determined by the slit and a line on the grating.

b) There is another method, however, efficient, simple to construct, and easy to use. This consists of a pair of optical flats close to the slit, but between it and the grating, with their ends properly separated and the pair so adjusted that the plane fixed by the slit and the central ruling on the grating shall be normal to their reflecting surfaces. Chromatic aberration is thus avoided.

The method is shown schematically in Fig. 2, which also gives a geometric means of determining the proper inclination of

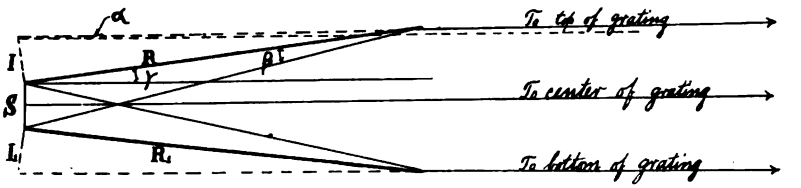


FIG. 2.

the reflectors to each other for any given slit-length and grating distance. For the sake of keeping the lines well separated in the drawing the angles are greatly exaggerated.

In the figure S is the slit, R, R_1 the two reflectors, and I, I_1 the slit's images in these reflectors. For a given position of the grating let a ray from the bottom of the slit be reflected from a point near the end of the upper reflector to the top of the grating. Other rays from the same point in the slit will go directly to the grating, others miss it entirely, but still others will be reflected from nearer points on R to parts of the grating below the top. The same thing holds for every other point in the slit, except that the higher it is, the nearer will be the parts of R that reflect its light to the grating. Similarly for the lower reflector.

To determine the proper setting of the reflectors, draw a line from the upper end of the image I, which should be of the same length as S, past the far end of R to the top of the grating, and another from the same point parallel to the line that joins the center of S with the middle point of the ruled surface. Let the

angle so formed be α, then if l is the length of the rulings on the grating, and L the distance from the slit to the grating,

$$\tan \alpha = \frac{l - 3S}{2L} \text{ nearly.}$$

Let the angle between R and the above ray to the top of the grating be β, and the length of the reflector a, then

$$\tan \beta = \frac{S}{a} \text{ nearly.}$$

Finally call the semi-angle between R and R_1 γ, then $\gamma = \alpha + \beta$, a determinate quantity.

The following table gives some of the numerical values of these constants with which I have worked.

Length of reflectors, 12.5 cm; focal length of grating, 6.4 m; length of rulings on the grating, 5 cm; distance of grating from the slit, a variable quantity, but as a somewhat extreme and unfavorable case say 4 m, which puts $\lambda 4400$, third order, at the center of the camera.

TABLE OF CONSTANTS.

Length of Slit	Distance Between Reflectors at Slit	Distance Between Reflectors at End from Slit	Required Radius of Circle of Light at Grating
1 mm	1 mm	4.47 mm	14 cm
1.5	1.5	5.92	17
2	2	7.38	22

From this it evidently is not necessary to use even a large condensing lens, or cone of light of wide angle; and, further, it clearly is easy to obtain a slit and its reflections practically three times the length of the slit alone, all sending light to the grating in such directions that it will produce superimposed, and therefore relatively brilliant, spectrum lines. Besides, the angle of incidence is so large that the images are nearly as bright as the slit itself. The reflectors with which I have worked consist of silver chemically deposited on glass optical flats, though this, in spite of the large angle of incidence, is probably not the best material for the ultra-violet.

In those cases where the image is practically a point and the

angle of the cone of light very large, better results might be obtained with suitable cylindrico-parabolic reflectors with their common foci at the center of the slit, but under all ordinary circumstances the flats are best and easiest to use.

I have found it convenient to fasten the reflectors respectively to an upper and a lower piece of brass, and to provide these with two sets of screws, one of which adjusts the reflectors relatively to each other, while the other, by resting on a small platform, adjusts the combination to the slit. The successful use of this attachment, instead of being difficult, as one might suppose, is very easy. None of the adjustments is difficult, and once the device is properly set, but little additional attention is required other than keeping the image at the proper place on the slit.

The general character of the results is shown in negatives VI and VII. Since these were taken only for the purpose of illustrating the principle, a very short slit, of about one millimeter, was used instead of a small image. It will be noticed that the lines of VI are triple, the outer components being broad and hazy; and that, if all three were made equally sharp and brought together, they would overlap through most of their length and produce a decidedly more intense line. This effect of tripling the lines and blurring the lateral ones was secured by tipping the mirrors, and thus putting the images out of line with the slit and also out of line with the rulings on the grating. Negative VII was taken when the mirrors were properly adjusted, and though the three sets of lines are here superimposed, it is free from noticeable defects of any kind.

III. SHAPE OF THE GRATING.

The general theory of reflecting gratings leaves great freedom of choice as to the shape of the surface, but the difficulties of construction, convenience of use, and other considerations so narrow the limits that, to the best of my knowledge, only flat and spherical concave surfaces are used.

If the light is sufficiently intense, the astigmatism of the concave grating as commonly mounted will do no harm, but in the case of feeble sources, or even faint lines in a bright source, this

will cause many parts of the spectrum to be overlooked that could easily be seen if all the energy was concentrated to closer limits. It would therefore be very desirable, while retaining normal spectra, to reduce the astigmatism to a minimum, and consequently it seems worth while to determine what surface will do this, and, if the theoretical one is difficult of construction, to find the nearest practical approach to it.

Let the slit, grating, and camera be, as in the Rowland mounting, on the vertices of a right-angled triangle, as shown in Fig. 3, where S is the slit, G the grating, and F the position of the camera or viewing telescope. Let the plane determined by S G F be horizontal, and the slit, which should be parallel to the rulings on the grating, vertical. Let the surface of the grating, concave as viewed from F, be a portion of a torus, and let the rulings be at right angles to the equator, and be divided into equal upper and lower halves by it. Further, let the radius of horizontal curvature of the grating be equal to $G F$, and its radius of vertical curvature equal to any desired value. Also let the rulings be equally spaced along a horizontal chord, so that the grating as completed and mounted will differ from the usual spherical concave grating only in having a different radius of vertical curvature. With these conditions it will be found, on making somewhat tedious but simple enough substitutions in Runge's formulæ,[1] *i. e*, by using the equations of a torus instead of those of a sphere, that the two concave gratings, spherical and toroidal, scarcely differ except in the matter of astigmatism.

The problem then reduces to that of finding the vertical curvature of a grating whose horizontal curvature is given, that will produce the required amount of concentration.

Let this requirement be that a point in the slit shall give a point image. To do this evidently the vertical curvature must be such that the sum of the distances from the given place in the slit to any point on a fixed ruling, say the middle one, and from that same point to the focus shall be a constant; that is, this particular ruling must be on the surface of an ellipsoid of revolution whose foci are at S and F.

[1] KAYSER'S *Handbuch der Spectroscopie*, I, p. 452 ff.

Let *A G B C* be the horizontal section of this ellipsoid. Then to determine the proper radius of vertical curvature of the torus, cut this ellipsoid by a vertical plane passing through *F* and *G;* that is, by a plane passing through one focus and the end of a latus-rectum through the other focus and normal to the plane determined by this latus-rectum and the major axis.

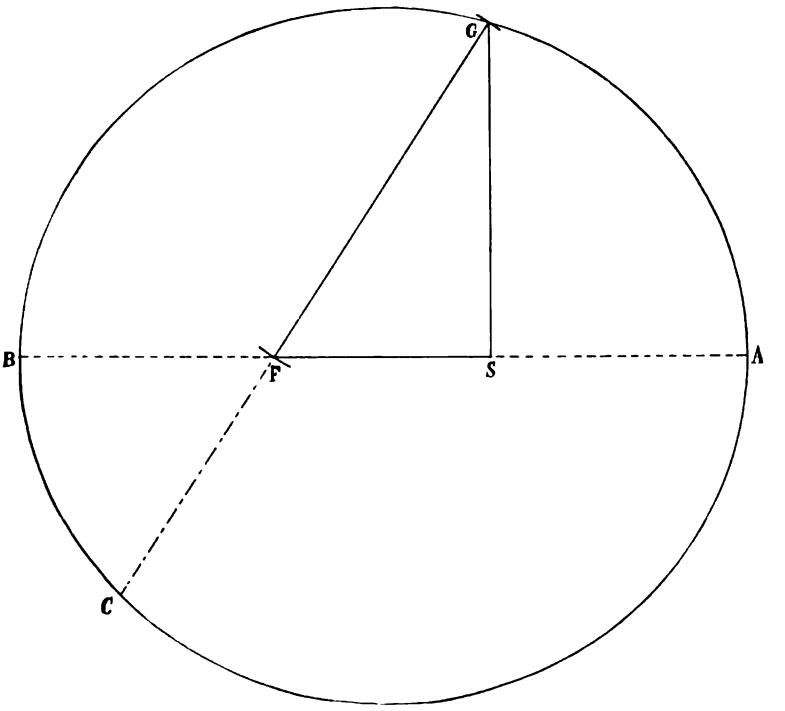


FIG. 3.

The curve thus cut out of the ellipsoid is an ellipse, and on revolving it about a vertical axis passing through *F* and parallel to its minor axis the required torus, a part of whose surface about *G* forms the grating, is generated.

Such an elliptical toroidal surface, however, is quite beyond the power of ordinary skill to form, but any uniform curve that closely fits this ellipse at the end of its major axis would give very nearly the same results, and doubtless among these the circle is the simplest; and, moreover, at the end of the major axis,

the place required, a circle whose radius is equal to the semi-latus-rectum has a very high order of contact with the ellipse, so that the circle is not only the simplest curve, but also one of closest approach to the ideally correct one.

To generate the required circular torus, revolve a circle whose radius is equal to the semi-latus-rectum of the elliptical section, as above described, at right angles to its plane, about a vertical axis passing through F, and have it so situated that the equator of the torus shall pass through G.

Of course, such a surface could give zero astigmatism at only one place, at F, where $S\,F$ is the distance between the foci of the ellipsoid of revolution. It will thus be necessary, after determining the radius of horizontal curvature $G\,F$, and the ruling to be used, to decide in what part of which spectrum the maximum intensity is desired. This will fix the value of $S\,F$, and through it, as shown above, the radius of vertical curvature, which in all cases will be equal to $S\,G$.

This same value for the vertical curvature, with the condition that a point source at S shall give a point image at F, is readily obtained from Mitchell's[1] and also from Runge's[2] general equation for astigmatism, but possibly the above direct and simple discussion of the problem is not superfluous, since it shows that the figure, as determined by these equations, is only an approximation, though an exceedingly close one, to the ideally correct surface.

Mitchell's equation is

$$C = -Z + Z\sqrt{r\left(\frac{\cos\mu + \cos\gamma}{\rho} - \frac{1}{R}\right)},$$

in which C is the half-length of a spectrum line due to a point source in the slit, Z the half-length of a ruling, r the radius of horizontal curvature, R the distance of the slit from the grating, ρ the radius of vertical curvature as I use it, or radius of the sphere as Mitchell uses it, γ the angle of incidence, and μ the angle of diffraction.

[1] ASTROPHYSICAL JOURNAL, 8, 107, 1898.

[2] KAYSER'S *Handbuch der Spectroscopie*, I, 464.

When the Rowland mounting is used, $R = r \cos \gamma$, and at the center of the camera $\mu = 0$; and therefore $C = 0$ when

$$r\left(\frac{1 + \cos \gamma}{\rho} - \frac{1}{r \cos \gamma}\right) = 1 ,$$

that is, when

$$\rho = r \cos \gamma = R.$$

Runge's expression for the entire length of a spectrum line is

$$\frac{r'}{r} s + \left[\rho - a' + \frac{r'}{r}(\rho - a)\right]\frac{l}{\rho} ,$$

where s = length of slit, l = length of ruling, ρ = radius of curvature, vertical as I use it, r = distance of slit to grating, r' = distance of camera from grating, a = the x co-ordinate of the slit when the center of the grating is taken as the origin, a' = the corresponding x co-ordinate of the spectrum line.

When the Rowland mounting is used, and the x axis is the line joining the grating with the camera, we have, where γ is the angle of incidence,

$$a' = r', \; a = r \cos \gamma, \text{ and } r' \cos \gamma = r;$$

and therefore in this case for point sources, that is with $s = 0$, the length of the spectrum lines becomes

$$\left(\rho - r' + \frac{r'}{r}\rho - r\right)\frac{l}{\rho} ;$$

and this vanishes when $\rho = r$.

Evidently, then, zero astigmatism may be obtained by having the radii of horizontal and vertical curvature of the grating equal respectively to its distances from the camera and the slit.

Finally, it would be extremely desirable so to set the cutting diamond point that the grating shall give its most brilliant spectrum at that place where the astigmatism is least.

Possibly a satisfactory approach to the above surface may lie beyond the skill of practical opticians, but, whether for the present this is true or not, it seemed to me worth while to discuss one of its valuable properties when used for a grating.

IV. CAMERA ATTACHMENTS.

Prism and plane-grating spectrographs, when illuminated by a line source, produce line images, while the spherical concave

grating, as usually mounted, gives lines, whether the source is also a line or only a point.

Therefore, in all these cases the energy in any given wave-length is spread out over a line of greater or less magnitude, and, if it happens to be very feeble, may not be detected at all. Clearly, then, it would be desirable to concentrate this energy within narrower limits, and for photographic purposes this is quite possible, since for a given wave-length the actinic effect depends rather on the quantity of light that reaches a given point than on small differences in direction.

In what follows I shall assume that the spectrum lines are straight, as they are when due to gratings, either plane or concave, and as they may be in prism instruments, provided the slit is properly shaped. Further, though the principles are exactly the same for all, I shall describe the methods as applied to a large concave grating spectrograph, the instrument with which I have worked.

a) One method of thus concentrating the spectrum lines is to place in front of the photographic plate a short-focus cylindric lens, whose axis is parallel to the focal curve. By suitably adjusting the distance between the plate and lens the normally long lines are reduced to very short ones, and the intensity greatly increased. This method, however, is open to several objections. The lines probably will not have exactly the same intervals between them that they would have without the lens, and, besides, they are not so clearly defined. If the lens is long, it must necessarily be made of glass, and therefore not be applicable to much of the ultra-violet. Still, even with all these objections, the cylindric lens is of service in hunting for very faint lines, especially if they happen to be somewhat hazy.

b) Another, and in most cases much better method is to place good reflectors in front of, but close to, the plate. This arrangement is shown in Fig. 4, where P is the photographic plate, R_1, R_2 the reflectors, and G the grating. Evidently some of the light will reach the plate directly, while two other portions will be reflected to the same place, one from R_1, the other from R_2, and the three acting together will correspondingly increase

the photographic effect. The reflected rays clearly pass over somewhat longer paths than do the direct ones, but this difference, one millimeter at most, is not enough, when long-focus gratings are used, materially to affect the definition.

To secure the best results the surfaces of the reflectors must be normal to a plane containing the spectrum line in question and passing through the center of the grating. Therefore, for the middle of the camera of a concave grating the reflectors should not be optical flats, but each should be a portion of a right cone whose vertex is on the tangent to the middle of the central ruling. For a short distance this very closely coincides

FIG. 4.

with a flat, and negative VIII was secured with such optical plates. A mere glance at this negative will show that the reflectors greatly increase the photographic effect, while a careful examination of it will reveal but little, if any, injury to the definition.

Much greater concentration could be had by using surfaces whose vertical sections through the axis give parabolas instead of straight lines as furnished by the cone. The photographic plate should, of course, in this case be placed near the latus-rectum. Each reflecting surface, according to this plan, would be part of a parabolic toroid.

Accurately to figure either of these surfaces, the conical or parabolic toroidal, would be difficult, but even for the best results accuracy of shape is not essential. It is only necessary that the reflected light should fall somewhere on a short straight line, and therefore it is sufficient if the horizontal sections of the reflectors are true circles. The vertical sections in the one case need not give perfectly straight lines, nor those of the other exact parabolas. With these conditions the surfaces would be very much more easily figured, and yet quite as efficient in every way.

Evidently in many cases two or more of these methods may be combined, with the result of correspondingly multiplying the final brilliancy of the spectrum lines.

While I have seen no reference to the actual use of any of these modifications or attachments, still from their very simplicity it would be surprising if no such application has been made of them. Indeed, Rowland[1] suggests the use of a cylindric lens, which from the context I take to be the same as IV, *a*, but does not say that he actually so used it. Besides, I know that I, *d*, has independently occurred to Wadsworth, but I have seen no account of its use in this way.

The material of the grating, various photographic processes, and manipulations, and even means of intensifying the brilliancy of the arc, flame, or other source, might all be discussed under the general title of this paper, but at present I have nothing to contribute on any of these important points.

UNIVERSITY OF VIRGINIA,
April 1903.

[1] *Physical Papers*, p. 489.

THE SPECTRUM OF *o CETI*.[1]

By JOEL STEBBINS.

ON account of the great instrumental power required for the observation of the spectra of faint objects, changes in the spectra of long-period variable stars have not been well studied. In fact, there is no star which undergoes a large variation in brightness whose spectrum has been systematically followed from maximum to minimum, or *vice versa*. It is proposed to give here the results of a study of the spectrum of *o Ceti*, or *Mira*, made, at the suggestion of Director Campbell, with the thirty-six-inch refractor of the Lick Observatory, from June 1902 to January 1903. During this period the star faded in brightness from 3.8 to 9.0 magnitude. The first photograph of the spectrum was obtained about three weeks after the predicted time of maximum, and a series of plates was secured covering the interval to minimum. No negatives were obtained after the star had again begun to increase in brightness.

The most important articles concerning the spectrum of *Mira* are those of Vogel,[2] Sidgreaves,[3] and Campbell.[4] Neither Vogel nor Sidgreaves followed the star long enough to find much change in its spectrum, and Campbell's work was mainly in connection with observations of the star for radial velocity, with the Mills spectrograph.

INSTRUMENTS AND METHODS.

The spectrograph used in my observations was the one employed by Messrs. Campbell and Wright in their work on

[1] "Dissertation in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the University of California," *Lick Observatory Bulletin* No. 41.

[2] H. C. VOGEL, "Ueber das Spectrum von *Mira Ceti*," *Sitzungsberichte der Berliner Acad.*, p. 143, 1896.

[3] WALTER SIDGREAVES, "The Spectrum of *o Ceti* as Photographed at Stonyhurst College Observatory," *Monthly Notices*, **58**, 344, April 1898.

[4] W. W. CAMPBELL, "Note on the Spectrum of *o Ceti*," ASTROPHYSICAL JOURNAL, **9**, 31, January 1899.

Nova Persei,[1] designated as "Spectrograph I." It is the regular Mills spectrograph converted into a one-prism instrument. It gives good definition, on the same photograph, of the region from $\lambda 3700$ to $\lambda 5600$. The length of this range of spectrum is 28 mm. Although this dispersion is only about one-fifth of that of the three-prism instrument, it is enough to yield a very fair determination of velocity.

When the seeing is good, an exposure of ten minutes will give a satisfactory negative of a star to which the *Draper Catalogue* assigns the photographic magnitude 5.0. If *o Ceti* were of about the tenth photographic magnitude at minimum, it would require, roughly, an exposure of one hundred times as long as a fifth-magnitude star, or about sixteen hours. Some of the light being in bright lines, we should expect a still longer exposure to be necessary. This estimate agrees with my experience. About July 1, 1902, an hour's exposure, just before daylight, could be made on the star, and it was bright enough to photograph in that time. An exposure of six hours at the time of minimum, using the fastest photographic plate obtainable, was not sufficient to produce a measurable image, although much could be seen in a qualitative way. On January 5, 1903, the date of my last negative, an exposure of five hours, beginning soon after sunset, was possible. Early in March 1903, the spectrum was again bright enough to record itself with a fairly short exposure; and in the absence of the writer, Messrs. Reese and Curtis were ready to make an attempt, but bad weather prevailed.

A Huggins reflecting slit is used with this instrument. A ninth-magnitude star can be followed accurately.

No flexure of the instrument was noticed on any of the earlier plates. The comparison spectrum was inserted at least four times during each exposure, and the definition was satisfactory. Several long exposures in September and October were imperfect, but the trouble was finally removed by taking greater precautions in tightening the various clamps and screws.

As there were no means of controlling electrically the

[1] *L. O. Bulletin* No. 8.

temperature of the instrument, changes of temperature must have effected the definition. The spectrograph when in use was covered with two thicknesses of woolen blanket.

The same emulsion of Cramer's "Crown" plates was used for most of the work. A few exposures in July and August were made on Cramer's "Isochromatic Instantaneous," but the "Crown" plates were more sensitive.

TABLE I.

o Ceti. 13E, 1902, July 16. $\lambda = 2123.10 + \frac{332873}{220.653 - R}$.

Description	Micrometer *R*.	$R_{0.00} - R$	$\frac{1}{R_{0.00} - R}$	λ From Formula	Correction to Formula	Reduction to Sun	λ
	20.706	199.947	500138	3787.91	+*0.14*		*3788.05*
	21.560	199.093	502278	3795.05	+*0.10*		*3795.15*
Bright, faint	21.950	198.703	503264	3798.33	+0.10	+0.36	3798.79
	26.124	194.529	514062	3834.27	+*0.09*		*3834.36*
Bright	26.301	194.352	514530	3835.83	+0.06	+0.36	3836.25
Bright, faint	28.241	192.412	519718	3853.10	+0.05	+0.37	3853.52
	29.012	191.641	521809	3860.06	*0.00*		*3860.06*
	32.111	188.542	530386	3888.61	+*0.06*		*3888.67*
Bright	32.208	188.445	530650	3889.52	+0.03	+0.37	3889.92
	32.870	187.783	532530	3895.75	+*0.05*		*3895.80*
	33.300	187.353	533752	3899.82	+*0.03*		*3899.85*
Bright, faint	33.950	186.703	535610	3906.00	0.02	+0.37	3906.39
	36.230	184.423	542232	3928.04	+*0.04*		*3928.08*
	40.364	180.289	554065	3969.43	−*0.02*		*3969.41*
H. line of Sun	40.345	180.308	554007	3969.24	0.00	+0.38	3969.62
Bright	40.479	180.174	555019	3970.61	0.00	+0.38	3970.99
5 wide	41.381	179.272	557812	3979.91	−0.01	+0.38	3980.28
5 wide	41.665	178.988	558697	3982.85	−0.01	+0.38	3983.22
3	42.396	178.257	560988	3990.48	−0.01	+0.38	3990.85
Bright place	43.116	177.537	563263	3998.05	−0.01	+0.38	3998.42
4	43.251	177.402	563692	3999.48	−0.01	+0.38	3999.85
Bright place	43.518	177.135	564541	4002.30	−0.01	+0.38	4002.67
	43.810	176.843	565473	4005.41	*0.00*		*4005.41*
Bright place	43.996	176.657	566069	4007.39	−0.02	+0.38	4007.75
4 wide	44.170	176.483	566627	4009.25	−0.02	+0.38	4009.61
4	45.282	175.371	570220	4021.21	−0.02	+0.38	4021.57
5 very wide	45.610	175.043	571288	4024.76	−0.02	+0.38	4025.12
4	45.850	174.803	572073	4027.38	−0.03	+0.38	4027.73
4 fine	46.436	174.217	573997	4033.78	−0.03	+0.38	4034.13
	47.546	173.107	577077	4046.03	−*0.05*		*4045.98*
3	48.394	172.259	580521	4055.50	−0.03	+0.39	4055.86
	49.130	171.523	583012	4063.79	+*0.15*		*4063.94*
	49.849	170.804	585466	4071.96	−*0.05*		*4071.91*
4 wide	50.414	170.239	587409	4078.43	−0.04	+0.39	4078.78
3 poor	50.850	169.803	588918	4083.45	−0.04	+0.39	4083.80
3 poor	51.274	169.379	590302	4088.36	−0.04	+0.39	4088.71

All of the plates taken in this work were measured and reduced with the iron spark-spectrum as comparison. From the ultra-violet to λ4415 there are many strong and sharp iron lines. From λ4415 to λ5600 the iron lines are fainter, and many of them have companions. The region from λ4800 to λ5600 of the iron spectrum was photographed with three prisms. It was found that there were several lines in this region which had no companions, and which could be used without hesitation. Close double lines were assumed to have positions depending upon the relative intensities of their components.

The plates were measured with one of the measuring microscopes in use for the regular line of sight work. Each plate was measured with both violet left and violet right in the eyepiece.

A plate of the sky was taken with a very long slit and the amount of the curvature of the comparison lines was determined in the same manner as by Campbell.[1] The curvature corrections were found to be insignificant.

The reductions from micrometer readings to wave-lengths were based on the Cornu-Hartmann formula, much of the computation being done with a Brunsviga calculating machine. The wave-lengths of the comparison lines were taken from Rowland's table. The example on page 343 shows the complete reduction of a plate after the constants of the formula had been derived. Italicized figures are used for the comparison lines.

DATA OF THE OBSERVATIONS.

In Table II is given a list of the plates secured with Spectrograph I. Mr. Wright obtained three plates in 1901, and he kindly turned them over to me for measurement and discussion. The width of the slit is expressed in terms of the divisions on the head of the screw, one division corresponding to 0.025mm. One plate, taken on September 16, was discarded on account of great flexure during the exposure. Something of interest was found on all other plates, except 57 A.

The brightness of the star at the time when each plate was

[1] ASTROPHYSICAL JOURNAL, 8, 144, 1898.

taken is shown in Fig. 1. The observations of visual magnitude are given at the end of this paper. As the plates are well distributed along the light curve until about the time of the minimum, no great change in the spectrum could have escaped observation.

ABSORPTION SPECTRUM.

As is well known, *o Ceti* and the other long-period variables have absorption spectra of Secchi's third type. Some observers have found the region from $H\gamma$ towards the red crossed by a series of dark bands, with edges sharp towards the violet, and they report that from $H\gamma$ to the ultra-violet the dark-line spectrum is very similar to that of the Sun. At first glance this seems to be verified by my plates, but a closer study shows the details to be very different. Fig. 2 shows an eightfold enlargement of the solar spectrum, photographed with Spectrograph I on a lantern-slide plate. The absorption spectrum of the star, Figs. 3–6, is seen to resemble little that of the Sun. The date of Fig. 4 should be August 11, instead of August 4, as is printed.

In comparing the star spectrum with the solar spectrum, it was not found best simply to measure the plates and then look for coincidences in Rowland's table. It is easy to find a line in the table which agrees in position with the one on the plate, but the intensities may be very different. The method adopted was to compare a plate of *o Ceti* with one of the sky. The two negatives, film sides together, were examined under the microscope with a low power.

The strong calcium lines *g*, H, and K, are present in the spectrum of the star, the *g* line being much more intense than in the solar spectrum. The strong iron lines of the Sun are not so prominent in *o Ceti*; in fact, they do not show with low dispersion. Of the large number of lines in the star spectrum, there are very few which coincide with lines of like intensity in the solar spectrum.

The absorption spectrum of *Mira* was measured accurately on seven plates. Each plate was measured and reduced independently, so that a faint line might be measured on one plate without

being noticed on any of the others. This does not mean that it had developed, or that its intensity had changed. Such lines, of which many were measured, could be easily obscured by an irregular arrangement of the silver grains. The best method of verifying changes is to examine different plates simultaneously, in pairs, under the microscope. This has been done, and all changes in intensity or character have been noted.

TABLE II.

Plates Secured With Spectrograph I.

Plate No.	Astronomical Date	Mt. Hamilton Sidereal Time, Middle of Exposure	Length of Exposure	Brightness of σ *Crti.* Mag.	Photo-Plate	Slit Width	Seeing	Remarks
	1902							
2B	June 27	22h 09m	0h 45m	3.8	Crown	1.4	Poor	
3C	27	22 39	0 10	3.8	Crown	1.4	Poor	
4E	27	22 46	0 03	3.8	Crown	1.4	Poor	
7C	July 6	22 47	1 15	4.1	Crown	1.4	Good	
8D	6	23 28	0 05	4.1	Crown	1.4	Good	
9E	6	23 35	0 02	4.1	Crown	1.0	Good	
13E	16	23 07	2 00	4.6	Iso.	1.4	Fair	
14F	16	0 12	0 05	4.6	Iso.	1.0	Fair	Shows bright lines only
18F	22	23 22	2 30	5.1	Iso.	1.4	Fair	
21F	29	23 30	2 40	5.3	Iso.	1.4	Fair	
25F	Aug. 4	0 07	3 00	5.4	Iso.	1.4	Good	
26D	4	1 44	0 07	5.4	Crown	1.0	Good	Shows bright lines only
27D	11	0 58	2 35	5.6	Crown	1.5	Fair	
28D	25	0 53	2 35	6.4	Crown	1.5	Poor	
33D	Sept. 6	0 45	3 00	7.0	Crown	1.5	Poor	
39D	22	2 20	4 40	7.1	Crown	1.6	Poor	Flexure during the exposure
42A	Oct. 4	1 47	5 53	7.8	Crown	1.8	Fair	Slight flexure
48A	20	4 10	5 40	8.5	Crown	1.8	Fair	Flexure
54A	Nov. 25	3 10	5 00	9.2	Crown	2.5	Poor	
57A	Dec. 21	3 18	5 25	9.0	Crown	3.0	Poor	Nothing found on this plate
	1903							
58A	Jan. 2	3 12	5 17	9.0	Crown	3.0	Poor	
59A	5	3 36	4 51	9.0	Crown	2.5	Fair	
	1901							
2211C	Aug. 3	1 15	0 20	...	Iso.	1.2	Good	
2212D	3	1 32	0 03	...	Iso.	1.2	Good	Bright lines only
2235D	17	1 26	1 20	...	Iso.	1.3	.....	

In Table III are given the wave-lengths of the absorption spectrum as derived from all the measures. The numbers expressing the intensity indicate, in a general way, the relative strength of the lines. It was intended, on each plate, to assign intensity 1 to the faintest lines distinguishable, and intensity 10 to the strong line at $\lambda 4255$. On this scale the *g*, H, and K lines might be 100 or 500, it matters little. The intensities assigned are merely relative, and no doubt the scale varies much in different parts of the spectrum. The lines on each plate were

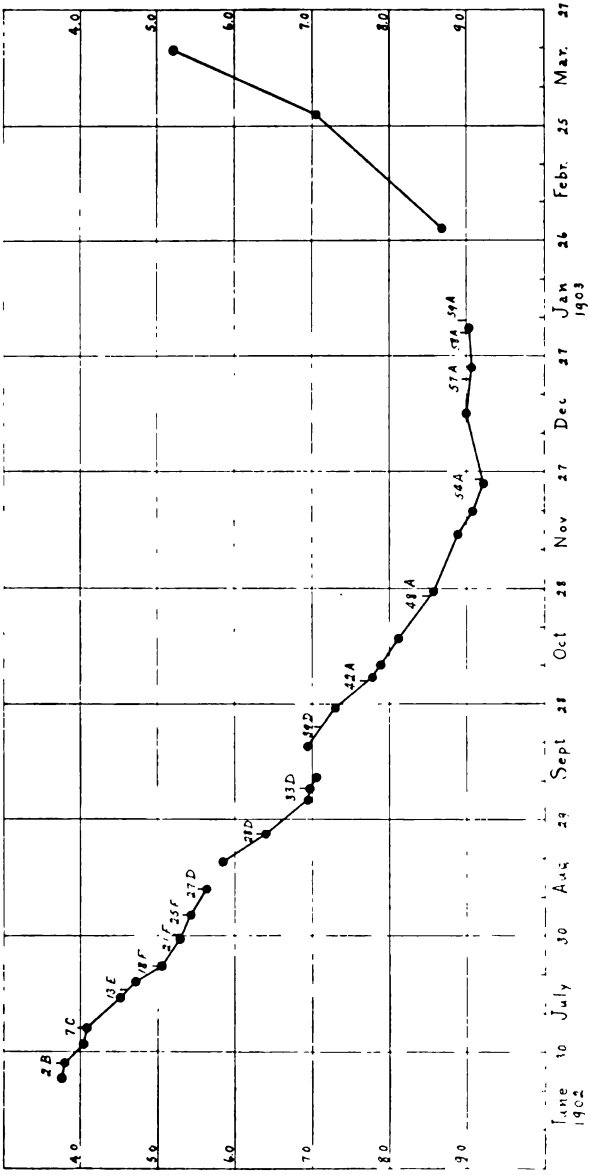


FIG. 1.—*o Ceti*. Light-Curve, 1902–3.

TABLE III.

Absorption Spectrum of *o Ceti*.

Intensity	Observed λ	Residuals in o.or t.m.							Remarks
		7C	13E	18F	25F	27D	28D	2235 D	
	3936.—	..	..	..	..	—	—	..	K line of Sun, very strong. Adjacent bright lines interfere with measurement of λ
5 wide	3945.09	04	.	..	..	*01*	*02*	..	
5 wide	3949.28	14	..	..	..	—	*14*	..	
2 fine	3951.65	..	..	..	..	..	—	..	
2	3957.27	..	..	..	..	..	—	..	
4	3958.98	—	..	..	..	*03*	03	..	
5 wide	3962.55	—	..	..	..	00	00	..	H line of Sun, very strong. Adjacent bright lines interfere with measurement of λ
	3969.6–	—	—	..	..	..	—	..	
4 wide	3980.20	..	08	..	..	..	*09*	..	
5 wide	3983.18	10	04	..	*19*	*01*	05	..	
5	3990.64	..	21	..	*24*	*04*	07	..	
2 fine	3997.61	—	..	..	..	..	..	..	
4	3999.72	09	13	..	*02*	*07*	*22*	11	
4 wide	4005.6–	..	..	..	—	..	..	..	Perhaps several
5 wide	4009.63	..	*02*	..	05	17	..	*20*	Perhaps several
4	4010.04	..	..	..	..	..	—	..	
3	4012.18	..	..	..	*15*	..	23	*09*	
1	4018.91	..	..	..	..	..	—	..	
4 wide	4021.41	*14*	16	..	00	*08*	10	*04*	
5 wide	4025.08	17	04	..	*17*	08	03	*14*	
3 wide	4027.68	20	05	..	10	..	*09*	*26*	
2 wide	4030.16	..	..	..	03	..	*04*	..	
6	4032.22	*19*	..	..	15	*05*	38	*27*	
7	4034.04	05	09	..	*01*	*02*	03	*11*	
1 fine	4035.71	..	..	..	—	..	..	..	
2	4036.22	..	..	..	..	..	—	..	
3	4045.16	..	..	..	*03*	02	..	..	
2 fine	4047.02	—	..	..	..	..	..	..	
	4043.—	..	..	..	—	—	..	—	Several lines run together
	4050.—	..	..	..	—	—	..	—	
3	4053.25	..	..	..	..	..	..	—	
6	4055.71	..	15	..	*01*	*09*	*19*	12	
2	4058.14	..	.	..	12	..	..	*11*	
2 wide	4060.43	..	..	..	03	..	..	*03*	Too wide and faint to measure. Probably several
	4064.—	..	..	..	..	—	—	..	
4 wide	4068.24	..	..	..	..	..	..	—	Too wide and faint to measure. Probably several
	4072.—	..	..	..	..	—	—	..	
5 wide	4072.28	.	..	..	09	*01*	..	*09*	
2 fine	4077.24	—	—	..	08	*11*	02	—	On plates 7C, 13E and 2235D these two lines run together
7 wide	4079.51	—	—	..	*02*	04	*03*	—	
4 wide	4083.85	*15*	*05*	..	12	*36*	06	39	
3	4088.88	20	*17*	..	*07*	11	*04*	*05*	
1 fine	4091.33	..	—	..	..	..	..	..	
5	4093.55	..	18	..	00	*12*	*21*	13	
3	4097.08	..	..	..	01	13	*08*	*04*	
3 fine	4100.56	..	..	..	..	..	—	..	
4	4105.88	..	..	..	*07*	..	..	08	
5	4110.34	..	*07*	..	*09*	..	08	09	
5	4112.97	..	*07*	..	07	..	..	..	
4 fine	4116.14	..	..	..	..	..	..	—	
4 fine	4117.71	..	00	..	*15*	..	18	*01*	
	4117.9	..	..	..	..	..	—	..	Several
	4119.1	..	..	..	..	..	—	..	

TABLE III.— *Continued.*

Absorption of Spectrum of *o Ceti.*

Intensity	Observed λ	Residuals in o.01 t.m.							Remarks
		7C	13E	18F	25F	27D	28D	2235D	
4 fine	4121.44	..	..	..	..	..	..	—	
4 fine	4122.62	..	..	..	..	..	..	—	
6	4124.66	01	21	..	*15*	..	*02*	*06*	
2 fine	4126.99	..	..	..	..	..	—	..	
3	4129.04	..	..	..	—	..	..	..	
3	4130.60	..	..	..	*04*	05	..	..	
4 fine	4133.00	..	..	..	..	.	—	..	
5	4135.32	*17*	05	..	*12*	*10*	17	15	
3	4137.77	11	..	..	..	..	..	*11*	
2	4140.96	..	..	..	00	..	..	*01*	
4 wide	4144.—	—	..	..	—	—	..	—	
3	4150.73	..	..	..	*12*	..	..	12	
3	4153.40	..	*04*	..	*01*	..	..	04	
3	4156.78	..	12	..	..	..	..	*11*	
4	4160.89	..	..	..	..	..	..	—	
3	4165.82	..	..	..	..	..	..	—	Poor
3	4169.27	..	..	..	17	..	00	*18*	
3	4173.56	..	—	..	..	..	..	..	
7	4175.41	11	28	..	*01*	03	*23*	*05*	
4	4178.32	*10*	13	..	*07*	*16*	23	*04*	
2	4180.96	..	..	..	..	*08*	07	..	
3	4183.73	..	..	..	..	..	..	..	
4 wide	4188.17	..	..	..	*16*	14	*23*	26	
5 wide	4191.79	*01*	08	..	07	*08*	*36*	32	
3	4199.79	..	..	..	..	..	..	..	
2 wide	4211.19	..	..	..	*27*	27	..	..	Poor
4 wide	4214.86	..	..	..	12	..	*11*	..	
	4227.84	00	28	..	—	*10*	*18*	—	*g* line of Sun, very strong
5	4235.37	..	*10*	..	04	..	07	..	
3	4242.82	..	..	..	*03*	..	..	04	
3	4251.60	..	02	..	*11*	..	..	10	
10	4255.46	06	33	..	*08*	*26*	*06*	01	
1	4259.64	..	..	..	..	..	..	..	
1 wide	4261.87	..	..	..	—	..	..	..	Poor
4	4273.15	..	*02*	..	00	..	..	02	
8	4275.84	00	17	..	*06*	*03*	*04*	*06*	
5	4285.28	..	—	..	..	..	..	.	
6	4290.72	*22*	..	..	13	03	*11*	17	
2 fine	4292.45	—	..	..	..	..	..	..	
1	4295.07	..	..	..	..	..	..	—	
2 fine	4297.56	..	..	..	..	..	..	—	
3	4300.16	06	..	..	..	..	..	*06*	
2	4304.06	..	—	..	..	..	..	..	
3	4307.23	*02*	*12*	..	..	..	..	15	
3	4310.57	..	07	..	..	..	..	*06*	
	4314.29	..	..	00	00	..	01	00	Head
2 fine	4320.03	—	..	..	..	..	..	..	
3	4326.97	06	..	..	*10*	..	..	05	
2	4331.13	..	..	..	..	..	..	..	
2	4348.12	.	..	..	..	..	..	..	
	4353.60	..	18	08	*06*	..	.	*18*	Head
4	4380.31	*03*	..	..	*06*	..	*25*	35	
5	4385.49	*12*	*09*	..	*04*	14	00	02	
4	4390.71	22	02	..	*14*	*11*	25	24	
	4395.89	24	..	24	*13*	—	..	*36*	Head

TABLE III.—*Continued.*

Absorption of Spectrum of *o Ceti.*

Intensity	Observed λ	Residuals in o.1 t.m.							Remarks
		7C	13E	18F	25F	27D	28D	2235D	
	4422.4	..	2	1	1	1	0	1	Head
4	4461.1	..	..	..	—	..	..	..	
	4463.0	..	4	1	2	3	4	0	Head
	4505.7	..	0	1	2	..	..	1	Head
4 wide	4514.7	..	..	..	..	—	..	..	
3 wide	4519.8	..	..	..	..	—	..	..	
2	4530.7	..	..	..	—	..	..	..	
4	4536.8	..	2	3	1	..	5	..	
	4548.8	..	4	2	4	..	..	6	Head
	4585.0	..	2	1	3	4	3	3	Head
	4626.8	..	1	4	2	6	2	0	Head
	4669.6	..	1	3	5	3	[21]	6	Head. Measure of Plate 28D
5	4709.2	..	2	0	..	1	..	3	rejected for discordance
	4714.2	..	4	3	1	..	..	..	Head
10	4739.5	..	1	2	3	1	3	4	
	4760.0	..	2	4	3	4	6	5	Head
	4804.2	..	0	4	3	0	·5	1	Head
	4842.8	..	..	..	..	2	2	..	Head
	4954.1	..	2	3	3	4	0	0	Head
	5167.0	..	2	8	2	..	..	5	Head
	5308.3	..	—	..	..	..	..	..	Head
	5358.8	..	—	..	..	..	..	..	Head
	5439.4	..	—	..	..	..	..	..	Head
	5447.9	..	2	0	7	..	..	5	Head
	5498.0	..	—	..	..	..	..	..	Head
	5568.7	..	..	..	..	..	..	—	Head

assigned intensities when the plate was measured; and when the different results were brought together, a sort of mean of the estimates was taken for each line. Where the estimates did not agree well, the line was compared directly on two or more plates to see if any change had taken place.

The residuals in the table were formed by subtracting the mean wave-lengths from the values given by each plate. All plates were assigned equal weight in forming the means. Periods (. . . .) signify that when the plate was measured nothing was noticed in the spectrum at that place. A dash (—) denotes that the line was observed, but could not be measured accurately. When a line was measured on only one plate there is no residual, and the dash indicates the plate referred to.

Plate 18F was measured only from λ 4300 towards the red.

The comparison on 7 C being underexposed to the red of λ 4415, the plate was not measured in that region.

From the residuals in Table III it is found that the probable error of the wave-length of a dark line derived from one plate is ±0.10 tenth-meter. When a line was measured on six plates, the probable error of its mean position is therefore ±0.04 t.m. In computing the probable error, the residuals from all lines to the violet of λ4400 were used.

Many of the lines being broad or unsymmetrical and difficult to measure, the residuals from different plates are not larger than would be expected. While the agreement of the results with each other is not a test as to systematic errors, yet I feel sure that the wave-lengths of but few of the dark lines measured on several plates can be in error by as much as 0.2 t.m.

A glance at the residuals shows that there is no evidence of variable velocity in the line of sight. Those from Mr. Wright's plate No. 2235 D, which was taken in 1891, indicate that the velocity was about the same during the corresponding phase of the star's light curve, a year previous. The residuals for each plate have been averaged; and if there were any change in the velocity it would appear in the mean residuals. The arithmetical means of the residuals to the violet of λ4400, and the corresponding velocities in kilometers per second, are as follows:

TABLE IV.

Date	Plate No.	t.m.	km
1902, July 6	7 C	+0.02	+1
16	13 E	+0.00	+4
Aug. 4	25 F	−0.03	−2
11	27 D	−0.02	−1
25	28 D	−0.03	−2
1901, Aug. 17	2235 D	+0.02	+1

Since many of the lines were not measured on all of these plates, the average residual of one plate is not strictly the average deviation of the measures on that plate from the mean of all the plates, but no sensible error is introduced by this assumption. When it is remembered that all lines, good and poor, were

included in the means, and that these plates were taken with a one-prism instrument giving a dispersion of but one-fifth of that of the regular Mills spectrograph, the range of only 6 km on six plates is very satisfactory. None of the plates taken later in 1902 are good enough to afford comparable conclusions as to the velocity; and it was thought best to measure only first-class plates in this connection.

The result that the star's velocity was constant over a certain period was derived without assuming the coincidence of any of the star-lines with lines of the solar spectrum, or with other known lines. A determination of the actual radial velocity requires an assumption as to what the wave-lengths of the star lines would be if they were not affected by velocity. Agreements of star and solar lines were looked for, by direct comparison of the star plates with a solar plate, and the only coincidences which seem certain are given in Table V. The wave-lengths and other data of the solar lines are from Rowland's table.

The H and K lines could not be measured accurately; the double manganese line at λ4030.92 gives a discordant result, which was rejected, and it looks as though both a strontium and a titanium line make up a stellar line at λ4079.51. The velocity of + 66 km was therefore derived from measures of only six lines, and none of these are fine and sharp enough to give the best determination.

TABLE V.

Coincidences of Dark Lines in the Spectra of o *Ceti* and the Sun.

o *Ceti*		Sun	Displacement	Velocity,	Sun
Intensity and Character	λ	λ	t.m.	km	Substance and Intensity
Strong	3936.—	33.82	———	———	*Ca* 1000 (K)
5 wide	3945.09	44.16	+0.93	+71	*Al* 15
5 wide	3962.55	61.67	+0.88	+67	*Al* 20
Strong	3969.6	68.62	———	———	*Ca* 700 (H)
6	4032.22	30.92	[+1.30]	[+96]	*Mn* 4 and 5
7	4034.04	33.22	+0.82	+61	*Fe-Mn* 7
7 wide	4079.51	77.88	———	———	*Sr* 8
		78.63	———	———	*Ti* 3
Strong	4227.84	20.90	+0.94	+67	*Ca* 20 (*g*)
10	4255.46	54.50	+0.96	+68	*Cr* 8
8	4275.84	74.96	+0.88	+62	*Cr* 7
				Mean +66 km	

Campbell in 1898 found +62 km with the three-prism instrument. The interval between August 29, 1898, the date of his first plate that year, and August 25, 1902, is equal to 130 days more than four periods of 331 days. The radial velocity has therefore been observed to be constant over about two-fifths of the period of light change.

To determine more coincidences, I have measured one of the high-dispersion plates of *o Ceti*, taken by Dr. Campbell in 1897. Although the plate is underexposed, about seventy lines were measured between λ4300 and λ4420, and more than twenty coincidences were found by direct comparison with a solar plate. The results are given in Table VI. As was done for the one-prism plates, the intensities are expressed on an arbitrary scale of 10.

TABLE VI.

Absorption Spectrum of *o Ceti*. High Dispersion. Plate 576 B. December 15, 1897.

o Ceti		Sun	Displacement t.m.	Velocity km	Remarks
Intensity and Character	λ				
Several	4299.7–	.....	.....	.....	Narrower in ⊙
	4300.8–	.....	.....	.....	
3	4301.58	.....	.....	.....	Closer double in ⊙
3	4302.17	.....	.....	.....	
2	4303.56	02.69	+0.87	+60.6	*Ca* 4
5, double?	4307.10	.....	.....	.....	Not ⊙
3	4308.16	.....	.....	.....	Not ⊙
4	4308.94	———	———	---	07.91 *Ca* 3, 08.08 *Fe* 6
4	4310.83	.....	.....	.....	Not ⊙
1	4312.90	.....	.....	.....	Not ⊙
Several	4315.2–	.....	.....	.....	Not ⊙
	4316.1–	.....	.....	.....	
2	4319.77	18.82	+0.95	+65.9	*Ca*, *Mn*? 4
Bright place	4325.93	.....	.....	.....	Same in ⊙
6	4326.85	25.94	+0.91	+63.0	*Fe* 8
1	4327.96	.....	.....	.....	Not ⊙
1	4328.69	.....	.....	.....	Not ⊙
1	4329.53	.....	.....	.....	Not ⊙
1	4330.30	.....	.....	.....	Not ⊙
5	4331.13	30.19	+0.94	+65.0	*V* 0 Stronger than in ⊙
5	4333.92	32.99	+0.93	+64.4	*Fe* 0
Several	4334.68	.....	.....	.....	Not ⊙
	4335.29	.....	.....	.....	
Poor 2	4335.91	.....	.....	.....	Possibly weak lines in ⊙
Poor 1	4336.38	.....	.....	.....	
1	4337.07	.....	.....	.....	Not ⊙
2	4338.16	37.22	+0.94	+65.0	*Fe* 5

TABLE VI.—*Continued.*

o *Ceti* Intensity and Character	λ	Sun λ	Displacement t.m.	Velocity km	Remarks
2	4338.61	37.72	+0.89	+61.5	*Cr* 3
1	4339.11	.....	.....	.....	Not ⊙
3	4345.56	44.67	+0.89	+61.4	*Cr* 4
4	4348.27	.....	.....	.....	Not ⊙
1	4352.16	51.22	+0.94	+64.8	*Cr* 3
Poor 2	4352.90	51.93	+0.97	+66.8	*Cr* 3
Head of band	4353.7-	.....	.....	.....	Not ⊙. Mean of 4 plates with one prism 4353.6
5	4353.91	52.91	[+1.00]	[+68.9]	*Fe* 4. This line always gives + residual in other stars
3	4356.97	.....	.....	.....	Perhaps ⊙, not certain
1	4349.70	.....	.....	.....	
1	4360.12	.....	.....	.....	
1	4360.65	.....	.....	.....	
1	4361.73	.....	.....	.....	All weak lines in star
Poor 1 wide	4363.05	.....	.....	.....	No coincidences certain
2	4365.56	.....	.....	.....	
2	4367.10	.....	.....	.....	
1	4367.87	.....	.....	.....	
4	4369.07	.....	.....	.....	Not ⊙
1	4370.48	.....	.....	.....	Not ⊙
1	4371.23	.....	.....	.....	Not ⊙
4	4372.22	——	——	——	(71.14 *Zr* 1, 71.22 −1, 71.44 *Cr* 4) same appearance
1	4373.17	.....	.....	.....	Not ⊙
Several to violet of	4376.4-	.....	.....	.....	Different in ⊙
Bright place	4376.68	.....	.....	.....	Not ⊙. This develops into a bright line
5	4377.06	76.11	+0.95	+65.1	*Fe* 6
1 wide	4378.06	.....	.....	.....	Different in ⊙
1 wide	4379.17	.....	.....	.....	
10	4380.27	79.40	+0.87	+59.5	*V* 4
1	4383.67	.....	.....	.....	Not ⊙
6	4384.62	83.72	+0.90	+61.6	*Fe* 15
7 double?	4385.90	——	——	——	84.87 *V* 3, 85.14 *Cr* 2
5	4390.26	89.41	-0.85	+58.1	*Fe* 2
4	4391.06	90.15	+0.91	+62.2	*V* 2 Stronger in star than in ⊙
Poor 2 wide	4392.76	91.92	+0.84	+57.4	*Cr* 1
7	4396.20	——	——	——	95.20 *Ti* 3, 95.41 *V* 2
2	4401 01	00.74	+0.87	+59.2	*V* 1
10 wide	4405.89	04.93	+0.96	+65.3	*Fe* 10
2	4407.02	.....	.....	.....	Not ⊙
4	4407.72	06.81	+0.91	+61.9	*V* 2
1	4408.70	——	——	——	07.81 V 2, 07.87 *Fe* 4
10	4409.41	——	——	——	08.36 *V* 2, 08.58 *Fe* 3, 08.68 *V* 2
Several to violet of	4410.7-	.....	.....	.....	Different in ⊙
4	4417.50	16.64	+0.95	+64.4	*V* 0. Stronger in star than in ⊙
Head of band	4422.5	.....	.....	.....	Not ⊙. Mean of 6 plates with 1 prism, 4422.4
	Mean of 20 lines			+62.7 km	

A comparison of Table VI with the corresponding part of Table III is a test of the reliability of the results obtained with one prism.

TABLE VII.

One Prism		Three Prisms	
Description	λ	λ	Description
3	4300.16	99.7– 00.8–	Several
2	4304.06	.....	Not measured
3	4307.23	07.10	5
3	4310.57	10.83	4
Head	4314.29	.....	Not measured
2	4320.03	19.77	2
3	4326.97	26.85	6
2	4331.13	31.13	5
2	4348.12	48.27	4
Head	4353.6–	53.7–	Head
4	4380.31	80.27	10
5	4385.49	84.62 85.90	6 7
4	4390.71	90.26 91.06	5 4
Head	4395.89	.....	Not measured
Head	4422.4–	22.5–	Head

As might have been expected, many lines observed as single with one prism are really made up of two or more components. This comparison also shows that the wave-lengths of Table III are probably not quite so accurate as the residuals indicate.

A brief summary of the number of dark lines of different elements observed in the spectrum may be of interest.

TABLE VIII.

Element	No. of Lines
Ca	6
Fe	11
Cr	9
V	11
Al	2
Sr	1
Mn	3 ?
Ti	2 ?

There can be no doubt of the presence of the first four elements in the list, and the aluminum and strontium lines are

prominent; but manganese and titanium must be considered as doubtful.

On account of the varying instrumental conditions it is easy to fall into error in judging as to changes in the intensity and character of dark lines. If all the negatives were of the same density and of uniform excellence, it would be easy to note such changes. There is one dark line which showed changes of which the reality is certain. This is the *g* calcium line at λ4227.84. Figs. 3–5 show how it broadened as the star grew faint. Measures of its width are necessarily rough, and must depend much upon the judgment of the observer. The measures of two plates are as follows:

TABLE IX.

Date	Plate	Width
1902, June 27	2 B	2 t.m.
September 6	33 D	9 t.m.

Other plates taken between these dates gave intermediate values. The intensity of continuous spectrum, in the neighborhood of λ4227, is almost the same on Plates 2 B and 33 D.

The general impression formed from examining the series of plates is that many other lines also grew broader as the brightness declined, but this is not certain. The effects of greater width of the slit, reduced intensity of the resulting negative, and flexure and temperature changes resulting from longer exposure, would all tend to make the lines wider and less sharply defined. The H and K lines are not shown on most of the plates, and nothing can be said as to changes in their character.

A few lines not visible on the early plates became prominent later. Four such lines have the following positions:

λ3990.64	λ4093.55
4045.16	4097.08

Lines of the solar spectrum which certainly coincide with these have not been found. The residuals in Table III indicate when these lines first appeared.

BANDS.

The prominent bands in the spectrum of *Mira* have been considered by some observers as a series of dark bands, with sharp edges towards the violet, and shading off toward the red. Others think them to be bright flutings like those of the arc spectrum of carbon. For convenience, they will be considered, in this paper, as dark absorption bands. On the plates of *o Ceti* and other third-type stars taken with Spectrograph I the bright portions of the banded spectrum are certainly brighter relative to the region above $H\gamma$, where there are no bands, than are the corresponding portions of the spectrum of a solar-type star. However, the dark portions are fainter than the same places in the solar type of spectrum.

In measuring the plates the micrometer wire was set on the division between a dark and a bright portion of the spectrum, so that the measurements remain the same whether the bands be considered as bright or dark. While the line of separation is probably sharp in most cases, its exact position is not easy to determine. No doubt the effect of irradiation plays an important part in direct visual observations. Since the bands considered as bright have their sharp edges towards the red, the effect of irradiation would be to make the observed wave-lengths of the heads too large. Corresponding to this visual error is the effect of spreading of the image on a photographic plate, causing an error in the same direction, and a slight allowance was made for it in executing the measures. At my request, Dr. Reese made several settings on the head of a band. The difference between his measures and mine was 0.3 t.m., my estimate being that much farther to the red. Although this was the only comparison made with another observer, it seems highly improbable that two persons should disagree by as much as one tenth-meter in the measurement of a sharply defined head.

No attempt was made to determine the position of the more diffuse ends of the bands. In many cases the intensity changes gradually from head to head, there being heavy absorption at the sharp edge, the spectrum growing uniformly brighter to the next line of demarcation.

Sidgreaves, on comparing the spectrum of *o Ceti* with that of *α Herculis* and other third-type stars, found a difference in the wave-length of the same band in different stars. For the head at λ5447 in *o Ceti*, he found the position of λ5458 in *α Herculis*, and intermediate values for *α Orionis* and *β Pegasi*. He suspected these discrepancies to be due to instrumental causes, but of this he was not certain. His work being done with an objective prism, he had no comparison spectrum with which to test the reality of the observed differences.

Along with the work on *o Ceti* in 1902, plates were taken of *α Herculis*, *β Pegasi*, *ρ Persei*, *α Ceti*, and *α Orionis*. As these stars are all bright, the exposures on them were comparatively short, from five minutes to one hour. Isochromatic plates were used, so that a range of spectrum as far to the red as λ5600 was covered. They were measured only for the positions of the heads of the bands. In the spectrum of *Mira* there are few dark lines shown in the region of the bands, but in the others many lines were recorded. Where there are more lines the bands are less prominent, and in the case of *α Ceti* and *α Orionis*, only three bands could be measured. All of these stars are on the regular Mills spectrograph program, and their radial velocities have been determined from their dark line spectra in the region of *Hγ*. As the wave-lengths of the bands must be affected by the radial velocities, it is necessary to apply to each observed wave-length a corresponding correction. In Table X is given, first the observed wave-lengths of the head of each band, already corrected for the orbital motion of the Earth. The accompanying correction is that which must be applied to each wave-length in order to reduce it to what it would be if the star's radial velocity referred to the Sun were zero. The mean result from the five stars has been taken after applying these corrections.

All bands visible on the plates were measured; and if the position of a band is not given for a star, it was not apparent. The results for any star are from all the plates of that star, except in the case of *α Herculis*, where one of the plates was a "Crown" plate and showed nothing to the red of λ4954.

The results obtained by Sidgreaves, Vogel, and others are

inserted for comparison. The corrections given by Sidgreaves in *Monthly Notices*, **59**, 509, have been applied to his first published wave-lengths. Vogel's photographs did not extend to the red of λ4800. The results of visual observations of the bands in third-type stars are taken from Frost's Scheiner.

TABLE X.

Measures of Bands.

o Ceti Several Plates +62 km		*α Herculis* 2 and 1 Plate −34 km		*ρ Persei* 2 Plates +27 km		*β Pegasi* 2 Plates +8 km		*α Orionis* 4 Plates +18 km		*α Ceti* 2 Plates −25 km		Corrected Mean, Excluding *o Ceti*	*o Ceti* Corrected for +62 km	Sidgreaves, *o Ceti*	Vogel, *o Ceti*	Vogel and Others, 3d-Type Stars, Visual
4314.3	−0.9	.....		.....		.....		.....		.....		.....	4313.4	....	....	....
4353.6	−0.9	.....		.....		.....		.....		.....		.....	4352.7	4352	....	....
4395.9	−0.9	.....		.....		.....		.....		.....		.....	4395.0	4395	....	....
4422.4	−0.9	4421.0	+0.5	.....		.....		.....		.....		4421.5	4421.5	4421	4422	....
4463.0	−0.9	.....		.....		.....		.....		.....		.....	4462.1	4460	4462	....
4505.7	−0.9	.....		.....		.....		.....		.....		.....	4504.8	4504	4506	....
4548.8	−0.9	.....		.....		.....		.....		.....		.....	4547.9	4546	4545	....
4585.0	−0.9	4583.9	+0.5	4584.8	−0.4	.....		.....		.....		4584.4	4584.1	4583	4581	4608
4626.8	−1.0	4625.7	+0.5	4626.8	−0.4	.....		.....		.....		4626.3	4625.8	4625	4622	....
4669.6	−1.0	4667.1	+0.5	4667.9	−0.4	.....		.....		.....		4667.6	4668.6	4669	4666	....
4714.2	−1.0	4713.6	+0.5	.....		.....		.....		.....		4714.1	4713.2	4714	4710	....
.....		4736.6	+0.5	.....		.....		.....		.....		4737.1	.....	4736	....	....
4760.0	−1.0	4760.0	+0.5	4761.3	−0.4	4761.8	−0.1	.....		.....		4761.0	4759.0	4758	4755	4767
4804.2	−1.0	4803.5	+0.5	4804.9	−0.4	4805.4	−0.1	.....		.....		4804.6	4803.2	4803	....	....
4842.8	−1.0	.....		.....		.....		.....		.....		.....	4841.8	4842	....	....
.....		.....		4848.8	−0.4	.....		.....		.....		4848.4	.....	....	....	....
.....		.....		.....		.....		.....		.....		.....	.....	4892	....	....
4954.1	−1.0	4953.0	+0.6	4954.7	−0.4	4954.6	−0.1	4955.4	−0.3	4954.7	+0.4	4954.5	4953.1	4951	....	4963
.....		.....		.....		.....		.....		.....		.....	.....	4998	....	....
.....		.....		.....		.....		.....		.....		.....	.....	5046	....	....
.....		.....		.....		.....		.....		.....		.....	.....	5074	....	....
.....		.....		.....		.....		.....		.....		.....	.....	5098	....	....
.....		.....		.....		.....		.....		.....		.....	.....	5135	....	....
5167.0	−1.1	5165.0	+0.6	5166.6	−0.5	5166.3	−0.1	5165.8	−0.3	5165.4	+0.4	5165.8	5165.9	5162	....	5169
.....		.....		.....		.....		.....		.....		.....	.....	5237	....	5244
5308.3	−1.1	.....		.....		.....		.....		.....		.....	5307.2	5306	....	....
5358.8	−1.1	.....		.....		.....		.....		.....		.....	5357.7	5356	....	....
.....		.....		.....		.....		.....		.....		.....	.....	5406	....	....
5439.4	−1.1	.....		.....		.....		.....		.....		.....	5438.3	....	....	....
5447.9	−1.1	5446.9	+0.6	5447.9	−0.5	5446.7	−0.2	5446.3	−0.3	5445.9	+0.5	5446.8	5446.8	5447	....	5453
5498.0	−1.1	5496.5	+0.6	5497.1	−0.5	.....		.....		.....		5496.8	5496.9	5498	....	....

Several objects measured as bands by the writer were not recorded by Sidgreaves; but they are not prominent, and we should not expect different observers with different instruments to agree as to details. Sidgreaves recorded more bands than I did to the red of λ 4954. The plates taken with Spectrograph I are underexposed in this region, and only the more prominent bands show.

A comparison of the corrected positions for *o Ceti* with the

means for the other stars shows that there is little difference in the wave-lengths, except that due to different radial velocities.

It is interesting to note what effect the application of the corrections for radial velocities has upon the agreement of the results. Residuals have been formed by subtracting the mean positions from those of each of the five stars. The sum of the squares of the residuals is reduced from 12.34 to 5.46 by the application of the corrections. This diminution of the residuals shows the validity of the assumption that the positions of the bands are the same in the five stars observed, and that the bands in each star have the same displacement, due to velocity, as have the fine, dark lines in the $H\gamma$ region.

However, the case is not clear with *o Ceti*. The correction for + 62 km changes the sum of the squares of the residuals, obtained by subtracting the mean wave-lengths of the other stars from those of the *o Ceti*, from 10.84 to 10.09. The corrections change the algebraic sum of the residuals from + 5.8 t.m. to − 5.3 t.m. This result is not surprising when it is remembered that all poorly defined bands were included in forming the residuals.

Judging by the accordance of the individual results, and allowing for any chance of personal error in the measures, it seems unlikely that the position of a sharply defined head of band should be in error by as much as one tenth-meter.

As far as I know, the identification of the bands of the third-type stars has not been accomplished. If the division at λ 5165.9 be considered as the head of a bright band, it is in close agreement with the head of the third carbon band at λ 5165.3, as measured by Kayser and Runge. The head at λ 4737.1, measured in *α Herculis*, may also correspond to their fourth carbon band, λ 4737.2. These two coincidences are the only ones I have found and we certainly need more evidence before drawing any conclusions from them.

VARIATIONS OF INTENSITY IN THE CONTINUOUS SPECTRUM.

Sidgreaves found that the continuous part of some regions of the star's spectrum changed in relative intensity as the star grew fainter. These changes have been verified in the present work.

In order to estimate the relative brightness of different parts, it was necessary to have a standard scale which should look like the spectrum of the star. An exposure on the sky of about one minute gives a strong solar spectrum not unlike that of a solar-type star. The telescope with Spectrograph I attached was pointed to the north pole at about noon on a clear day, and a series of twenty-five plates with carefully timed exposures varying from 1^s to 80^s was taken. These plates, all of the same emulsion, were developed simultaneously in the same tray, so that all would receive the same photographic treatment. The 1^s exposure gave a faint image and the 80^s an overexposed one. As on any one plate the amount of the silver deposit varies in different parts of the spectrum, the region between the iron lines at λ 4046 and λ 4072 was adopted as standard.

These plates were used for the purpose of comparing the intensities on different plates of *o Ceti* and also on different parts of the same plate. The method of using the scale plates was as follows: The plate of the star was placed film up on the table of the measuring microscope. The different scale plates were then successively placed film down on the star plate and viewed with a low power. The scale plate whose spectrum equaled the star spectrum in density was selected, and the number of seconds which it had taken to produce the image on the scale plate was called the intensity of the star spectrum. Of course, the lines in the star spectrum interfered with the estimates, but it was usually possible to find a space where there were apparently no absorption lines. In all cases the darkest patch of spectrum in the immediate vicinity on the negative was used.

Since we do not know the exact relation between time of exposure and density of the photographic image, the estimates made with the scale plates do not give us absolute intensities. There are also many errors introduced by instrumental causes. Among other sources of error is that due to the non-achromatism of the large objective. The proportions of light of different wave-lengths which enter the slit must vary with the quality of the seeing, and the care used in guiding. The star image which falls on the slit is not homogeneous, but is composed of a central

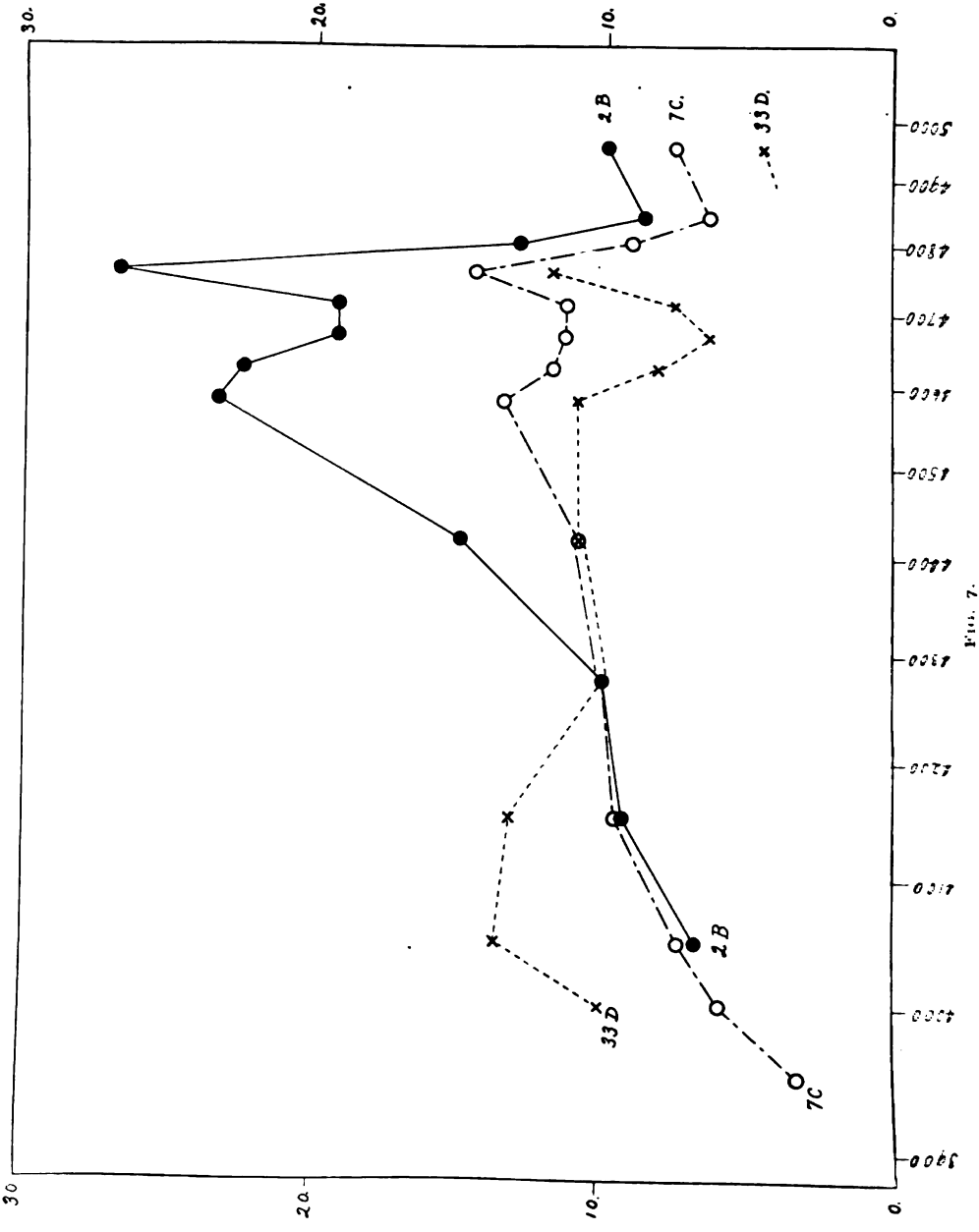

30.
20.
10.
0.
2B
7C.
33D.
5000
4900
4800
4700
4600
4500
4400
4300
4200
4100
4000
3900
2B
33D
7C

FIG. 7.

PLATE XV.

FIG. 2.—Solar Spectrum (sky).

FIG. 3.—o *Ceti*, 1902 July 6. 7 C.

FIG. 4.—o *Ceti*, 1902 August 4. 27 D.

PLATE XVI.

FIG. 5.— *o Ceti*, 1902 September 6. 33 D.

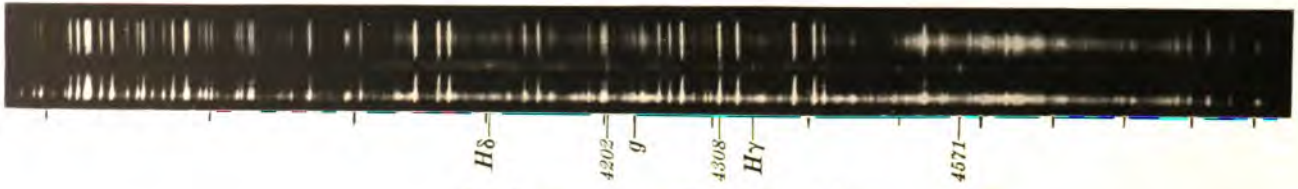


FIG. 6.— *o Ceti*, 1902 October 4. 42 A.

disk of $H\gamma$ light surrounded by concentric circles of light of longer and shorter wave-lengths.

The effects of different conditions of seeing and guiding were determined from plates taken of other stars, and while some differences were noticed, they were small compared with the changes observed in the case of *o Ceti*.

Fig. 7 gives the "intensity curves" of *o Ceti* from three plates, which were all of the same emulsion. The intensities of the same portions of the star spectrum were estimated on each plate by means of the scale. These estimates were plotted, and, in the figure, the lines join the estimates without reference to the intensities of the other parts of the spectrum. The images on the different plates being as a whole unequal in density or blackness, the portion near λ 4275 was made equal to 10, and the estimated intensities of other portions were changed to correspond. All estimates to the red of λ 4400 refer to the brighter portions, which may be heads of bright bands.

Sidgreaves's plates showed that as the star declined in brightness, the intensity of the bright portions of the spectrum between λ 4300 and λ 5000 grew less relatively to that of the region near λ 5500. My plates show a decrease in intensity of the region from λ 4300 to λ 5000 relative to the continuous spectrum from λ 4000 to λ 4300.

Figs. 3 and 5 (Plates XV and XVI) show the changes well. Plate No. 2B is not in excellent focus and is therefore not reproduced.

It should be remembered that the solar spectrum, Fig. 2 (Plate XV) is from a negative of the sky, on a lantern-slide plate whose sensibility curve is very different from that of the Crown plates. The maximum intensity of the continuous spectrum on a Crown plate is about λ 4600 for a solar-type star.

These changes in intensity have been described as changes in the continuous spectrum, but they may be simply the fading out of some bright bands.

BRIGHT LINES.

The most noticeable and interesting features in the spectrum of *o Ceti* are the bright lines. The great brilliancy of some of

the hydrogen lines, when the star was near its maximum, has been recorded by several observers.

The peculiar fact was noticed that *Hα*, *Hβ*, and *Hε* were apparently missing, while others of the hydrogen series were very bright. Photographs, which showed *Hγ* and *Hδ* as intense, gave no trace of *Hβ* and *Hε*. Sidgreaves, in 1898 and 1899, found something which from its position might be the bright *Hβ*, but he did not consider it as certain. It has been seen bright on some plates at Harvard.[1] Mr. Wright found *Hε* distinctly bright on a plate which he had taken in August 1901. Figs. 3 to 5 show additional evidence on this point. *Hβ* and *Hε* were recorded as bright lines on all the dense negatives taken with Spectrograph I. They seem to have grown stronger relatively to the other hydrogen lines and also to the continuous spectrum as the star grew faint.

An attempt was made to observe *Hα* visually, but without

TABLE XI.

Measures of Bright Lines.

λ	7C	9E	13E	14F	18F	25F	27D	28D	2235D
3751.2	..	..	..	..	..	..	—	..	..
3771.52	..	..	..	..	..	..	—	..	..
3708.76	*04*	..	03	..	..	..	02	..	..
3836.20	*04*	..	05	..	..	*07*	08	00	*03*
3853.51	..	..	01	..	..	..	00	00	..
3889.01	03	..	01	11	..	*08*	03	02	*12*
3900.36	00	..	03	..	..	*04*	03	09	*11*
3908.18	..	..	..	..	..	..	02	*03*	..
3933.15	..	..	..	..	..	..	*09*	09	..
3939.10	..	..	..	..	..	..	*07*	07	..
3968.49	..	..	..	..	..	.	*10*	10	..
3970.87	11	..	12	..	..	*04*	*01*	03	*20*
4007.74	*01*	..	01	..	..	..	..	..	..
4102.66	*03*	00	08	00	..	*03*	03	*01*	*03*
4202.91	*04*	..	06	..	..	00	*08*	*06*	14
4210.71	..	..	..	..	..	11	*20*	09	..
4234.12	..	..	*03*	..	..	02	*13*	03	10
4308.70	..	..	..	..	..	*02*	*03*	06	..
4341.33	09	*04*	10	13	..	*15*	*08*	00	*04*
4373.01	..	..	..	..	..	..	..	—	..
4376.78	..	..	..	..	..	*05*	07	*01*	..
4571.82	..	..	..	..	..	*20*	*07*	28	..
4862.34	..	..	23	..	*09*	*14*	02	*03*	..

[1] *Harvard Annals*, **28**, Part 1.

success, on the night of September 6, 1902, when the instrument was changed to adapt it for visual work. Corresponding to the bright well-defined *Ha* of the comparison spectrum there was continuous spectrum, but no bright line in the star. Mr. Wright was present and verified the observation. *Ha* should be looked for when *Mira* is again at a maximum.

Table XI gives the measures of all the bright lines which appeared, sooner or later, during the course of the work. The same scheme of residuals is used as in Table III.

Below is given a list of places which have the appearance of bright lines, but which, in the judgment of the observer, are bright spaces between absorption lines. Some of the places were measured on only one plate, but the wave-lengths are probably as accurate as those of the absorption lines.

λ 3978.7	λ 4213.0
3998.5	4427.9
4002.7	4434.5
4031.5	4512.6
4179.6	4777.6

From the residuals of Table XI the probable error of the wave-length of a bright line, determined from one plate, is found to be ±0.06 t.m. The wave-lengths of the bright lines show no more change than do those of the dark lines. The average residual for each plate has been computed and there is no sign of variable position. A comparison of these average residuals with those of the dark lines is as follows:

TABLE XII.

Date	Plate No.	Dark Lines	Bright Lines
1902, July 6	7 C	+0.02	+0.01
16	13 F	+0.06	+0.04
August 4	25 F	−0.03	−0.03
11	27 D	−0.02	−0.03
25	28 D	−0.03	+0.03
1901, August 17	2235 D	+0.02	−0.04

These average residuals are all so small that the agreement in sign for some cases is of little significance.

Table XIII gives the identification of some of the bright lines. There can be no doubt that the hydrogen series is present. Metallic lines which seem to coincide in five cases are given, but, in spite of the accuracy of the measures, we must consider the question of their identity to be still open.

TABLE XIII.

Identification of Bright Lines.

o *Ceti* λ	Tabular λ	Displacement t. m.	Substance	Authority for λ
3751.2–	3750.15	+1.0–	*Hκ*	Ames
3771.52	3770.7–	+0.82	*Hι*	Ames
3798.76	3798.0–	+0.76	*Hθ*	Ames
3836.20	3835.6–	+0.60	*Hη*	Ames
3853.51				
3889.91	3889.15	+0.76	*Hζ*	Ames
3906.36	3905.66	+0.70	*Si* 12	Rowland
3908.18				
3933.45				
3939.10				
3968.49				
3970.87	3970.18	+0.69	*Hε*	Rowland
4007.74				
4102.66	4101.89	+0.77	*Hδ*	Wright
4202.91	4202.20	+0.71	*Fe* 8	Rowland
4216.71				
4234.12				
4308.70	4308.08	+0.62	*Fe* 6	Rowland
4341.33	4340.63	+0.70	*Hγ*	Rowland
4373.61				
4376.78	4376.11	+0.67	*Fe* 6	Rowland
4571.82	4571.26	+0.56	*Mg* 5	Rowland
4862.34	4861.53	+0.81	*Hβ*	Rowland

It should be noticed that a bright line developed on each side of each of the strong dark calcium lines, *g*, H and K. Without looking at the plates, this might seem to be due to a double reversal of the calcium lines. Their appearance is not such, however. Mr. Wright also examined the plates with this in mind, and in his judgment the lines are separate bright lines, and the phenomenon is not one of double reversal.

Several attempts were made, in June and July 1902, to secure photographs of the spectrum of *Mira*, with the regular three-prism instrument. It was impossible, however, to make an exposure long enough to record the continuous spectrum. The only features recorded were *Hγ* and two other bright lines. *Hγ*

was single on all the plates. It seemed nearly monochromatic, but was a little sharper on the violet than on the red side. It had the same appearance as that found by Campbell in 1898 at about the same interval after the star's maximum. In 1898 he found that $H\gamma$ was triple from five to two weeks before maximum. Before observations of *o Ceti* were begun in 1902, it had been intended to make polariscopic tests for Zeeman effects in the bright lines; but they were found single on the first photographs and no observations for polarization were attempted. These should certainly be made when it is again possible to observe the star at maximum. Measures of the three-prism plates give the following displacements of $H\gamma$ in tenth-meters.

TABLE XIV.

Date	Plate No.	t.m.
1902, July 2	2446 F	+0.65
2	2447 A	+0.68
21	2470 D	+0.65
August 18	2505 E	+0.61
		Mean +0.65 t.m.

It is interesting to compare the measures of plates taken in 1902 with those of 1898. Campbell also observed the bright lines near λ 4308 and λ 4376. They were also measured on the high-dispersion plate of August 18, 1902. In the following scheme, under the heading "Bright Lines" is given the actual observed displacement of each bright line, corrected for the

TABLE XV.

Line	Campbell, 1898		Stebbins, 1902		
	Three Prisms		Three Prisms. Bright Lines		One Prism Dark Lines, +66km
	Bright Lines t.m.	Dark Lines, +66km t.m.	t.m.	t.m.	t.m.
Hδ 4101.89	+0.64	+0.85	.....	+0.77	+0.90
Fe? 4308.08	+0.60	+0.89	+0.61	+0.62	+0.95
Hγ 4340.63	+0.64	+0.90	+0.65	+0.70	+0.96
Fe? 4376.11	+0.61	+0.91	+0.66	+0.67	+0.96

earth's motion. Under "Dark Lines" is given an assumed displacement which corresponds to that of the absorption lines in the spectrum.

The dark lines in this region of the spectrum are apparently displaced about 0.25 tenth-meter farther to the red than are the bright lines. The results obtained with one prism are systematically larger than those obtained with three prisms. This may be partly due, in the case of $H\gamma$ and $H\delta$, to overexposure on many of the plates. Since the lines seem to shade off towards the red, greater exposure probably slightly increases the apparent wave-lengths. However, since the displacement of the other hydrogen lines, which where not overexposed, is about the same as that of $H\gamma$ and $H\delta$, this effect is probably not large. The difference between the results of 1898 and 1902 is due, no doubt, to personal errrors.

The bright lines at $\lambda\lambda$ 4308 and 4376, marked as possibly due to iron, are of peculiar interest. They were recorded in Table VI as dark lines. The appearance of each, on some of the plates, is of a bright line with an adjacent dark one on the red side. If the bright lines be due to iron, they are displaced by the same amount as the bright hydrogen lines; and if iron produces the absorption components, the displacements are equal to those of other dark lines.

A glance at the series of plates showed that there were many changes of intensity among the bright lines, both relative to each other and to the continuous spectrum. Lines not visible on the earlier plates became more intense than the hydrogen lines which were so bright near maximum.

In order to estimate the amounts of the changes, a scale was made by the following method: An occulting strip was arranged in front of the photographic plate in the three-prism Mills spectrograph in such a manner that only the line λ 4308.081 of the iron spectrum reached the plate. A two-second exposure gave just a trace of an image of this line, whereas ten minutes produced an overexposed image of greater breadth and blackness than was ever obtained of the bright $H\delta$ star line. The plate-holder was moved by small successive steps along in its cell,

and a series of carefully timed exposures, varying from one second to ten minutes, was made. At intervals the current was switched to the other side of the comparison apparatus and an image of the same line due to an exposure of five seconds was recorded on the plate. These extra images served as a rough test of the constancy of the light, and as reference points, for the scale. The exposures were all made on the same plate, and the different images received the same photographic treatment.

The bright lines of *o Ceti* were compared with this scale, and to each line was assigned as intensity, the number of seconds required to produce the equal line in the scale. Independent estimates of the same line made in this manner at different times are accordant with each other.

As the density of the star spectrum, as a whole, varied greatly on the different plates, the intensities of the same line on different plates are not comparable, unless they are referred to a common standard. If the intensity of a bright line, as estimated with the scale, be divided by the intensity of the continuous spectrum on the same plate, we get what we might call the intensity of the bright line referred to the continuous spectrum. The quotients formed in this manner will be referred to as the intensities of the bright lines. It is evident that long and short exposures of the star taken on the same night should give approximately the same intensities of the bright lines. As has already been shown, there is evidence that the intensity of the continuous spectrum varied in different parts as the star faded. The intensities of bright lines were, therefore, all referred to the portion of the continuous spectrum between λ 4102 and λ 4227.

The development of the bright lines is shown in Table XVI. It happens that the faintest bright line to which an intensity was assigned is called 1. On plate 27D the line at λ 3908.18 was estimated with the bright line scale to be of intensity 3.5. The intensity of the continuous spectrum between $H\delta$ and g was called 24. Dividing 3.5 by 24 and multiplying by 10, an arbitrary factor, the resulting intensity of the bright line is 1. On

the plate 2B the scale value of the $H\delta$ line was 250, and the continuous spectrum was of intensity 6. The resulting intensity of $H\delta$ is therefore 420. These numerical operations are crude, but they are as accurate as necessary.

In Table XVI a period (.) indicates that the image on the plate is dense enough, and that the definition is good enough to show the line as bright, had it existed when the plate was taken, but that it was not visible. A dash (—) means that the line did not appear on the plate, but that, judging from the evidence furnished by other plates, it would have been recorded with a longer exposure. Where the observer is unable to state whether the line existed or not, the space has been left blank. Plates numbered 13E, 18F, 21F, and 25F, and those of Mr. Wright, are Isochromatic, and are therefore not strictly comparable with the Crown plates.

The short exposures 3C and 4E give results fairly consistent with the long exposure 2B, taken on the same night. The plates 8D and 9E do not agree at all with the denser negative of 7C. This discordance shows that the estimates are very rough.

Another source of error is the effect of widening the slit for the long exposures. This might easily produce a progressive change in the apparent intensity of the lines relative to the continuous spectrum, but it certainly could not produce such great variation among the lines themselves.

The changes of the lines are shown graphically in Figs. 8–13. The intensities are represented by ordinates. It will be seen how the hydrogen lines, $H\gamma$ and $H\delta$, decreased in intensity as the star grew faint. In Fig. 8, Plate 2B, the intensity of $H\gamma$ has been arbitrarily reduced to accord with the evidence furnished by the other plates taken on the same night. Figs. 8–12 truthfully indicate the development of the bright lines from $H\delta$ to the red. In Fig. 13 and to the violet of $H\delta$ on the other plates the data are insufficient on account of underexposure. The line at λ 4007 was the only one which certainly disappeared during the interval covered by these six plates. The development of the line at λ 4571 was remarkable. This line, which is perhaps due to magnesium, did not appear until the star was of about

TABLE XVI.

Intensities of Bright Lines.

	Crown Plates						Isochromatic Plates				Crown Plates						Isochromatic	
Date	1902 June 27			July 6			July 16	July 22	July 29	Aug. 4	Aug. 11	Au. 25	Sept. 6	Sep 22	Oct. 4	Oct. 26	1901 Aug. 3	Aug. 17
Plate Number	2B	3C	4E	7C	8D	9E	13E	18F	21F	25F	27D	28D	33D	39D	42A	48A	2211C	2235D
Mag. of Star		3.8			4.1		4.6	5.1	5.3	5.4	5.6	6.4	7.0	7.0	7.8	8.5	—	—
Intensity of Continuous Spectrum	6	2.0	1.5	9.5	1.4	2.0	7.5	7	7	11	24	17	6.5	7	5	2.0	2.9	9
*H*κ 3751.2–	—	—	—	—	—	—	—	—	—	—	faint	—	—	—				—
*H*ι 3771.52	—	—	—	—	—	—	—	—	—	—	2	—	—	—				—
*H*θ 3798.76	—	—	—	3	—	—	3	—	—	—	2	—	—	—				—
*H*η 3836.20	9	10	—	12	—	—	13	8	4	2	6	4	4	—				6
3853.51	.	.	.	.	.	.	3	—	—	—	4	4	—	3				.
*H*ζ 3889.91	33	40	20	34	27	—	41	26	21	14	17	11	13	7	5		14	16
3900.36	4	—	—	7	—	—	6	4	—	2	5	5	6	5				4
3908.18											3	3	—	—				
3933.45											1	2	—	3				
3939.10											4	5	7	8				
3968.49											4	5	8	5				
*H*ε 3970.87	4	—	—	9	—	—	12	13	13	11	17	14	17	10				4
4007.74	6	—	—	9	—	—	—	4	.	.	.	.	.	.				.
*H*δ 4102.06	420	600	410	220	140	50	290	390	390	260	120	100	90	40	30	13	140	190
4202.01	10	—	—	9	—	—	13	14	19	21	12	21	32	31	30	40	21	17
4210.71	.	.	.	.	.	.	.	.	.	6	4	6	14	14	13			.
4234.12	.	.	.	.	.	.	8	—	7	7	4	6	11	6	12			7
4308.70	.	.	.	.	.	.	.	.	12	13	9	13	35	37	48	60		.
*H*γ 4341.33	420	520	280	130	90	34	160	240	190	170	80	70	70	30	24	13	80	60
4373.61	.	.	.	.	.	.	.	.	.	3	2	5	9	8	6			.
4376.78	.	.	.	.	.	.	.	11	8	8	6	8	15	15	12			.
4571.82	.	.	.	.	.	.	.	.	.	12	7	15	28	60	90	75		.
*H*β 4862.34	17	—	—	10	—	—	8	5	5	6	8	9	11	12	11			3

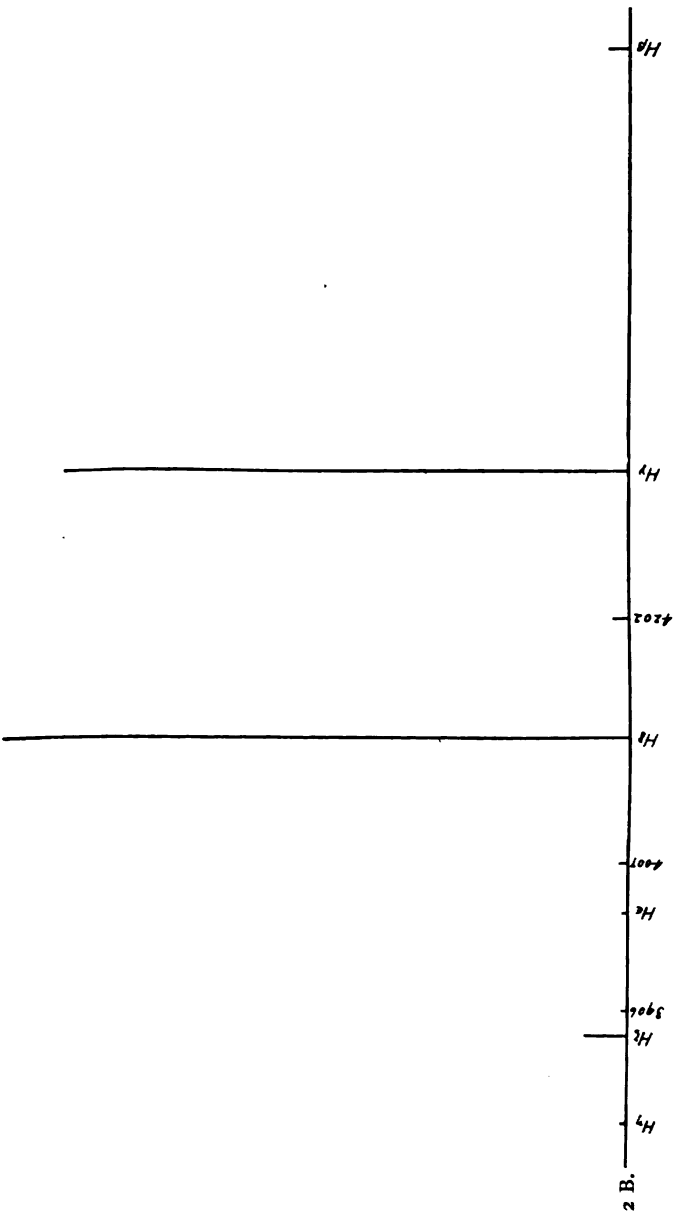


FIG. 8.

magnitude 5.4, and as the star grew fainter it became the most prominent object in the spectrum.

The images on the plates following 48A are not strong enough to give results which can be entered in Table XVI. A brief summary of what was found on the plates may be of interest:

54A. Bright lines 4202, 4308, apparently present, 4571 certain. No trace of any continuous spectrum.

58A. Bright lines 4202, 4308, 4571 all certain. Faint continuous spectrum from λ 4000 to λ 4800.

59A. Bright line 4202 not found. 4308, 4571 present. Faint continuous spectrum from λ 4000 to λ 4150, also from λ 4200 to λ 4585.

On 58A and 59A the continuous spectrum was visible in the $H\gamma$ and $H\delta$ regions, but no trace of a bright line was seen in either case. The evidence furnished by these later plates therefore strengthens the conclusion that the bright hydrogen lines disappeared at minimum. $H\gamma$ and $H\delta$ would certainly have been recorded if they had remained as intense as they were a month preceding minimum. Figs. 8–13 show that $H\gamma$ and $H\delta$ were growing fainter as the star declined, and the lines λ 4202, λ 4308, and λ 4571 were increasing in intensity as far as Plate 48A. I am not prepared to say whether the intensities of these last three lines increased or decreased after that time, but they did not change much. It is certain that no new bright lines appeared which were as prominent as those already under observation, for the plates were carefully examined. In fact the lines on Plate 54A were entirely overlooked on the first examination.

While the method of estimating intensities here presented is subject to many errors, it affords a better idea of the changes than be could given by a mere description. All the estimates with the bright-line and continuous-spectrum scales were made in such a way as to avoid personal bias. The observer did not know, when using the scales, how the final intensities would come out, and the relative changes observed in the continuous spectrum and in the bright lines are real.

Until this star has been successfully followed through all its phases, it is obviously too soon to advance any theory to account for the observed changes in its spectrum or in its brightness.

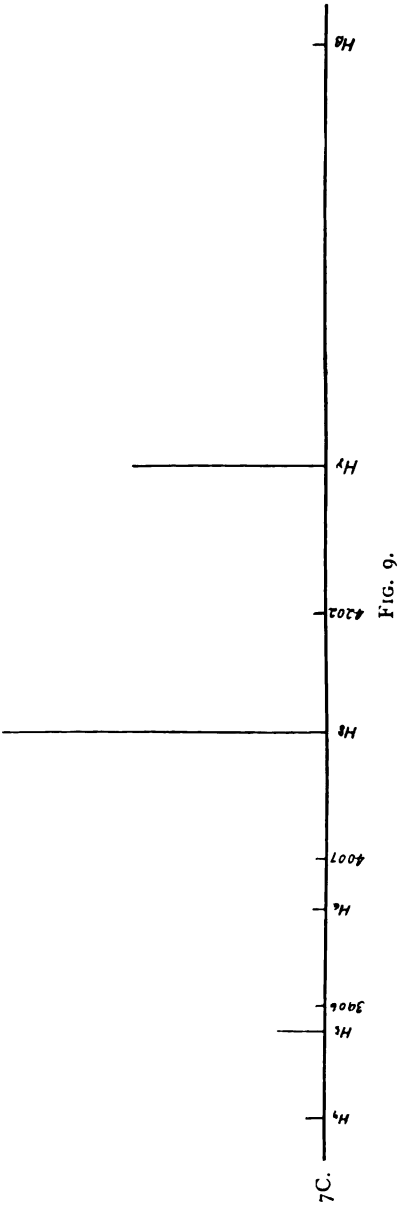

H8
Hγ
4202
H8
4007
Hε
3906
Hζ
Hη
7C.

FIG. 9.

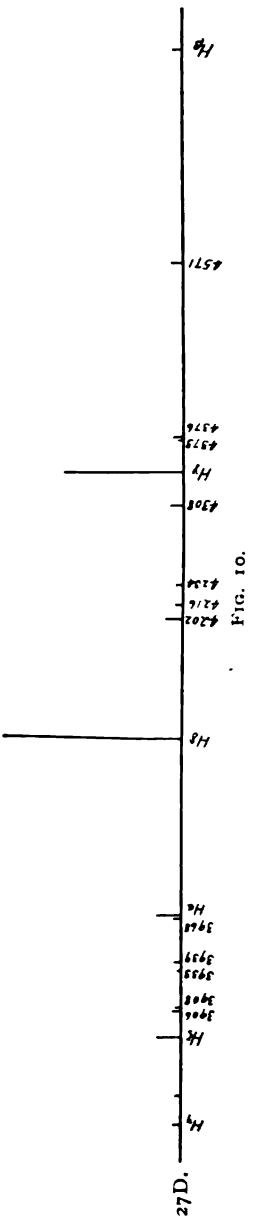

H8
4571
4576
4575
Hγ
4308
4234
4216
4202
H8
Hε
3942
3933
3929
3908
3906
Hζ
Hη
27D.

FIG. 10.

The light gathering power of the great refractor is not sufficient to follow the spectrum satisfactorily, at least not on the dispersive scale used by me. An ideal equipment for work on variables would consist of a large reflector which could be used for this purpose alone, with spectrographs of various dispersive powers arranged for use in all parts of the spectrum.

The apparent constancy of radial velocity is strong evidence that the variations in brightness are not due to the influence of a companion star, unless, indeed, the companion were of very small relative mass, were moving in a very eccentric orbit and approached very close to the primary. The displaced system of bright lines could not belong to such a companion, as their wavelengths are apparently constant. The large irregularities in the period of the light curve practically preclude the question of a binary system, though perhaps not absolutely so.

The remarkable distribution of light in the hydrogen series seems as yet impossible to explain. Miss Clerke has explained the apparent absence of $H\epsilon$ by assuming the hydrogen to be at a lower level in the star's envelope than the calcium layer, the H calcium absorption destroying the $H\epsilon$ radiations. This theory may still be true, though the existence of a faint bright $H\epsilon$ has been proven. However, $H\beta$ and $H\alpha$ are much more reduced in intensity than is $H\epsilon$; and with them there is no evidence of over lying absorption strata. It is usually expected that the bright hydrogen series will diminish in intensity from red towards the violet. This appears to hold even for new stars, in which the disturbance has been very sudden, though exception should perhaps be made for $H\alpha$ when new star spectra approach the nebular form. The supposition that in *Mira* the hydrogen series may regularly decrease in brightness from violet to red is not tenable, as the diminution from $H\gamma$ to $H\beta$ is entirely too great. Inasmuch as the bright lines are strongly shifted (to the violet with reference to the dark-line system) from their normal positions apparently by causes other than radial velocity and pressure, it seems probable that the peculiarities of the hydrogen-line intensities are due also to causes now unknown to us. The presence of the bright iron lines λ 4308 and λ 4376, the absence of others

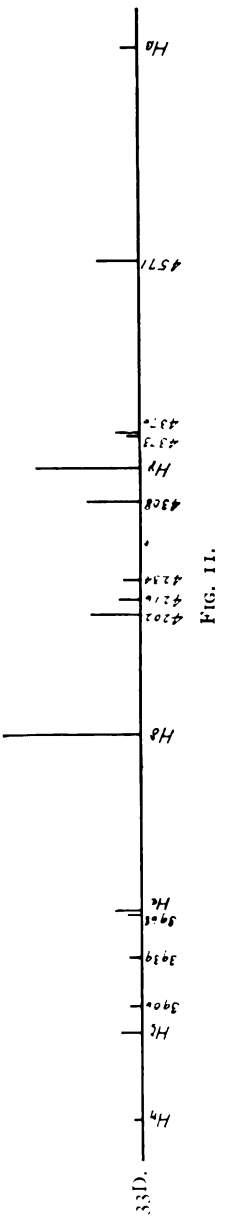


FIG. 11.

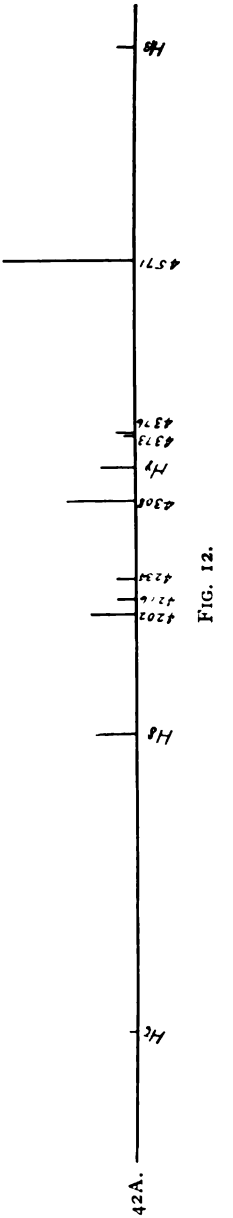


FIG. 12.

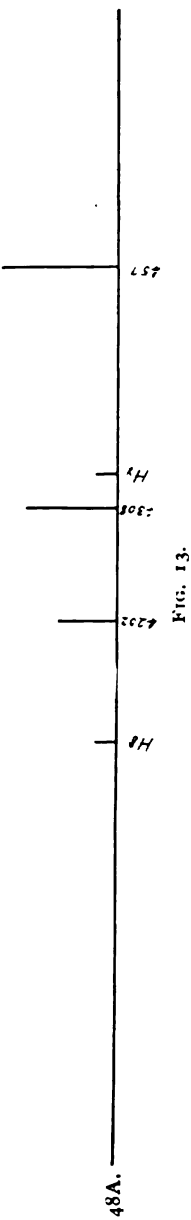


FIG. 13.

of the same element, and the diversity of structure observed by Campbell in the triple $H\gamma$ and $H\delta$ bands are perhaps due to similar unknown causes.

The great variations of relative intensity observed in the hydrogen and other bright lines, and in the continuous spectrum, show that the star's decrease in light is produced by other causes than general absorption.

Considering all the evidence, it seems reasonably certain that the star's variation in brightness is due to the action of internal forces.

I beg to acknowledge my indebtedness to Director Campbell, who provided the necessary apparatus and made valuable suggestions during the course of the work; to Messrs. Wright and Reese, for continual advice and assistance, and to Dr. H. D. Curtis, for enlarging the negatives for reproduction.

OBSERVATIONS OF THE BRIGHTNESS OF *o CETI* IN 1902–1903.

The brightness of *o Ceti* was observed by Argelander's method about once a week, beginning with June 23, 1902. At first the star's magnitude was determined with the naked eye or opera glass. After it had become too faint to be seen with an opera glass, the three-inch finder of the twelve-inch telescope was used.

In order to avoid prejudice, the observations in "grades" were made, recorded, and then laid aside, to be reduced a month or two later. When observing, I did not know the magnitude of any of the comparison stars, and in but few cases did I remember what the estimate on any previous night had been.

The magnitudes of all stars brighter than 7.00 were taken from the *Harvard Photometry*, *Harvard Annals*, **45**. For the fainter stars the adopted magnitudes are those of H. M. Parkhurst, given in **29**, 144 of the same publications. No stars that are not in one of these two sources were used for the comparisons.

The value of one "grade" has been determined from the observations to be 0.11 magnitude. However, most of the comparisons were made and reduced in such a manner that the resulting magnitude is independent of the value of a grade. For the sake of uniformity, all estimates were made in grades, and it

was tried to keep the value of one grade as nearly constant as possible. In reducing observations where *o Ceti* was compared with two stars, one brighter and one fainter, the magnitude of the variable has been taken proportionally between them. In taking the mean of several estimates, an estimate derived from two stars has been given twice the weight of an estimate based

TABLE XVII.

Reference Letters and Magnitudes Adopted for the Comparison Stars.

Letter	Mag.	*B. D.*	Name	R. A. 1900	Decl. 1900
a	3.64	−10°240	*η Ceti*	1h 03m6	−10° 43′
b	3.64	+ 2°422	*γ Ceti*	2 38.1	+ 2 49
c	3.71	−16°295	*τ Ceti*	1 39.4	−16 28
d	3.86	− 8°244	*θ Ceti*	1 19.0	− 8 42
e	3.90	− 0°406	*δ Ceti*	2 34.4	− 0 06
f	3.91	+ 2°317	*α Piscium*	1 56.9	+ 2 17
g	3.96	−11°359	*ζ Ceti*	1 46.5	−10 50
h	4.30	+ 7°388	*ξ² Ceti*	2 22.8	+ 8 01
i	4.33	+ 9°359	*μ Ceti*	2 39.5	+ 9 42
j	4.46	+ 8°273	*o Piscium*	1 40.1	+ 8 39
k	4.62	+ 4°293	*ν Piscium*	1 36.2	+ 4 59
l	4.73	+ 8°455	*λ Ceti*	2 54.4	+ 8 31
m	4.88	+ 2°290	*ξ Piscium*	1 48.4	+ 2 42
n	5.05	+ 4°418	*ν Ceti*	2 30.6	+ 5 09
p	5.22	− 4°260		1 37.7	− 4 11
q	5.54	− 1°353	75 *Ceti*	2 27.1	− 1 28
r	5.63	− 3°336	66 *Ceti*	2 07.7	− 2 52
s	5.64	− 1°322	70 *Ceti*	2 17.1	− 1 20
t	5.66	− 7°393	67 *Ceti*	2 12.0	− 6 53
u	5.73	− 1°377	84 *Ceti*	2 36.1	− 1 08
w	5.76	+ 1°410		2 12.8	+ 1 17
x	6.00	− 2°375	63 *Ceti*	2 06.5	− 2 18
y	6.30	− 3°374	71 *Ceti*	2 20.0	− 3 14
z	6.47	− 5°438		2 14.6	− 4 48
A	6.71	+ 0°370		2 10.1	+ 0 15
B	6.8–[1]	− 4°394		2 20.0	− 4 20
C	7.27	− 3°372			
D	7.33	− 3°340			
E	7.86	− 2°389			
F	8.12	− 4°379			
G	8.44	− 3°375			
H	8.65	− 4°390			
I	8.80	− 3°371			
J	8.95	− 3°363			
K	9.35	− 3°347			
L	9.44	− 3°355			
M	9.48	− 3°373			
N	9.99	− 3°362			

[1] Harvard magnitude 7.06, has 8.5 magnitude companion. Combination of the two assumed 6.8 magnitude.

upon a single comparison star. As a hundredth of a magnitude is of little significance in observations of this kind, the final means have been rounded off to the nearest tenth of a magnitude, but the light-curve, shown in Fig. 1, was drawn with the means taken to hundredths. Evidence of the Purkinje phenomenon shows in the change from opera glass to the three-inch finder.

TABLE XVIII.

Observations of the Brightness of *o Ceti*.

Astronomical Date	G. M.T.	Comparisons	Mag.	Mean	Astronomical Date	G. M.T.	Comparisons	Mag.	Mean
1902					1902				
June 23	23^h5	v = b	3.64		Aug. 25	22^h2	x 2 v 2 B	6.42	6.4
		v 2 g	3.74		Sept. 3	20.8	B 2 v 2 D	7.06	
		a 2 v	3.86				v = A	6.71	6.9
		v 2 f	3.69		6	21.6	B 2 v 2 D	7.06	
		v = d	3.86	3.8			y 6 v 4 C	6.88	7.0
27	22.8	b 2 v	3.86		9	22.5	v 2 C	7.05	7.0
		f 1 v	4.07		17	19.5	v = B	6.80	
		v 2 g	3.74				y 8 v 3 C	6.98	
		v 2 c	3.49				v 3 D	7.00	6.9
		v 4 e	3.46		27	19.8	v = C	7.27	
		d 2 v	4.08	3.8			v = D	7.33	7.3
July 2	22.9	b 2 v 2 e	3.77		Oct. 5	18.5	v = C	7.27	
		v = f	3.91				E 4 v 2 F	8.04	7.8
		g 2 v 2 i	4.14		8	19.0	E 5 v 2 F	8.05	
		d 2 v 3 m	4.27	4.0			C 2 v 3 G	7.74	7.9
6	22.8	v = e	3.90		15	19.3	v 2 G	8.22	
		f 1 v 3 m	4.15				E 5 v 2 F	8.05	8.1
		d 2 v 2 h	4.08		27	19.0	F 3 v 2 J	8.61	
		g 2 v 3 i	4.11	4.1			v 1 H	8.54	
14	22.2	e 3 v 2 m	4.40				v 2 I	8.58	8.6
		i 1 v 1 l	4.53		Nov. 11	19.8	v = J	8.95	
		v = k	4.62				v = I	8.80	
		v = j	4.46	4.5			H 2 v 4 M	8.93	8.9
18	23.1	v = k	4.62		17	17.8	I 2 v 3 M	9.07	
		v = m	4.88				J 3 v 6 L	9.11	9.1
		j 2 v 3 n	4.70	4.7	24	19.8	J 5 v 5 N	9.17	
22	23.1	v = n	5.05	5.0			v 4 L	9.00	
29	22.6	v = p	5.22				v 2 K	9.13	
		n 2 v 2 t	5.36				I 2 v 2 M	9.11	9.2
29	23.2	m 5 v 5 s	5.46	5.3	Dec 12	19.5	J 2 v 5 L	9.09	
Aug. 4	23.1	v = t	5.06				v = I	8.80	9.0
		n 2 v 2 s	5.34		24	17.8	J 2 v 2 K	9.15	
		p 3 v 2 q	5.41	5.4			v 5 L	8.80	9.1
11	22.7	v = s	5.64		1900				
		v = n	5.73						
		q 2 v 2 r	5.58	5.6	Jan. 3	16.8	J 3 v 3 K	9.15	
18	23.8	r 2 v 2 x	5.81				v = I	8.80	9.0
		v = w	5.76		29	16.2	v = J	8.95	
		s 3 v 3 y	5.97	5.9			G 2 v 2 H	8.54	8.7
25	22.2	v = y	6.30		Feb. 28	16.0	V 2 C	7.05	7.0
		v = z	6.47		March 17	15.5	m 5 v 5 q	5.21	5.2

After January 10, 1903, the observer was in residence at Berkeley, and a few observations of brightness were made there with various instruments.

REMARKS.

June 27: Fifth comparison assigned ½ weight on account of interval of 4 grades.

August 18: First of opera glass series.

September 3: Could see v with naked eye.

September 17: First with three-inch finder.

October 5: Identifications certain.

October 8 and 15: Remembered comparison with E and F on October 5.

January 29: Berkeley. One-inch telescope.

February 28: Berkeley. Two-inch telescope.

March 17: Berkeley. Opera glass.

MAY 1, 1903.

ON THE SPECTRUM OF THE AURORA.

By C. RUNGE.

IN his report on the aurora Paulsen[1] compares its spectrum with the spectrum of the bluish light near the cathode of a vacuum tube filled with oxygen and a little nitrogen and monoxide of carbon. Paulsen comes to the conclusion that there is a close connection between the two spectra:

Les tableaux ci-dessus semblent donc révéler un accord intime entre le

Krypton	Intensity	Aurora	Intensity	Oxygen Tube near Cathode	Intensity
				603.5–589.0	1
				598.0	2
587.1	8			589.0–575.0	3
				575.0–553.7	1
				569.3	1
557.0	8	558.0–554.4	10	561.8–556.8	12
556.2	4			543.3	1
				534.3	5
				528.7–513.5	1
				526.5	3
				523.0	5
				520.0 518.3	8
				508.0–506.5	1
				500.0	3
				496.0	2
				492.0	1
				485.5–480.0	5
				476.2	1
467.1	2	470	10	470.2	10
462.4	1	463	10	464.8	10
				458.8	1
		455	10	456.8	1
				450.5	2
450.2	4				
446.4	5	449	10	448.8	2
445.4	4				
440.0	1	441.5–439.0	1	441.6	10
				437.5	1
437.6	3			436.5	2
436.3	2	436.0–430.5	1	435.2–433.6	10
432.0	4				
431.9	2			431.7	2
427.4	4	428.5–425.0	10	428.5–426.0	10

[1] *Rapports présentés au congrès international de physique*, 3, 438. Paris, 1900.

spectre de l'aurore boréale et celui de la lumière qui entoure la cathode d'un tube contenant de l'oxygene et de l'azote.

The conclusion seems to me misleading. For if we compare the spectrum of the aurora with the spectrum of krypton, the coincidences are at least as striking as in the case considered by Paulsen. In the preceding table the three spectra are written side by side as far as krypton lines have been observed. The first column gives the spectrum of krypton in a vacuum tube without Leyden jar and spark-gap, according to my observations. The second column contains the spectrum of the aurora, and the third the spectrum near the cathode of an oxygen tube, both as given by Paulsen. The wave-lengths are given in $\mu\mu$.

I do not maintain that these coincidences prove the spectrum of the aurora to be that of krypton. I wish, on the contrary, to draw the inference that these comparisons of spectra have very little value as long as the wave-lengths of the auroral lines are not measured more accurately. The only auroral line which has been measured with a considerable amount of accuracy is the green line, and here the coincidence seems to be in favor of krypton, as I pointed out some years ago.[1]

In Scheiner's *Astronomical Spectroscopy*, translated, revised, and enlarged by E. B. Frost, the following determinations of the wave-length of the green line are considered the most accurate:[2]

	λ		λ
1867, Ångström	5568	1874, Huggins	5572
1872, Vogel	5572	1880, Copeland	5573
1872, Wijkander	5573	1882, Gyllenskiöld	5569
1873, Lemström	5570	1894, Campbell	5571.6

My determination of the green krypton line is:

$$\lambda = 5570.417, \text{ mean error } 0.015.$$

The broad band at λ561.8–556.8 that Paulsen has measured in the spectrum of the oxygen tube is probably the green band of carbon monoxide.

KIRCHRODE BEI HANNOVER,
October 1903.

[1] *Nature*, 59, 29, 1898, p. 320.

[2] I have reduced the wave-lengths to Rowland's scale.

TEN STARS WHOSE RADIAL VELOCITIES VARY.

By EDWIN B. FROST and WALTER S. ADAMS.

THE systematic observations of stars having spectra of the *Orion* type, which have been a part of our program during the past two years, continue to yield, as an interesting by-product, a large proportion of spectroscopic binaries. The present list brings the number so far found with the Bruce spectrograph up to twenty-three (aside from four having spectra of other types). We have at present obtained the minimum number of three good plates for only sixty-three of these *Orion* type stars, so that the ratio of those whose radial velocities are variable is at the least greater than 1 : 3 for those so far observed by us. The fact must be considered that the lines in the spectra of many of these stars are so broad and ill-defined that only rough determinations of radial velocity are possible, whence variations of small amplitude must escape detection with the present appliances. Further, the interval of time covered by our observations, particularly of the fainter stars included in our present list, is too short to permit the recognition of variations having periods longer than a few days or weeks. Finally, three observations are by no means sufficient to establish the constancy of the radial velocity of any star, even during a short interval of time. Accordingly the striking inference must be drawn that one out of every two or three stars with spectra of the *Orion* type constitutes a dual (or perhaps multiple) system.

Most of the spectrograms referred to below were obtained with the dispersion of one prism and with the triple camera lens of 607 mm focus. They are designated as series IB, followed by the current number. Since our last communication, in the June number of this JOURNAL, the outer temperature case has been altered by the insertion of two doors, so that it can be used with one prism or two or three prisms, as may be desired, and the temperature can be maintained practically as steadily as when

all doors are closed and the three prisms are used according to the original design of the spectrograph.

Reference has already been made[1] to the advantages derived from the use of low dispersion for stars with this type of spectrum. It would probably not be too much to say that fully ha' of the stars in the list below can be studied to better advantage with one prism than with three. A notable illustration of this is *ι Orionis*, in the spectrum of which, upon negatives taken with high dispersion, the lines are barely recognizable as very faint brightenings in the continuous spectrum, and are practically immeasurable. Accordingly, while it may appear that in the list of measures given below rather large differences are to be found between the values obtained by the different observers from the same plate, these differences are to be considered as due rather to the inherent character of the spectra than to the fact that insufficient scale has been employed to enable accurate measurement.

π Andromedae ($\alpha = 0^h\ 32^m$; $\delta = +33°\ 10'$; Mag. = 4.4).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 92	1903, Sept. 25	$17^h\ 31^m$	F.	− 6	+ 3	2	5	− 2
106	Oct. 10	14 7	A.	+31	+33	4	3	+32
129	Oct. 17	19 30	F.	+58	+62	5	5	+60

The lines in the spectrum of this star are rather sharper than in the case of most stars of this class. The first plate is poor, but the other two are rated in our notes as good.

ξ Cassiopeiae ($\alpha = 0^h\ 37^m$; $\delta = +49°\ 58'$; Mag. = 4.8).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 107	1903, Oct. 10	$14^h\ 50^m$	A.	−20	− 8	3	3	−14
153	Oct. 24	17 34	A.	−33	−37	5	3	−35
175	Nov. 7	14 35	F.	0	− 9	3	5	− 5

[1] "Some Miscellaneous Radial Velocity Determinations with the Bruce Spectrograph," ASTROPHYSICAL JOURNAL, 18, 67, 1903.

The lines in the spectrum of this star, although not excessively broad, are very ill-defined and diffuse, and the measures are probably subject to more uncertainty than those for any star in the list.

o Orionis ($\alpha = 5^h\ 17^m$; $\delta = -0^\circ\ 29'$; Mag. $= 4.6$).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
C 18	1903, Feb. 19	$15^h\ 59^m$	A.	....	+31	..	4	+31
C 26	Feb. 25	13 33	F.	+29	....	4	..	+29
A 413	Mar. 6	14 25	A.	....	+31	..	6	+31
IB 75	Sept. 5	21 51	A.	+18	+21	4	5	+19
87	Sept. 18	21 46	A.	+29	+34	6	6	+32
103	Sept. 26	21 55	A.	....	+27	..	6	+27
114	Oct. 10	20 34	A.	....	+32	..	3	+32
133	Oct. 17	22 47	F.	....	+27	..	7	+27
156	Oct. 24	20 05	A.	....	+33	..	5	+33

The values given by C 18 and C 26 of the above list are entitled to low weight in view of the quality of the spectra. The chief evidence of variation in the star's velocity is furnished by IB 75: the excellent character of this plate and the satisfactory agreement of the two sets of measures upon it lead us to the conclusion that the variation is real. The spectrum is very well adapted, for a star of this type, to accurate measurement, most of its lines, in particular those due to helium, being strong, narrow, and well defined.

χ *Aurigae* ($\alpha = 5^h\ 26^m$; $\delta = +32^\circ\ 8'$; Mag. $= 5.0$).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 73	1903, Sept. 5	$19^h\ 48^m$	A.	+31	+25	5	5	+28
112	Oct. 10	18 46	A.	+15	+ 9	5	6	+12
120	Oct. 16	20 16	A.	+11	+15	5	5	+13

The spectrum of this star is similar to that of *o Orionis*, although its lines are rather less sharply defined. λ 4267 is exceptionally strong.

ι *Orionis* ($\alpha = 5^h\ 30^m$; $\delta = -5^\circ\ 59'$; Mag. = 3.0).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 76	1903, Sept. 5	$22^h 29^m$	A.	+18	+23	2	4	+21
97	Sept. 25	21 58	F.	+44	+35	4	4	+40
104	Sept. 26	22 33	A.	+59	+55	2	3	+57
134	Oct. 17	23 19	F.	....	+35	..	3	+35
147	Oct. 23	23 37	F.	+42	+41	3	3	+42
157	Oct. 24	20 38	A.	....	....	..	..	....
167	Oct. 30	20 00	A.	+89	+91	2	2	+90

The spectrum of this star appears to be decidedly complex. In the case of the majority of the plates the helium lines and $H\gamma$ consist of exceedingly broad and diffuse lines upon which are superposed maxima sometimes to the number of two or three, and usually of considerable intensity. The effect of these is greatly to complicate the determinations of velocity. Sufficient evidence has not yet been obtained to determine whether these maxima are due to the other member or members of the system producing the variation in velocity, or to physical conditions in the star. The determinations of velocity given in the table are, in all cases, derived from the broad diffuse lines, although it has also been our practice to measure the position of the various maxima as well. In the case of IB 157, however, the complication is so great that it has seemed best to us to omit the value obtained from it until further study of the star's spectrum has enabled us to form more definite conclusions as to the relationships involved in the various lines.

ν *Orionis* ($\alpha = 6^h\ 2^m$; $\delta = +14^\circ\ 47'$; Mag. = 4.4).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			
A 303	1903, Jan. 22	$20^h 23^m$	F.	+82	+80	2	4	+81
IB 173	Oct. 31	23 32	F.	+20	+23	6	5	+21
187	Nov. 14	21 31	A.	+11	+13	5	5	+12

The first plate, taken with three prisms and camera A, had such broad and hazy lines that it was laid aside without measure-

ment. The second plate, however, indicated so different a radial velocity that an attempt was made to measure the first, with the above result, which is only very rough. On the second plate measures were made on the carbon line at $\lambda 4267$ and *Mg* $\lambda 4481$, in addition to $H\gamma$ and three helium lines.

18 *Aquilae* ($\alpha = 19^h\ 2^m$; $\delta = +10^\circ\ 55'$; Mag. = 5.1).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 46	1903, June 13	$18^h\ 10^m$	A.	+15	+10	3	3	+12
52	July 4	18 37	F.	−17	−25	3	3	−21
68	Sept. 5	14 12	A.	..	−28	..	4	−28
77	Sept. 12	14 28	A.	− 7	− 3	4	6	− 5

We have found the lines in this star difficult to set upon, and the difference in the values obtained by the two observers is unusually large.

2 *Lacertae* ($\alpha = 22^h\ 17^m$; $\delta = +46^\circ\ 2'$; Mag. = 4.8).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 64	1903, Aug. 8	$17^h\ 56^m$	F.	− 82	− 89	4	4	− 86
71	Sept. 5	17 43	A.	− 19	− 13	4	5	− 16
71	Second Component			−199	−171	3	3	−185
88	1903, Sept. 25	13 27	F.	+ 1	+ 1	4	4	+ 1

The spectrum on the first plate is much fainter than on the other two, but on re-examination it gave indications of the presence of lines due to a second luminous component, which were measurable on the second plate. These additional lines cannot be seen on Plate 88.

6 *Lacertae* ($\alpha = 22^h\ 26^m$; $\delta = +42^\circ\ 37'$; Mag. = 4.6).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 50	1903, June 26	$19^h\ 45^m$	F.	−23	−24	4	3	−24
65	Aug. 8	18 47	F.	−14	−14	4	4	−14
83	Sept. 18	18 6	A.	− 6	0	3	4	− 3

In this spectrum the silicon lines appear in addition to those of hydrogen and helium, but settings could be made on only one of these. All the plates are rated as good.

1 Hev. *Cassiopeiae* ($\alpha = 23^h\ 25^m$; $\delta = +58°\ 0'$; Mag. = 4.8).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 151	1903, Oct. 24	$15^h\ 48^m$	A.	−58	−75	5	5	−66
168	Oct. 30	21 07	A.	..	−60	..	3	−60
169	Oct. 31	15 27	F.	−75	−65	4	4	−70
177	Nov. 7	16 52	F.	+ 3	− 6	3	3	− 2

Plate 168 was obtained through clouds and is underexposed. The lines in the spectrum are very broad and diffuse, and are perhaps complicated by maxima.

AN *ORION* STAR WITH A GREAT RADIAL VELOCITY.

Five spectrograms have yielded the following values for the velocity of the star:

ξ *Persei* ($\alpha = 3^h\ 53^m$; $\delta = +35°\ 30'$; Mag. = 4.1).

Plate	Date	G. M. T.	Taken by	Measured by	No. of Lines	Velocity
						km
IB 93	1903, Sept. 25	$18^h\ 27^m$	F.	A.	2	+88
101	Sept. 26	20 38	A.	A.	3	89
119	Oct. 16	19 20	A.	A.	2	80
141	Oct. 23	18 41	F.	A.	2	80
160	Oct. 24	23 17	A.	A.	2	88
				Mean		+85

The spectrum of this star is excessively difficult of accurate measurement, owing to the breadth, and still more to the extremely ill-defined character of its lines. Consequently we do not regard the above range of 9 km in the measures as a real variation. Observations covering a longer interval may, however, show the star to be a spectroscopic binary, and, in view of the very low radial velocities which seem in general to be characteristic of stars having the *Orion* type of spectrum, a result of this nature is, in fact, rather to be expected.

BRIGHT-LINE SPECTRA.

Recent plates which we have obtained of the following stars of the *Orion* class show them to contain bright lines:

c Persei,	$\alpha = 4^h\ 1^m$;	$\delta = +47°\ 27'$;	Mag. = 4.3.	3 plates by F.
25 *Orionis*,	5 20	+1 45	4.6.	2 plates by F. and A.
β Piscium,	22 59	+3 17	4.6.	4 plates by F.

As is characteristic of the stars with this peculiar variety of the *Orion* type spectrum, the hydrogen lines have a double bright component, which is superposed about centrally on the broader dark line or band. Bright *Hβ* is especially conspicuous on the plates, partly from its intrinsic intensity, and partly because of the weakness of the continuous spectrum at that region on our plates. Bright *Hδ* is only faintly visible on the broad absorption line, following the usual decline in intensity of such bright lines as the violet is approached. The helium lines at λ4388 and 4472 and *Mg* 4481 are dark, but are dim and diffuse in each star. The separation of the components of the bright hydrogen lines is least for *β Piscium*, and is very wide for 25 *Orionis*.

Eight plates of *c Persei* were used in the *Draper Catalogue*, and the spectrum was classed as A. Five each were included for 25 *Orionis* and *β Piscium*, which were respectively assigned to classes B and A. The exposure times were probably not suited to bring out the peculiarities of the spectra. These stars were not included among those studied by Miss Maury and by Miss Cannon in Vol. 28 of the *Harvard Annals*.

The determination of the true radial velocity is very difficult for such spectra, but, as far as we have yet measured the plates, we have not obtained certain evidence of a variation of the radial velocities of these three stars.

YERKES OBSERVATORY,
November 15, 1903.

FURTHER OBSERVATIONS ON THE SPECTRUM OF THE SPONTANEOUS LUMINOUS RADIATION OF RADIUM AT ORDINARY TEMPERATURES.[1]

By Sir William Huggins and Lady Huggins.

In the plate accompanying our paper on the spectrum of the glow of radium bromide,[2] at least seven lines are seen to agree, both in position and in intensity, with corresponding lines in the band spectrum of nitrogen. We called attention to other lines, of which some traces may be detected on the plate, and we suggested that with a longer exposure a more complete spectrum would be obtained. One strong line in the radium bromide glow spectrum, about λ 3914, has no similar line corresponding to it in the band spectrum of nitrogen as given on the plate.

We have since taken photographs, with longer exposures, of two specimens of radium bromide, one prepared by Buchler & Co., and the other received from the Société Centrale de Produits Chimiques. In these photographs lines only faintly glimpsed in our earlier photographs can be seen distinctly. A photograph taken of the French radium bromide with an exposure of 216 hours is reproduced on the accompanying plate.

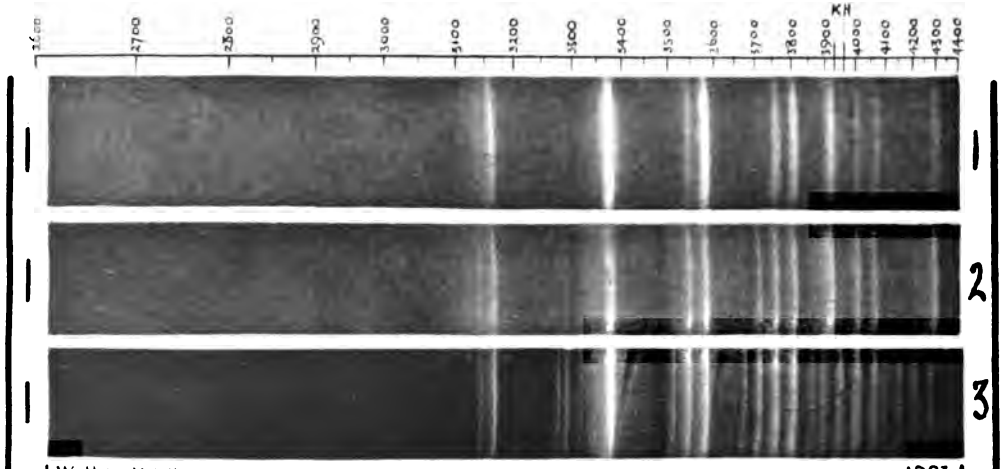
The coincidence of the spectrum with the band spectrum of nitrogen is shown to be even more complete by the presence of a faint trace of the next more refrangible band, beginning at λ 2976.7. In addition, some of the fainter single lines of the nitrogen spectrum now come out in the radium bromide spectrum.

At the same time that the coincidence down to minuter details with the nitrogen band spectrum is brought out, the strong outstanding line, about λ 3914, is now seen to be accompanied by a second, but less intense, outstanding line at about λ 4280; neither of which is present in the ordinary band spectrum of nitrogen, which was the one reproduced on the plate of our first paper.

[1] From advance proofs of a paper communicated to the Royal Society, October 29, 1903.

[2] Astrophysical Journal, 18, 151, 1903; *Proc. R. S.*, 72, 196, 1903.

Spectra of Radium Bromide and of Nitrogen



1 Radium Bromide : Glow Spectrum.
2 Nitrogen : Negative Band Spectrum.
3 Nitrogen : Positive Band Spectrum.

This nitrogen band spectrum is the one distinguished by Deslandres as that of the positive pole, but it appears at all parts of a vacuum tube, and is also produced when a suitable induction coil discharge, without capacity, is taken across air at the ordinary density. The nitrogen spectrum that was measured by Ames was taken by using an end-on vacuum tube closed with a quartz plate; in his list no lines are given at the places of the two outstanding lines in the glow spectrum.

When, however, the spectrum is taken of the aureole about the negative pole of a vacuum tube containing a residuum of atmospheric air, the ordinary, or positive-pole spectrum becomes enriched by a new spectrum of bands; and in this additional spectrum the heads of the two strongest bands in the photographic region, occur at the positions of the two outstanding lines of the radium glow spectrum.[1] On the plate are given, below the more complete radium spectrum now obtained, the ordinary band spectrum of nitrogen, and also the same spectrum enriched with the bands peculiar to the aureole of the negative pole. This latter spectrum corresponds to that of the radium glow. The peculiar conditions, whatever they may be, which determine the presence of these additional negative-pole bands must find their counterpart in the nitrogen molecules when under stimulation by the radium bromide. The additional bands which show themselves in the spectrum of nitrogen when taken from the glow at the negative pole of a vacuum tube are usually believed to be associated with the stimulation of the very rapidly moving corpuscles of the cathode stream. Accordingly the presence of these negative-pole bands in the spectrum of nitrogen when excited by radium naturally suggests whether the β rays, which are analogous to the cathode corpuscles, may not

[1] DESLANDRES's measures, reduced to Rowland's scale, of the heads of these two bands are λ 3914.4 and 4279.6 (*Thèses*, 1888, GAUTHIER-VILLARS, and *Comptes Rendus*, **101**, 1256). ÅNGSTROM and THALÉN give 4281.6 for the less refrangible band (*Nova Acta Upsal.* (3), **9**, 1875). HASSELBERG'S measure for the head of the less refrangible band is 4378.6 (*Mem. de l'Acad. St. Petersb.*, **32**, No. 15). PERCIVAL LEWIS on "Some New Fluorescence and Afterglow Phenomena in Vacuum Tubes Containing Nitrogen" (ASTROPHYSICAL JOURNAL, **12**, 8) found fluorescent nitrogen to give a band spectrum; and, in some conditions of the fluorescence, the most intense bands were those of wave-lengths 3576.9 and 3371.2.

be mainly operative in exciting the radium glow. On this surmise it would be reasonable to expect some little extension of the glow outside the radium itself. We are unable to detect any halo of luminosity outside the limit of the solid radium bromide; the glow appears to end with sudden abruptness at the boundary surface of the radium. It may be that it is only at molecular distances, and at the moment of their formation, that the rays can excite the nitrogen molecules.

As the glow spectrum is produced by the influence of the radium on nitrogen at the atmospheric pressure, it seemed to be of interest to find out whether the negative-pole spectrum could be obtained in air at the ordinary pressure. It has already been stated that when a suitable discharge of an induction coil, without capacity in the circuit, is taken between electrodes in air, the ordinary band spectrum of nitrogen appears. Separate photographs, therefore, were taken of the parts of the discharge in the close neighborhood of the two electrodes, which were about three-eighths of an inch apart. The bands peculiar to the negative-pole of a vacuum tube were found upon the plate taken of the negative electrode.

As the radium glow consists of light from nitrogen molecules stimulated into luminosity by the presence of the more active radium molecules, it was reasonable to suppose that the bromine molecules, chemically associated with the latter, might also be sufficiently stimulated to reveal their presence by the lines in the spectrum, peculiar to them. Photographs were accordingly taken of the poles of a vacuum tube containing traces of atmospheric air together with bromine vapor. The band spectrum of nitrogen appeared alone upon the plates when no capacity was introduced; but with the intercalation of a jar, the lines of bromine came out in the photopraphs, in addition to the lines of air. The experiment was then repeated at atmospheric pressure by enclosing platinum electrodes in a glass bulb communicating with the atmosphere by a narrow tube. Photographs of the coil discharge taken between them revealed the ordinary band spectrum of nitrogen. A few drops of bromine were then introduced into the bulb, filling it with bromine vapor. Photographs were again

taken of the discharge in the air now heavily laden with bromine, but the spectrum remained precisely the same as before the bromine was introduced, namely, that of nitrogen only.

We find in this experiment possibly a sufficient reason for the absence of any of the lines of bromine in the glow spectrum: it may be that stimulation from the active radium molecules affects preferentially the nitrogen molecule, so that this molecule can be shaken into luminosity by a stimulation which is insufficient to excite the bromine molecule to a comparable extent.

The experiment then suggested itself whether, under similar conditions of discharge, radium itself, when placed upon the electrodes, would be able to show its presence by its characteristic lines in the spectrum of the discharge taken between them. The result was negative; as in the case of bromine, no lines other than those of nitrogen appearing upon the plate. A small jar was then put into the circuit and another photograph taken, when the complete spectrum of radium came out strongly, but without the band spectrum of nitrogen.

If, as suggested by Rutherford, the α rays are connected with helium, the experiment seemed worth making of taking a photograph of the spectrum arising from their bombardment upon a zinc sulphide screen. It seemed possible, though not very probable, that the encounters of these bodies, at the enormous speed at which they travel, with the molecules of air, and their final collision with the screen, might on that hypothesis give rise to some of the radiations peculiar to helium and so produce its spectrum on the plate. Fortunately the strong continuous spectrum due to the fluorescence of the screen ends abruptly in the violet a little before the place, at λ 3889, of the strongest line of helium in the photographic region, and so leaves the spectrum quite free for the detection of this line, even if it were only faintly present. The result of the experiment, so far as concerns helium, was negative; which must not of course be interpreted as excluding the presence of helium, but only as showing that, if present, the conditions are not favorable to the appearance of its spectrum.[1]

[1] M. HENRI BECQUEREL has quite recently investigated the scintillation observed on a phosphorescent screen when excited by radium. He comes to the conclusion:

On the first photograph that was taken, the two strongest lines of the nitrogen band spectrum were faintly seen, but a photograph with a new screen and a longer exposure showed no trace of the nitrogen bands. In the first case it might be that some very minute particles of radium bromide had attached themselves to the screen, and by their independent glow had given rise to the lines of nitrogen which were on the photographic plate.

About one centigram of French radium bromide, which was in the form of small particles, was put into a very small glass tube scarcely larger than was necessary to contain it. The tube was securely closed and left for two months. As the α rays being unable to escape, would probably occupy the interstices between the radium bromide particles, it seemed desirable to examine whether as helium, or still in some precedent condition, they would show their presence in the glow spectrum. The tube was exposed, immediately in front of the slit, for 168 hours. The spectrum shows a strong continuous spectrum from the fluorescence of the glass, and faintly the bands of nitrogen, but no other lines with certainty. We intend to photograph again the spectrum of the glow from this tube, after a longer time has passed for an accumulation of the α rays, and of the gas-like emanation.

When the radium bromide is covered with a plate of quartz, the continuous spectrum, due to the fluorescence of the quartz, is not only strong, but extends a long way into the ultra-violet. It can be traced on the photograph as far as $\lambda 2500$.

After a few hours the quartz darkens under the action of the radium bromide, the brown stain extending through the complete substance of a plate one-tenth of an inch in thickness. The stain is due probably to the reduction of silicon.

Experiments were made in the hope of throwing light upon the shift found in the photograph of the radium glow spectrum,

(1) "Ce sont les rayons α qui provoquent la phosphorescence scintillante;" (2) "Ces faits établissent sinon une démonstration, du moins une grande présomption en faveur de l'hypothèse qui attribuerait la scintillation à des clivages provoqués irrégulièrement sur l'écran cristallin par l'action continue plus ou moins prolongée des rayons α." *Comptes Rendus*, 137, 633, 634; October 27, 1903.

reproduced on the plate of our first paper. As subsequent photographs of this spectrum were entirely free from any trace of shift, the shift found on the first plate must have been accidental. Repeated photographs, taken with the spectroscope in different positions, failed to show the smallest trace of shift from flexure. The only suggestion we can make in explanation is that the piece of solid radium bromide accidentally shifted in its cell, so as no longer to be directly under the slit, and in consequence the collimator lens was not wholly filled with light.

The results of the experiments described in this paper would appear to show generally, if analogy with electric stimulation may be assumed, that the radium stimulation, whether we take the operative cause to lie in the β rays, or in the encounters of nitrogen molecules with the active molecules of radium—by which, for the first time, a spectrum of bright bands in the ultra-violet region has been obtained at ordinary temperatures, and without the intervention of an electric discharge—from the very circumstance of its being of such a nature as to give rise to the band spectrum of nitrogen, is not of a kind which can elicit from either the molecules of bromine, or of radium their characteristic line spectra.

The question suggests itself whether or not the same inability may hold in respect of the helium molecule, which is easily stimulated by an electric discharge; we have not as yet made experiments on this point.

Reviews

Lehrbuch der Physik. Von O. D. Chwolson. Bd. I, pp. 791. Translated from the Russian into the German by H. Pflaum. Braunschweig: Vieweg, 1902.

The appearance of a new compendium of physics, filling no fewer than four octavo volumes, is certain to make one curious as to the features which distinguish it from other treatments of a similar nature. And since in this particular case, alas! no similar treatment exists in the English language, we are driven to the larger German treatises, such as Wüllner and Müller-Pouillet, for a basis of comparison. For, while the *Text-book of Physics* by Poynting and Thomson may well be reckoned in this class, and while, in a very true sense, it will cover the entire field of physics, it is nevertheless largely devoted to the discussion of discrete problems, and begins by omitting the entire subject of dynamics.

As compared with Wüllner's *Lehrbuch*, which may perhaps be taken as a typical, connected discussion of the whole subject, the most striking contrasts are perhaps the following:

1. Chwolson gives a much more elaborate philosophical introduction, devoting more than fifty pages to questions such as the classification of the sciences, the distinction between theoretical, mathematical, and experimental physics, the characteristics of a good hypothesis, etc. The author has evidently a keen appreciation of the unity of his science, for this same philosophical vein runs through the entire volume. To illustrate, the spectrometer, essentially an instrument for measuring angles, and usually handled under the head of optics, here finds discussion in a chapter on "Measuring Instruments," where it logically belongs.

2. As compared with Wüllner or with Violle's *Cours de Physique*, the volume under review contains much more elaborate references to the literature of each topic. These valuable bibliographies are placed at the end of each chapter, and are there grouped under the headings of the sections, so that really each section is provided with its own bibliography. The amount of Russian work here cited will be surprisingly large to those who still imagine that the energies of this

great nation are exclusively devoted to the arts of war and to the care of Siberian exiles.

3. The mathematical discussion is the very simplest — too simple, we venture to think. Differential equations, even the most elementary, are studiously avoided. In America, certainly no student would be reading so advanced a treatment of general physics without the preparation to enjoy more elegant mathematical methods. As illustration may be cited the derivation — or rather the lack of derivation — of the equations for damped vibrations, p. 156.

4. Most American readers will probably agree that some of the chapters are too full. For example, the ninth chapter devotes twelve octavo pages to the "dimensions" of physical quantities. Is it not easily possible to put all the essentials of this subject into one quarter of this space?

Your reviewer is not alone in thinking that a great service would be rendered English-speaking students by any competent physicist who will give them a treatise on general physics, along the lines of Chwolson — a treatise which will fill the great gap lying between the college and university text-books, on one hand, and the great compendium of Winkelmann, on the other hand.

H. C.

A Popular History of Astronomy During the Nineteenth Century. Fourth edition. By AGNES M. CLERKE. London: A. & C. Black, 1902. Pp. 489.

WITH the fourth edition Miss Clerke brings her well-known history down to the summer of 1901. As with previous new editions, fresh material has been wisely chosen and skilfully woven into the text, so that neither its value as a book of reference nor its literary charm has been impaired. With great wealth of material to be sifted, it is really remarkable that so little that is questionable has been incorporated and that so few researches of significance have been overlooked. Perhaps the most important omission is that of Nichols's investigations on the heat-radiation of the stars, which were made during the summers of 1898 and 1900. One has the impression also that German astronomers do not receive their full share of attention. At any rate, the great preponderance of English over German references seems disproportionate to the relative merits of German and English investigators. Of a short list of errors, typographical and otherwise, in the third edition, called

to the reviewer's attention by Dr. Schlesinger, the greater part have been corrected in the fourth edition. Among those that remain might be mentioned that of the parallax of 61 *Cygni*, which is still given as between $0''.43$ and $0''.47$, although more recent determinations by Davis, Kapteyn, Wilsing, and others make it practically certain that the average parallax of the two components is less than $0''.40$. There is still an error in correcting the series contained in footnote 5 on page 71 — the first member being $1\frac{1}{2}$, instead of $\frac{1}{2}$ as given.

The illustrations are much improved in quality and are supplemented by a half-tone of the comet of May 1901, reproduced from a photograph taken at the Cape of Good Hope. To Table IV has been added a list of radial and tangential velocities of selected stars.

S. B. BARRETT.

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